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Forecasting FPM Transfers in Small Municipalities: a Comparative Time Series Approach

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ABSTRACT

The Municipalities Participation Fund (FPM) is the main revenue source for small Brazilian municipalities. This study compares the predictive capacity of three time series approaches, a seasonal naïve model, Exponential Smoothing (ETS), and SARIMA, applied to the monthly FPM transfers to the municipality of Realeza-PR, from January 1997 to May 2026, confronting the best projection with the National Treasury's official estimate for 2026. A series of 353 observations was used, with automatic parameter selection under the AICc criterion. The ETS(M,A,M) model showed the best out-of-sample performance (MAPE of 7.00%; MASE of 1.695), outperforming SARIMA and the seasonal naïve model. However, the Ljung-Box test and the cumulative periodogram of residuals indicated remaining autocorrelation. It is concluded that ETS outperforms SARIMA in forecasting the FPM, but no model, without treating structural breaks and outliers, is fully reliable.

Keywords: FPM; Time Series; Exponential Smoothing; SARIMA; Budget Planning.
JEL Code: C53; R51; C22.

SUMMARY

INTRODUCTION

The Municipal Participation Fund (FPM) is the main source of revenue for most small Brazilian municipalities. Composed of a portion of federal revenue from the Income Tax (IR) and the Tax on Industrialized Products (IPI), the fund redistributes federal resources to subnational entities according to population and income criteria, functioning as a central mechanism for fiscal equalization within the Brazilian federal system (National Confederation of Municipalities, 2014). For municipalities with a population of around 20,000, such as Realeza (PR), the FPM often accounts for more than half of current revenues, making its projection a critical step in annual budget planning (Realeza. City Hall, 2026) .

The preparation of the Annual Budget Law (LOA) requires municipal administrators to estimate, months in advance, the amount of transfers the municipality will receive in the following fiscal year. In practice, this estimate is made under structural uncertainty, as FPM transfers depend on federal tax revenue, which is sensitive to the economic cycle, tax policy, and unforeseeable events (National Confederation of Municipalities, 2014). The National Treasury publishes an official annual estimate of FPM growth to support municipal planning. The lack of accessible technical tools to validate or challenge this estimate leaves municipal managers in small municipalities dependent on information from the federal government.

Given this scenario, time-series models have been applied to forecast fiscal transfers and municipal revenues in Brazil. Brito et al. (2021) used time-series models to project FPM transfers at the state level, Silva et al. (2017) used the ARIMA model family to forecast ICMS tax revenue in the state of Pará. Rodrigues and Junior (2018) explored econometric approaches for forecasting municipal revenues. These studies demonstrate the potential of quantitative tools for fiscal planning. However, few studies combine, within a single framework, predictive validation using out-of-sample data, a focus on small municipalities with high dependence on the FPM, and a direct comparison between statistical projections and the National Treasury's official estimates. This methodological combination is necessary to assess whether a predictive model effectively adds useful information to the local manager's decision-making process or whether, when applied directly and without additional treatment of the time series, it proves insufficient.

Given this gap, this study aims to compare the predictive power of three time-series forecasting approaches: a naive seasonal reference model, Exponential Smoothing (the ETS family, which includes the *Holt-Winters* method), and the Seasonal Autoregressive Integrated Moving

Average (SARIMA) model, applied to the monthly FPM transfers to the municipality of Realeza (PR), to determine which has the greatest ability to forecast revenue, and to compare the results with the government's official forecast.

METHODOLOGY

The data used in this study refer to the monthly transfers from the Municipal Participation Fund (FPM) to the municipality of Realeza (PR), covering the period from January 1997 to May 2026, totaling 353 monthly observations. The information was obtained from the National Treasury's "Transparent Treasury" portal, using the feature that provides details on constitutional transfers made (Brazil. National Treasury Secretariat, 2026b) . For the purpose of comparing projections, the official transfer estimate released by the government was used (Brazil. National Treasury Secretariat, 2026a).

A specific adjustment procedure was necessary due to a change in the portal's reporting format starting in 2025. Historically, the FPM consists of the regular transfer, set at 22.5% of federal revenue from the Income Tax (IR) and the Tax on Industrialized Products (IPI), plus three additional transfers of 1% each, established by Constitutional Amendments No. 55/2007 (Brazil, 2007) (December), No. 84/2014 (Brazil, 2014) (July), and No. 112/2021 (Brazil, 2021) (September). Until December 2024, these additional payments were accounted for as part of the first ten-day period of the regular FPM. Starting in January 2025, the portal began recording them as a separate line item, labeled "FPM 1%." To ensure the historical comparability of the series, the monthly aggregate combines the "FPM" and "FPM 1%" lines within each month. Without this procedure, the months of July, September, and December 2025 and beyond would show artificially reduced values, compromising the seasonal pattern of the series.

For the purpose of comparing projections, the official transfer estimate released by the National Treasury was used, which projects nominal growth of 8.97% for Realeza's FPM in 2026, corresponding to an absolute value of R$ 38,118,772.56 (Brazil. National Treasury Secretariat, 2026a) .

To enable an assessment of real purchasing power over the three decades analyzed, all nominal values were deflated using the Broad National Consumer Price Index (IPCA), calculated by the IBGE, using the *deflateBR* package, with April 2026 as the reference date (Meireles, 2018) . The real value is given by:

$$VR_t = VN_t \times \frac{IPCA_{\frac{abril}{2026}}}{IPCA_t} \#(1)$$

where VR_t is the deflated value, VN_t is the nominal value, and $IPCA_t$ is the index number for month t.

The deflated series serves an exclusively interpretive and descriptive purpose; it is not used in the predictive model or for comparison with the official estimate from the National Treasury, which publishes its projections in nominal terms—the unit in which the municipal budget is executed.

The sample was divided into two subsets. The training set covers the period from January 1997 to December 2024, totaling 336 observations, and is used to estimate the model parameters. The test set covers the period from January 2025 to May 2026, with 17 observations, and is used to evaluate the predictive capacity of the models using data not observed during training. This out-of-sample validation strategy is the procedure described by Hyndman and Athanasopoulos (2018) for time series forecasting and ensures that error metrics reflect the model's actual performance in real-world situations, where the accuracy of forecasts can only be determined by considering the model's performance on new data that was not used during its fitting.

Figure 1 - Visualization of training and test data.

Source: Adapted from Hyndman and Athanasopoulos (2018).

The series was converted to a monthly time series ($m = 12$), starting in January 1997, and subjected to multiplicative decomposition, which is suitable for economic series in which the amplitude of seasonal variations increases proportionally to the level of the series over time, as described in Hyndman and Athanasopoulos (2018) . As shown in Equation 2.

$$Y_t = T_t \times S_t \times R_t \# (2)$$

Where T_t, S_t and R_t are, respectively, the trend, seasonal, and irregular components in period t.

To assess whether the adoption of a statistical model adds useful information to fiscal planning, three forecasting approaches were estimated and compared using the same nominal time series: the naive seasonal model, Exponential Smoothing (ETS family), and the SARIMA model.

The first approach is the naive seasonal model, which uses the observed value for the same month of the previous year as the forecast for each month.

$$\hat{y}(t) = y(t - m)\#(3)$$

Since it has no estimated parameters, it serves as the baseline that any more sophisticated model must outperform to justify its adoption; it is also the reference model used in the calculation of the MASE (Hyndman; Koehler, 2006).

The second approach is Exponential Smoothing, in which forecasts are derived from weighted averages of past observations, with weights that decay exponentially as the observations become more distant in time (Hyndman; Athanasopoulos, 2018) . For series with trend and seasonality, the multiplicative *Holt-Winters* method decomposes the forecast into three equations: level (ℓ), trend (b), and seasonality (s):

Suavização Exponencial # (4)

$$l_t = \alpha\cdot\left[\frac{y_t}{s_{t-m}}\right] + (1-\alpha)\cdot\left[l_{t-1} + b_{t-1}\right] b_t = \beta\cdot[l_t - l_{t-1}] + (1-\beta)\cdot b_{t-1} s_t = \gamma\cdot\left[\frac{y_t}{(l_{t-1} + b_{t-1})}\right] + (1-\gamma)\cdot s_{t-m} \hat{y}_{t+h|t} = \left[l_t + h\cdot b_t\right]\cdot s_{t+h-m(k+1)}$$

where α, β and γ, belonging to the interval [0,1], are the smoothing parameters for the level, trend, and seasonality, respectively. The ETS (Error, Trend, Seasonal) state-space formulation was adopted, in which the specific form of the model—including the additive or multiplicative nature of the error and seasonality and the possible attenuation of the trend—is automatically selected by minimizing the AICc criterion using the *ets()* function from the *forecast* package (Hyndman; Athanasopoulos, 2021). The multiplicative structure of the error and seasonality directly addresses the heteroscedasticity observed in the series. The third approach, the SARIMA model, is described below.

The Seasonal Autoregressive Integrated Moving Average $(\text{SARIMA})_m$, is an extension of the ARIMA model for series with seasonality, as per the notation presented by Hyndman and Athanasopoulos (2018) :

$$ARIMA \quad \underbrace{(p,d,q)}_{\text{Parte não sazonal do modelo}} \quad \underbrace{(P,D,Q)_m}_{\text{Parte sazonal do modelo}}$$

where m is the number of observations per year. We use uppercase letters for the seasonal components of the model and lowercase letters for the non-seasonal components.

For the ARIMA method, the data must exhibit stationarity (mean, variance, and autocovariance invariant over time). This condition was assessed using the Augmented Dickey-Fuller (ADF) test, with the null hypothesis being the presence of a unit root (Dickey; Fuller, 1979) , implemented using the *urca* package(Pfaff, 2006) , with a specification including a deterministic trend and lag selection based on the AIC criterion.

The ADF provides evidence regarding the behavior of the time series. The effective determination of the non-seasonal differentiation order *d* is performed internally by the automatic selection algorithm via the *auto.arima* function of the *forecast* package (Hyndman; Athanasopoulos, 2018), which selects the orders of the components by minimizing the AICc criterion and applies the KPSS test (Kwiatkowski et al., 1992) , whose null hypothesis is stationarity, unlike the ADF.

The seasonal part of the model consists of terms similar to the non-seasonal components of the model, but involving backward shifts of the seasonal period

Modelo SARIMA # (5)

$$\left(1-\phi_1 B-\cdots-\phi_p B^p\right)\left(1-\Phi_1 B^{12}-\cdots-\Phi_P B^{12P}\right)(1-B)^d\left(1-B^{12}\right)^D y_t=\left(1+\theta_1 B+\cdots+\theta_q B^q\right)\left(1+\Theta_1 B^{12}+\cdots+\Theta_Q B^{12Q}\right)\varepsilon_t$$

The estimation and validation process was conducted in two stages. In the first stage, the three approaches were estimated on the training sample (January 1997 through December 2024) and evaluated on the test set, comparing their out-of-sample predictive performance. In the second stage, the approach with the best out-of-sample performance was re-estimated on the complete time series (January 1997 to May 2026), which was then used for projections from June 2026 to December 2027. These two steps may result in different specifications, as the validation model is optimized for the training sample, while the final model incorporates additional information from the most recent 17 months.

Accuracy was assessed using the Mean Absolute Percentage Error (MAPE) and the Root Mean Square Error (RMSE) (Hyndman; Koehler, 2006) :

$$MAPE=\frac{100}{n}\sum_{t=1}^{n}\left|\frac{y_t-\hat{y}_t}{y_t}\right| \qquad RMSE=\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(y_t-\hat{y}_t\right)^2}\ \#(6)$$

Since MAPE is expressed as a percentage, it allows for direct comparison with the official estimate of nominal growth released by the National Treasury. RMSE, expressed in the unit of the time series, penalizes errors of greater magnitude and is relevant for municipal cash flow planning. As a complementary metric, we also report the Mean Absolute Scaled Error (MASE), proposed by as a robust alternative to MAPE because it does not exhibit asymptotic behavior near zero and is scaled relative to a naive reference model—that is, simply repeating the previous value.

The three metrics were calculated for the three models on the test set, forming the basis for the predictive comparison between the approaches. Because it is scaled relative to the naive reference model, MASE allows for a direct assessment of the extent to which each approach exceeds—or fails to exceed—this minimum performance threshold.

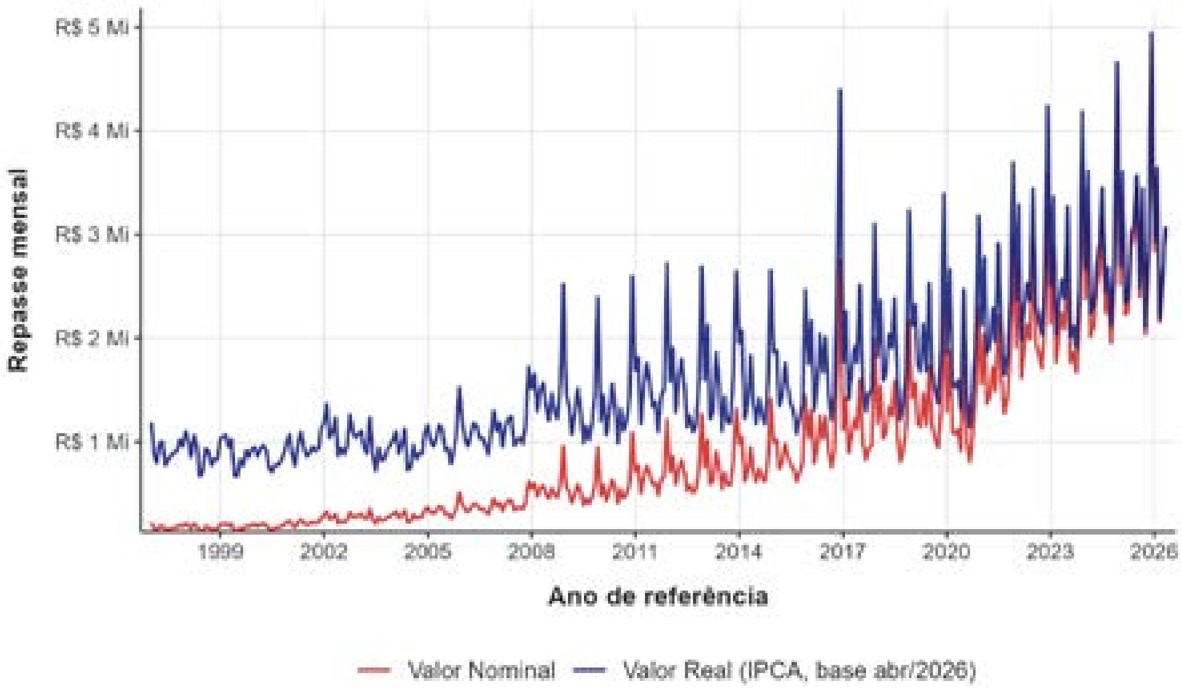
Model fit was assessed by analyzing the residuals, including a visual inspection of the residual plot over time, the autocorrelation function (ACF) of the residuals, and the histogram. The formal Ljung-Box test was applied under the null hypothesis of no serial autocorrelation in the residuals (Ljung; Box, 1978) , with $h = 24$ lags, corresponding to the standard *checkresiduals* function from the *forecast* package for time series with frequency $m = 12$.

As a complementary diagnostic for the presence of uncaptured systematic structure, the integrated (cumulative) periodogram of the residuals from the best-performing model was applied. Under the assumption that the residuals are consistent with white noise, the cumulative periodogram grows approximately linearly across frequencies, remaining within the Kolmogorov-Smirnov confidence bands. Deviations from these bands indicate that part of the periodic structure of the series was not captured by the model (Brockwell; Davis, 2016) .

RESULTS

Figure 2 shows the evolution of monthly transfers from the FPM to Realeza between January 1997 and May 2026. The nominal series shows significant growth over the three decades, with transfers—which did not exceed R$500,000 per month in the late 1990s—regularly surpassing R$3 million starting in 2022. When the figures are adjusted for inflation (IPCA, April 2026 base), however, real growth is more modest, as much of the nominal increase reflects the currency’s loss of purchasing power over the period. Furthermore, the real data series shows greater volatility starting in 2015, a period marked by more intense fluctuations in the transfers, which foreshadow the challenges of adjusting the statistical model.

Figure 2 - Comparison of nominal and real values.



Source: Authors, 2026.

Table 1 presents the multiplicative seasonal indices calculated from the training series. An index above 1 indicates that the month in question receives, on average, more than the monthly average for the year, while an index below 1 indicates the opposite. The results are consistent with the FPM transfer schedule and with the legislative changes that have expanded the transfers over the years. December stands out with the highest index (1.459); that is, December transfers are, on average, 45.9% higher than the annual average, a direct reflection of the 1% constitutional surcharge established by Constitutional Amendment No. 55/2007 (Brazil, 2007) to bolster municipal coffers for the payment of civil servants' 13th-month salary. February is the second-highest month (1.181). Conversely, September has the lowest index (0.794), historically and structurally marked by a slowdown in federal tax revenue, which may have prompted the creation of the 1% surcharge through Constitutional Amendment No. 112/2021 (Brazil, 2021) . July (0.944) exhibits similar behavior, which is why it was subject to the additional adjustment under Constitutional Amendment No. 84/2014 (Brazil, 2014).

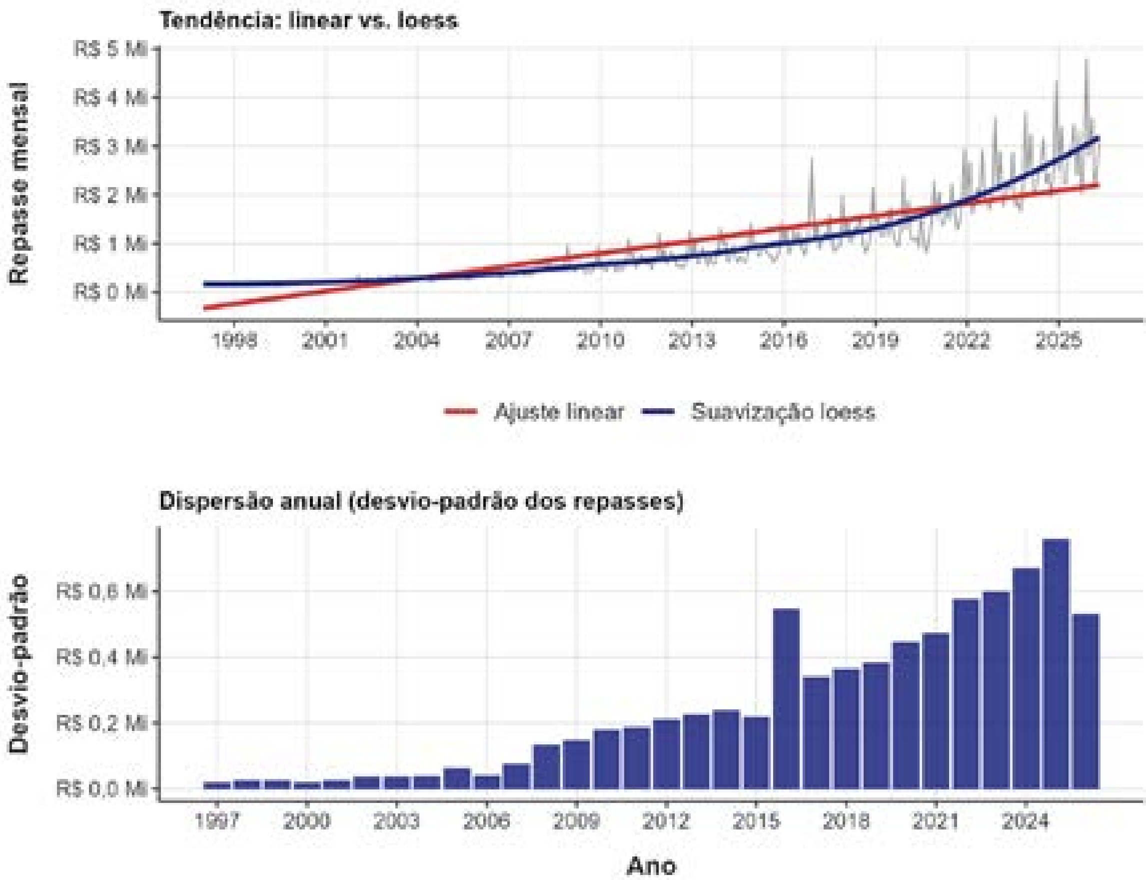
Table 1 - Effect of the month on the mean.

Month	Seasonal Index	Effect on the annual average
January	1.063	6.26%
February	1,181	18.12%
March	0.87	-13.03%
April	0.961	-3.92%
May	1.104	10.35%
June	0.923	-7.66%
July	0.944	-5.57%
August	0.887	-11.28%
September	0.794	-20.64%
October	0.834	-16.60%
November	0.981	-1.91%
December	1.459	45.88%

Source: Authors, 2026.

Figure 3 characterizes the time series in terms of the two aspects raised in the methodological assessment: the shape of the trend and the stability of the variance. In the upper panel, the linear fit and the local *loess* smoothing reveal that the growth in on-lending is not linear. The *loess* curve remains relatively flat until the early 2010s and, particularly starting in 2021, accelerates sharply, surpassing the trend line. This convex behavior is characteristic of nominal fiscal series, in which growth combines the cumulative effect of inflation with the real expansion of tax revenue. Although the linear fit explains 72.8% of the variance ($R^2 = 0.728$), it underestimates the transfers in the recent period. In the lower panel, the annual standard deviation of the pass-throughs increases significantly over time, with the average dispersion for the 2021–2025 five-year period being about 26 times higher than that of 1997–2001. This result confirms the presence of heteroscedasticity (variance that increases proportionally to the level of the series) and justifies both the multiplicative decomposition and the adoption of an exponential smoothing model with multiplicative components, the ETS(M,A,M).

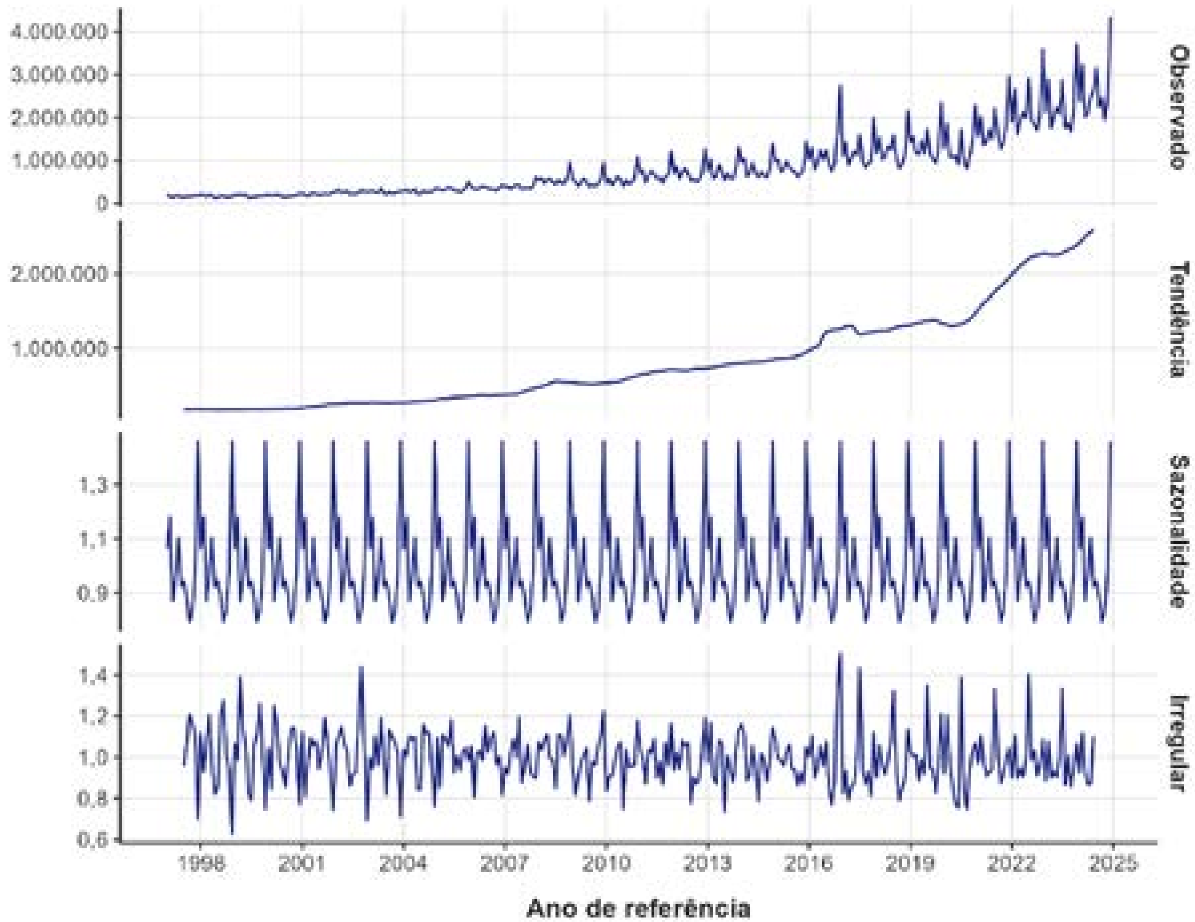
Figure 3 – Characterization of the trend and variance of the time series.



Source: Authors, 2026.

The multiplicative decomposition (Figure 4) visually confirms the seasonal fluctuations. As the level of transfers increases over the decades, the seasonal fluctuations also become larger in absolute terms. The random component, which represents what is not explained by either the trend or seasonality, exhibits increasing variance, especially starting in 2015. This means that the transfers have become progressively more difficult to predict using a fixed-parameter model, such as SARIMA, indicating that the time series has begun to incorporate external disturbances.

Figure 4 - Multiplicative decomposition.



Source: Authors, 2026.

The ADF test assessed whether the FPM transfer series is stationary—that is, whether its statistical properties (mean and variance) remain constant over time—a necessary condition for applying the SARIMA model. The test yielded a statistic of *tau*3 = -5,91, which is more negative than the 5% critical value (-3.42), leading to the rejection of the null hypothesis of a unit root; that is, the ADF test suggests that the series does not wander randomly without direction but exhibits stationary behavior around a deterministic upward trend. The joint statistics *phi*2 and *phi*3, which test restrictions on the deterministic trend, also reject their null hypotheses (Table 2). As descriptive evidence, the ADF suggests that the series is stationary around a deterministic trend.

Table 2 - ADF test values.

Statistic	Calculated value	Critical 1%	Critical value (5%)	Critical 10%
tau3	-5.91	-3.98	-3.42	-3.13
phi2	12.19	6.15	4.71	4.05
phi3	17.91	8.34	6.3	5.36

Source: Authors, 2026.

The KPSS test indicated that the time series requires differentiation (d = 1) prior to modeling, a result adopted by *auto.arima* in its internal order selection routine. This conclusion does not contradict the possible indication of stationarity from the ADF test, where the two null hypotheses are opposite (unit root in the ADF and stationarity in the KPSS) and may point in different directions when the series combines a deterministic trend (structural growth of the FPM) with a stochastic component that accumulates over time. In this scenario, first-order differentiation is the standard solution of the *Box-Jenkins* group because it eliminates both components simultaneously.

As further evidence of the inadequacy of the automatic fit without break handling, Table 3 shows that the selected model included a non-seasonal moving average term (ma1) that was statistically insignificant at the 5% level (p = 0.2518). The inclusion of insignificant parameters is a result of models that attempt to accommodate unaccounted-for structural shocks by oversizing the stochastic component, which compromises the quality of the model and exacerbates the out-of-sample forecast error.

Table 3 - Estimated coefficients of the test SARIMA model.

Term	Coefficient	Standard Error	z-statistic	p-value
ar1	-0.6767	0.1086	-6.23	0
ma1	0.1146	0.1	1.15	0.2518
ma2	-0.565	0.0605	-9.34	0
sar1	-0.4159	0.0585	-7.11	0
sar2	-0.1509	0.0581	-2.6	0.0094

Source: Authors, 2026.

Table 4 compares the out-of-sample predictive performance of the three approaches. Exponential Smoothing ETS(M,A,M) achieved the best results across all metrics, outperforming both the naive seasonal model and SARIMA. MASE compares each model to the average error of the naive reference during the training period, which repeats the value from the same month of the previous year; thus, a value greater than 1 indicates an error larger than that of this trivial reference. The Naive Seasonal model itself, evaluated during the test period (January 2025 to May 2026), exceeded its own average training error (1997 to 2024), with a MASE of 2.682. With a MASE of 2.18, the error of the "$SARIMA(1,1,2)(2,1,0)_{12}$" model during the test period was more than double the average error that the Naive Seasonal model made during the training period. Even the best model, the ETS, had a MASE of 1.695—still above unity—which highlights the limitations of fixed-parameter approaches for forecasting FPM in Realeza and the need for additional data processing.

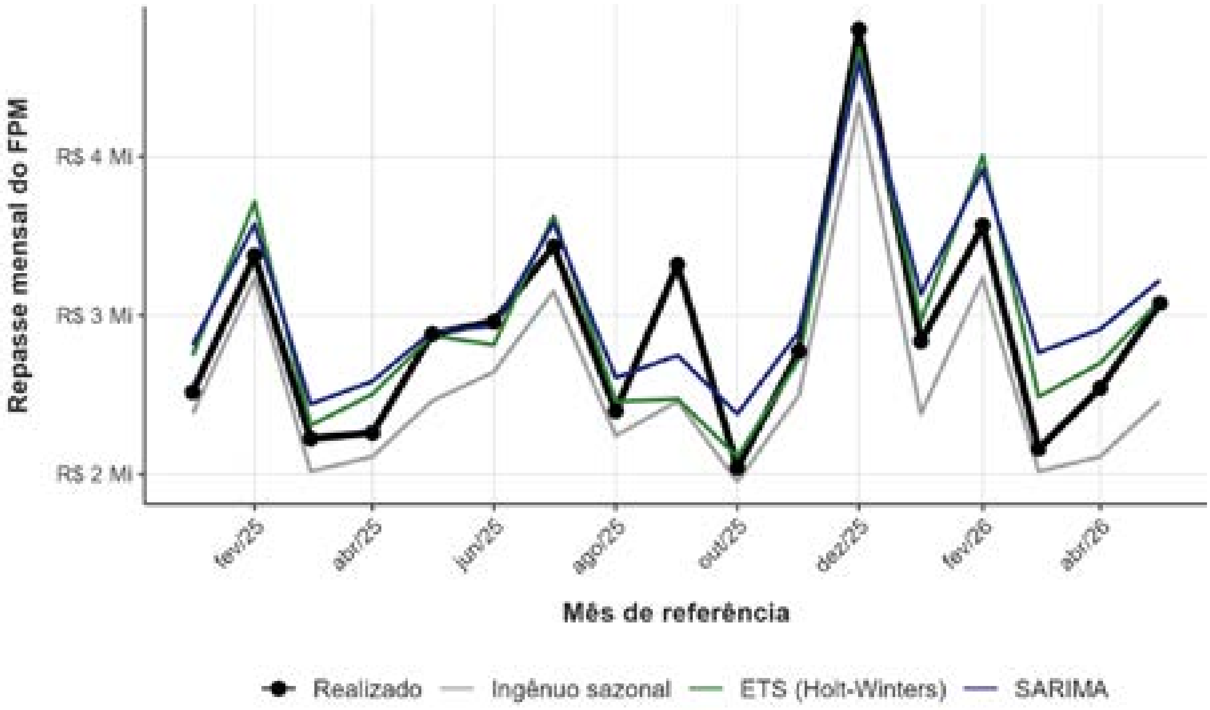
Table 4 - Comparative predictive performance (out-of-sample validation).

Model	RMSE (R$)	MAPE (%)	MASE
Naive seasonal	377,494.00	10.83	2.682
Exponential Smoothing ETS(M,A,M)	284,493.00	7.00	1.695
$SARIMA(1,1,2)(2,1,0)_{12}$	305,634.00	9.81	2,184

Source: Authors, 2026.

Figure 5 visually compares the forecasts from the three approaches with the actual transfers made during the test period. The ETS and SARIMA curves accurately capture the seasonal pattern of the transfers, but both tend to fall above the actual values in most months, with SARIMA showing the most pronounced overestimation, consistent with a cumulative deviation of 5.79% during the test period, compared to 2.28% for ETS. The naive seasonal model (gray) remains systematically below the actual figures, as it reproduces only the value from the same month of the previous year without incorporating the growth trend. The figure illustrates, in a qualitative manner, what Table 4 quantifies, showing that the ETS(M,A,M) forecasts are closest to the observed transfers.

Figure 5 – Actual transfers versus forecasts during the test period.



Source: Authors, 2026.

Table 5 details the month-by-month performance of the selected model (ETS). The cumulative deviation over the 17 months was R$ 1,118,439.55, equivalent to just 2.28% of the total realized amount, well below the 5.79% for SARIMA, which reinforces the predictive superiority of Exponential Smoothing. Nevertheless, some months exhibit high deviations, notably September 2025 (25.46% error), March 2026 (15.22%), and February 2026 (12.42%), associated with the structural breaks and outliers discussed below.

Table 5 - Month-by-month performance of the selected model (ETS).

Month	Forecast (R$)	Actual (R$)	Difference (R$)	Absolute error (%)
Jan/25	2,746,382.39	2,515,773.92	-230,608.47	9.17
Feb/25	3,714,049.01	3,375,878.09	-338,170.92	10.02
Mar 25	2,306,141.19	2,223,171.05	-82,970.14	3.73
Apr/25	2,502,961.30	2,259,121.14	-243,840.16	10.79
May 25	2,871,752.27	2,881,829.33	10,077.06	0.35
Jun 25	2,813,640.24	2,960,034.66	146,394.42	4.95
Jul/25	3,625,615.47	3,434,711.76	-190,903.71	5.56
Aug 25	2,455,668.41	2,397,337.64	-58,330.77	2.43
Sep 25	2,473,996.91	3,318,992.45	844,995.54	25.46
Oct/25	2,107,371.85	2,040,374.24	-66,997.61	3.28
Nov/25	2,724,489.06	2,773,561.77	49,072.71	1.77
Dec. 25	4,681,737.53	4,800,440.02	118,702.49	2.47
Jan 26	2,964,612.59	2,835,096.61	-129,515.98	4.57
Feb 26	4,007,330.14	3,564,565.75	-442,764.39	12.42
Mar 26	2,487,117.97	2,158,560.32	-328,557.65	15.22
Apr 26	2,698,174.48	2,541,495.10	-156,679.38	6.16
May 26	3,094,358.52	3,076,015.92	-18,342.60	0.60
TOTAL (17 months)	50,275,399.32	49,156,959.77	-1,118,439.55	2.28

Source: Authors, 2026.

After estimating the selected model, the residual errors (the difference between what the model predicted and what actually happened) should behave like random noise, with no recognizable pattern. The Ljung-Box test checks for exactly that. For Exponential Smoothing ETS(M,A,M), with a statistic of $Q^* = 41.92$ (24 degrees of freedom) and a p-value of 0.0132, the test rejects the hypothesis that the residuals are random, indicating patterns in the errors that the model failed to capture (Table 6).

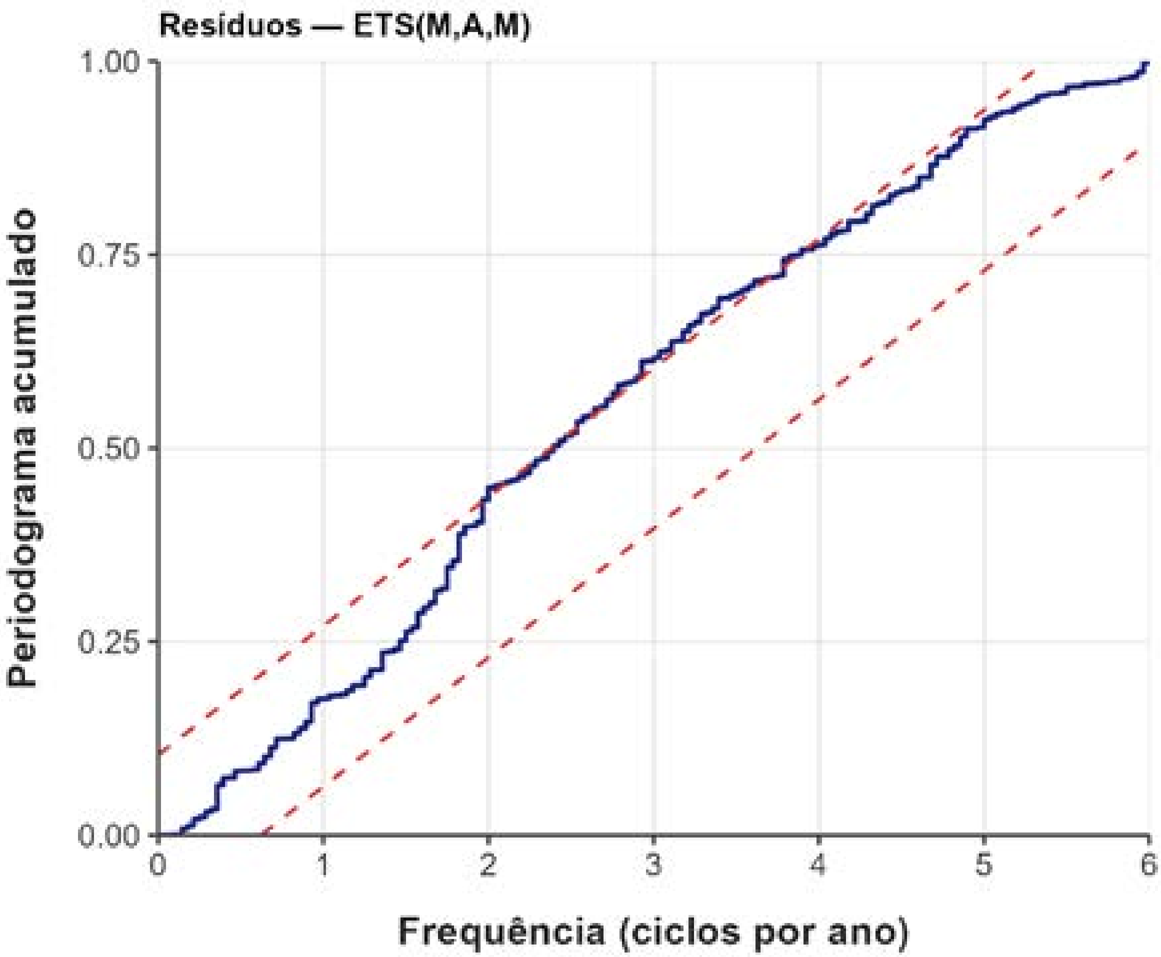
Table 6 - Ljung-Box Test.

Model	Q* (Ljung-Box)	Degrees of freedom	p-value	Conclusion (5%)
ETS(M,A,M)	41.92	24	0.0132	H0 is rejected: autocorrelation remains

Source: Authors, 2026.

This evidence is corroborated by the integrated periodogram, which shows the cumulative curve of the residuals extending beyond the Kolmogorov-Smirnov confidence bands, indicating an uncaptured periodic structure (Figure 6). Although it is the approach with the best predictive performance among the three evaluated, the selected model does not fully capture the systematic component of the series.

Figure 6 - Integrated periodogram of the selected model's residuals.

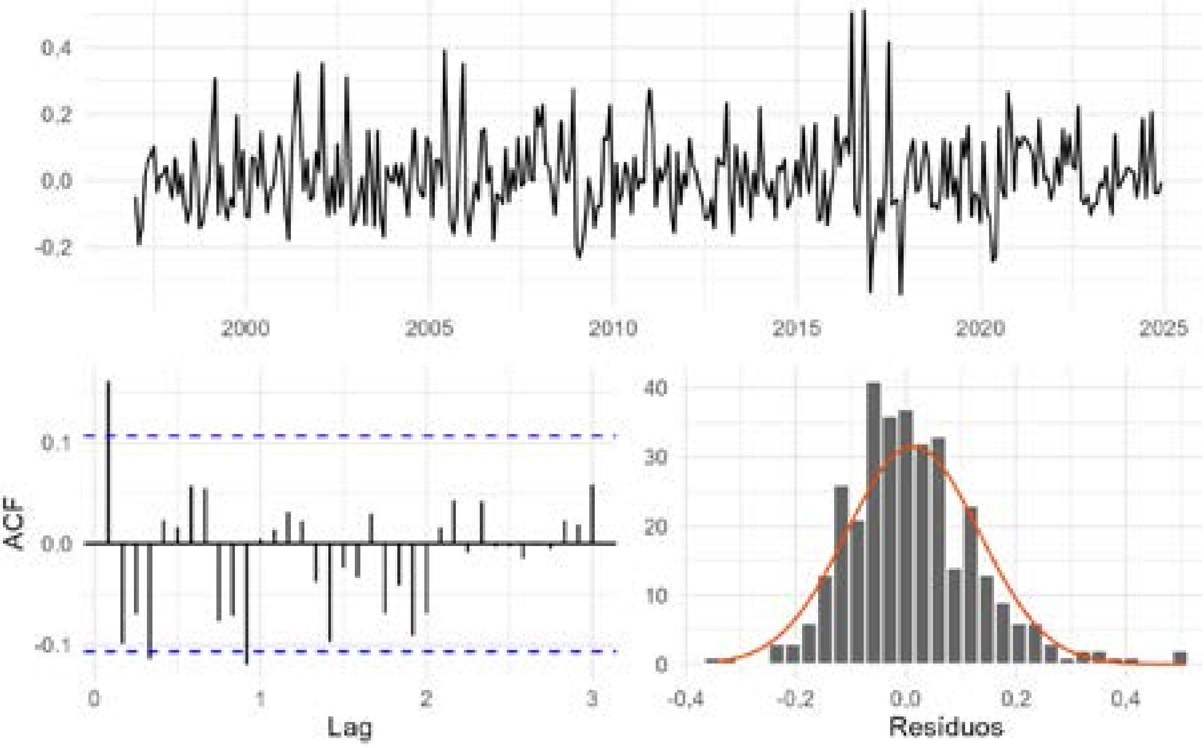


Source: Authors, 2026.

Since this is a model with multiplicative error, the ETS(M,A,M) residuals are expressed in relative terms, not in absolute values. Even so, two anomalous behaviors stand out. Between 2016 and 2018, a positive error of nearly 50% is followed, a few months later, by a negative error of similar magnitude—a delayed reflection of the federal fiscal crisis that reduced revenue from income tax and the IPI , which serve as the calculation bases for the FPM. The variance of the residuals remains high throughout the 2020s, reflecting the shock of the COVID-19 pandemic, the subsequent economic recovery, and the gradual implementation of the 1% supplement effective in September under Constitutional Amendment No. 112/2021 (Brazil, 2021) . These events represent structural breaks—abrupt and permanent changes in the behavior of the time series—that fixed-parameter models, estimated once for the entire sample, are unable to accommodate dynamically; neither SARIMA nor ETS can do so. The rejection of H_0 by the Ljung-Box test confirms that the residuals of the selected model do not constitute white noise; even though the ETS approach had the best predictive performance among the three evaluated, it did not fully capture the time-dependence structure of the series.

In Figure 7, the top panel displays the residuals over time, where the magnitude of the errors is noticeably greater starting in 2016 than in the previous period, indicating that the variance of the residuals does not remain constant throughout the series. The lower-left panel presents the autocorrelation function (ACF) of the residuals, with dashed bands delimiting the 95% confidence interval, calculated as $\pm \frac{2}{\sqrt{T}}$ (Hyndman and Athanasopoulos, 2021). The horizontal axis is in annual units, such that each whole unit corresponds to 12 monthly lags. Some autocorrelations extend beyond these bands, indicating correlation in the residuals. The lower-right panel shows the histogram of the residuals with the theoretical normal curve superimposed. The distribution is approximately centered at zero but exhibits a more elongated right tail, reflecting episodes in which the model underestimated atypically high transfers.

Figure 7 – Residuals from the Exponential Smoothing ETS(M,A,M) model.



Source: Authors, 2026.

Once the validation stage was completed, the selected model was re-estimated using the complete time series to generate the projections, a procedure that resulted in the ETS(M,A,M) specification, whose smoothing parameters are presented in Table 7 ($\alpha = 0.2561$; $\beta = 0.0042$; $\gamma = 0.4582$; AICc = 10,040.52). The continued failure of the Ljung-Box test, both during the validation stage and upon reestimation using the complete time series, reinforces the conclusion that the unaccounted-for residual structure stems from characteristics of the time series itself—particularly legislative structural breaks and macroeconomic *outliers*—rather than from the specific choice of a model.

Table 7- Smoothing parameters for the selected model ETS(M,A,M), re-estimated on the complete time series.

PARAMETER	ESTIMATE
ALPHA	0.2561
BETA	0.0042
GAMMA	0.4582
AICc	10,040.52

Source: Authors, 2026.

After re-estimating the selected model using the full time series, the projected nominal growth of the FPM for Realeza is presented in Table 8. For 2026, the National Treasury projected a transfer 8.97% higher than in 2025, while the selected model (ETS) projects growth of only 3.42%, a more conservative estimate. However, even the best model—without *outlier* handling and further refinement of data processing—still exhibits residual autocorrelation and should not be considered in isolation for the formulation of municipal budgets in their current form.

Table 8 - Model forecasts and comparison with the Treasury.

Projection Method	Base: Nominal FPM 2025 (R$)	Nominal growth 2026	Projected FPM for 2026 (R$)
National Treasury (official forecast)	34,981,226.07	8.97%	38,118,772.56
Selected model — ETS(M,A,M)	34,981,226.07	3.42%	36,177,474.57

Source: Authors, 2026.

The objective is not to assert that these figures are reliable as a fiscal management tool, but rather to demonstrate what the model produces and to what extent it differs from the National Treasury's official estimate—an exercise that is part of the study's objectives.

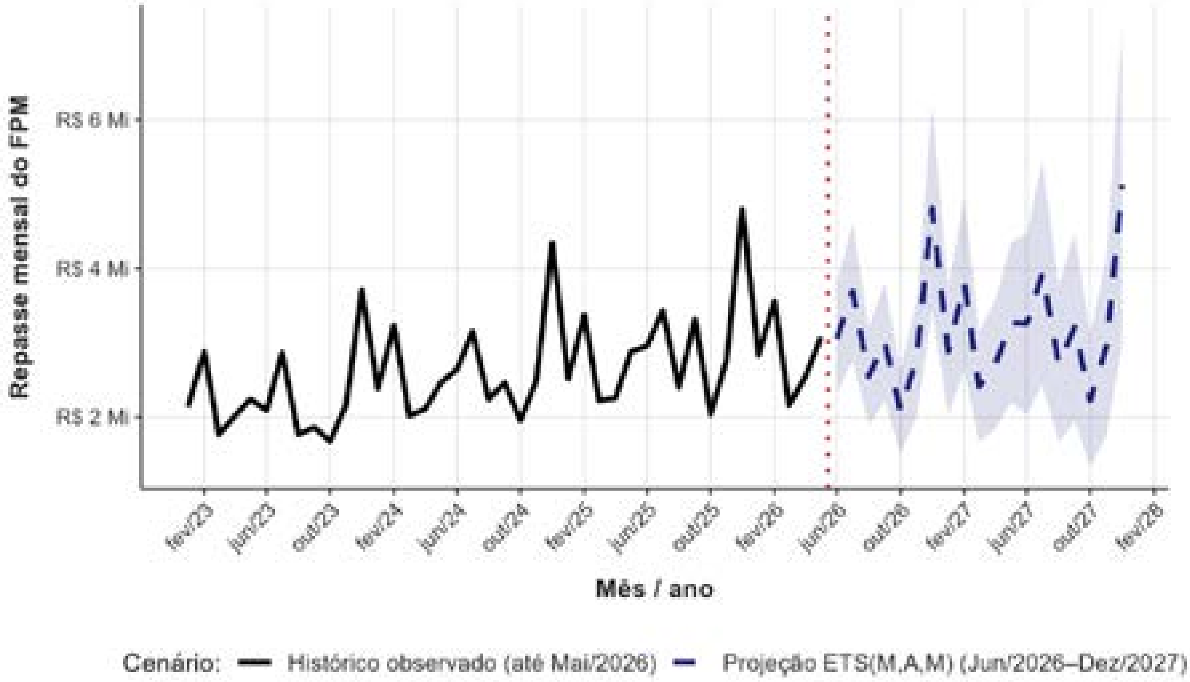
The selected model, ETS(M,A,M), estimated based on the complete time series, projects a total of R$ 36,177,474.57 for fiscal year 2026, combining the R$ 14,175,733.70 realized between January and May with the R$ 22,001,740.87 projected for June through December. This figure implies a nominal growth of 3.42% over the FPM realized in 2025 (R$ 34,981,226.07), falling 5.55 percentage points short of the National Treasury's official projection, which estimates growth of 8.97% (R$ 38,118,772.56). The absolute difference between the two projections is R$ 1,941,297.99, a significant amount for a municipality the size of Realeza (Figure 8).

Figure 8 - Revenue projection using the ETS(M,A,M) model for 2026.

Source: Authors, 2026.

For 2027, the selected model projects a total of R$ 38,504,959.26, with a 95% confidence interval ranging from R$ 24,517,978.79 to R$ 52,491,939.74, implying a nominal growth of 6.43% over the estimated 2026 closing figure. The wide range of the interval (over R$ 27 million) reflects the uncertainty accumulated over the twelve-month projection period and the multiplicative nature of the ETS model; it should be interpreted with caution, especially since the probabilistic validity of these intervals presupposes residuals consistent with white noise-—a condition not verified in this study (Figure 9).

Figure 9 - Revenue trends according to the ETS(M,A,M) model in 2027.



Source: Authors, 2026.

FINAL CONSIDERATIONS

A comparison of the three approaches shows that Exponential Smoothing, in the form of ETS(M,A,M), outperforms the SARIMA model and the naive seasonal model in forecasting monthly FPM transfers to the municipality of Realeza during the analyzed period. Nevertheless, none of the fixed-parameter models is, on its own, sufficient to produce statistically valid forecasts. The MASE outside the sample exceeding 1 in all three approaches indicates that all of them made more errors during the test period than the naive seasonal model did on average during the training period. For the selected model, the Ljung-Box test and the integrated periodogram reject the hypothesis that the residuals are consistent with white noise, confirming that part of the time-dependence structure of the series remains unaccounted for even by the best-performing approach.

Two factors, identified throughout the analysis, explain this limitation and point to the necessary approaches for future research. First, structural legislative changes—the gradual introduction of the three 1% surcharges (Constitutional Amendment No. 55/2007, Constitutional Amendment No. 84/2014, and Constitutional Amendment No. 112/2021)—introduce subtle and permanent shifts in the seasonal patterns of the time series that are not captured by fixed-parameter models. Second, macroeconomic *outliers*— , the episodes of 2015–2016 and 2020–2021—produce deviations that inflate the residual variance and distort parameter estimates.

The contribution of this study lies in empirically demonstrating, for a small municipality with high dependence on the FPM, that exponential smoothing methods can outperform SARIMA in fiscal forecasting; however, the adoption of any fixed-parameter model—without identifying and addressing structural breaks and *outliers*—still yields estimates with a margin of error that is significant for budgetary planning. Looking ahead, the diagnosis using the integrated periodogram indicates that the remaining systematic structure could be captured by hybrid models—which combine a linear component and a nonlinear component—as a promising avenue for future research.

BIBLIOGRAPHICAL REFERENCES

BRASIL. Emenda Constitucional nº 55, de 20 de setembro de 2007. Altera o art. 159 da Constituição Federal. Diário Oficial da União: Seção 1, 20 set. 2007. Disponível em: <https://www2.camara.leg.br/legin/fed/emecon/2007/emendaconstitucional-55-20-setembro-2007-559897-publicacaooriginal-82343-pl.html> Acesso em: 10 de março de 2026.

BRASIL. Emenda Constitucional nº 84, de 2 de dezembro de 2014. Altera o art. 159 da Constituição Federal para aumentar a entrega de recursos pela União para o Fundo de Participação dos Municípios. Diário Oficial da União: Seção 1, 3 dez. 2014. Disponível em: <https://planalto.gov.br/ccivil_03/constituicao/emendas/emc/emc84.htm> Acesso em: 10 de março de 2026.

BRASIL. Emenda Constitucional nº 112, de 27 de outubro de 2021. Altera o art. 159 da Constituição Federal para disciplinar a distribuição de recursos pela União ao Fundo de Participação dos Municípios. Diário Oficial da União: Seção 1, 28 out. 2021. Disponível em: <https://www.planalto.gov.br/ccivil_03/constituicao/emendas/emc/emc112.htm> Acesso em: 10 de março de 2026.

BRASIL. SECRETARIA DO TESOURO NACIONAL. Previsão anual de transferências FPM/FPE, IPI-Exportação e CIDE-Combustíveis. Tesouro Nacional Transparente. Disponível em: <https://www.tesourotransparente.gov.br/publicacoes/previsao-anual-de-transferencias-fpm-fpe-ipi-exportacao-e-cide-combustiveis/2026/114>. Acesso em: 7 jun. 2026a.

BRASIL. SECRETARIA DO TESOURO NACIONAL. Transferências Constitucionais Realizadas. Disponível em: <https://www.tesourotransparente.gov.br/consultas/transferencias-constitucionais-realizadas>. Acesso em: 5 jun. 2026b.

BRITO, Thazia Karine De Souza; SOUZA, Jose Antonio Nunes de; FERREIRA, Francisco Danilo da Silva; SILVA, William Gledson e. Previsão mensal dos valores do Fundo de Participação dos Municípios (FPM) Norte-Rio-Grandenses a partir de um modelo de séries temporais. Economia & Região, v. 10, n. 3, p. 33–48, 6 dez. 2021. DOI 10.5433/2317-627X.2022v10n3p33

BROCKWELL, Peter J.; DAVIS, Richard A. Introduction to Time Series and Forecasting. Cham:

Springer International Publishing, 2016.

CONFEDERAÇÃO NACIONAL DE MUNICÍPIOS. Fundo de Participação dos Municípios (FPM). Brasília, 2014.

DICKEY, David A.; FULLER, Wayne A. Distribution of the Estimators for Autoregressive Time Series with a Unit Root. Journal of the American Statistical Association, v. 74, n. 366a, p. 427–431, jun. 1979. DOI 10.1080/01621459.1979.10482531

HYNDMAN, Rob J.; ATHANASOPOULOS, George. Forecasting: principles and practice. 2nd edition ed. Melbourne: OTexts, 2018.

HYNDMAN, Rob J.; ATHANASOPOULOS, George. Forecasting: Principles and Practice (3rd ed). 3a ed. Melbourne, Australia: OTexts, 2021.

HYNDMAN, Rob J.; KOEHLER, Anne B. Another look at measures of forecast accuracy. International Journal of Forecasting, v. 22, n. 4, p. 679–688, out. 2006. DOI 10.1016/j.ijforecast.2006.03.001

KWIATKOWSKI, Denis; PHILLIPS, Peter C. B.; SCHMIDT, Peter; SHIN, Yongcheol. Testing the null hypothesis of stationarity against the alternative of a unit root. Journal of Econometrics, v. 54, n. 1–3, p. 159–178, out. 1992. DOI 10.1016/0304-4076(92)90104-Y

LJUNG, G. M.; BOX, G. E. P. On a measure of lack of fit in time series models. Biometrika, v. 65, n. 2, p. 297–303, 1 ago. 1978. DOI 10.1093/biomet/65.2.297

MEIRELES, Fernando. deflateBR: Deflate Nominal Brazilian Reais. , 28 set. 2018. Disponível em: <https://CRAN.R-project.org/package=deflateBR>. Acesso em: 6 jun. 2026

OLIVEIRA, Kelly. Arrecadação do governo registra queda de 5,62% em 2015. Disponível em: <https://agenciabrasil.ebc.com.br/economia/noticia/2016-01/arrecadacao-do-governo-registra--queda-de-562-em-2015>. Acesso em: 16 jun. 2026.

PFAFF, Bernhard. Analysis of integrated and cointegrated time series with R. New York, NY: Springer, 2006.

REALEZA. PREFEITURA MUNICIPAL. Portal da Transparência: Receitas e Renúncias. Prefeitura Municipal de Realeza. Disponível em: <https://realeza.pr.gov.br/transparencia/receitas-renuncias>. Acesso em: 17 jun. 2026.

RODRIGUES, Francisco; JUNIOR, Antonio Erivando Xavier. APLICAÇÃO DO MODELO KOYCK PARA PREVISÃO DA RECEITA PÚBLICA: UM ESTUDO DA PREVISÃO DO ISS, COTA-PARTE DO ICMS E DO FPM NO MUNICÍPIO DE MOSSORÓ/RN. 2018.

SILVA, Alexandre César Batista da; ALBUQUERQUE, F. S.; BEZERRA, M. S. C.; SILVA, W. B. Utilização de séries temporais na previsão de arrecadação de ICMS no estado do Pará. In: Recife: UFPE, 2017.