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Average Tax Rates On Consumption And Factor Income In Brazil 2000q1-2022q4

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ABSTRACT

The aim of this paper is to calculate average effective macroeconomic tax rates levied on consumption, labor income, and capital income for the Brazilian economy using data on tax revenue and national accounts for the period from 2000 to 2022. We follow the methodology proposed by Mendoza, Razin & Tesar (1994) , and other that is based ond the adjust on income distribution proposed by Gollin (2002). Both allows estimating those tax rates consistently with the distortionary taxation faced by the representative agent in a general equilibrium framework. We apply temporal disaggregation techniques in order to get those tax rates also on quarterly basis. The results show that taxation in Brazil burdens especially labor income and consumption.

Keywords: Tax Rates, Temporal Disaggregation, Functional Distribution of Income
JEL: E62; H20; H22.

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1. INTRODUCTION

Public finance and macroeconomic analysis emphasize the importance of taxation in the options available to economic policymakers. Changes in taxation are a powerful tool not only in the execution but also in the evaluation and implications of a given policy on other macroeconomic aggregates. In addition, taxation imposed on income-generating factors and indirect taxation on goods and services constitute a set of constraints affecting not only the choices of policy planners, but also the intertemporal choice of the representative agent in the structure of dynamic stochastic general equilibrium (DSGE) models.

The bases of taxation are consumption and factor income. Consumption tax leads the representative agent in the economy to substitute an excessively taxed good for another that is different from his optimal choice. Income tax, on the other hand, affects the decisions of economic agents between work and leisure. The higher the income tax, the lower the incentive for the worker and, consequently, the lower the labor supply. Taxes on wealth and income also affect the return on economic activities, which causes capital shifts between federal entities, while tax incentives accentuate the large imbalances that exist between the states of the federation. By altering the choices of economic agents, distortive taxation also generates efficiency losses in the allocation of resources, reducing the well-being of society. Advances in the adequate measurement of aggregate taxation help to improve general equilibrium models, which serve as laboratories for deciding on policies that minimize economic distortions.

The existing literature proposes different strategies for combining tax codes, tax collection, and data on income distribution among the generating factors. The seminal work by Araújo Neto and Souza (2003) calculated the average tax rates on consumption, labor income, and capital income for Brazil from 1975 to 1999. The results revealed that consumption taxation is predominant, while low capital taxation would suggest the existence of tax substitution, a theory according to which countries that tax consumption and labor income more heavily tend to tax capital income less.

Azevedo and Fasolo (2015) estimated time series for effective tax rates on consumption and factors of production in Brazil. According to the authors, the estimated tax burden shows a positive trend over time, with a slight interruption during the 2009 recession. In addition, the authors conducted studies relating the estimated effective rates and their relationship to the real business cycle. The time horizon analyzed covers the period 1999-2014. Regarding the level of rates, taxation on capital is lower relative to consumption and labor. However, the difference

is not as pronounced as in Araújo Neto and Souza (2003). This is because the authors treat the income of self-employed workers as labor income.

Almeida et al. (2017) estimated quarterly time series for effective tax rates on consumption and production factors in Brazil for the period from 1997 to 2013. The authors' work differs from that of Azevedo and Fasolo (2015) due to the use of methods of temporal disaggregation of annual series without corresponding high-frequency periods. Nevertheless, the results obtained show that taxation in Brazil mainly burdens labor and capital income compared to consumption taxation. It could be said that the divergence in the level of tax rates relative to the work of Azevedo and Fasolo (2015), which analyzes the same time horizon, is the wage bill created by Almeida et al. (2017), resulting from the product of the average wage of formally registered employees and the employed population. In addition, the authors followed Mendonza et al. (1994) and Araújo Neto and Souza (2003) using property income and the operating surplus of producing families as capital income.

It is also worth considering works such as that of Lledó (2005). In this case, the author does not seek to estimate a time series for tax rates, but only to calibrate a general equilibrium model that studies the tax system under fiscal adjustment. In his work, the author considers that the income of self-employed workers is the result of labor income and that the operating surplus is capital income in the economy. Nevertheless, as in Azevedo and Fasolo (2015), the high tax burden on formal workers is automatically diluted, and greater equity in the taxation of factors is observed.

Finally, according to the brief literature review, there are two assumptions that directly interfere with the estimation of tax rates in the economy. The first concerns the categorization of the tax. For example, while Azevedo and Fasolo (2015) consider the IOF to be a consumption reducer, Almeida et al. (2017) consider it a capital accumulation reducer[1] . In addition to the categorization of taxes, however, the researcher must also consider the tax base. After all, when statistics are sought for a general equilibrium model, it must be considered that all producing/consuming families hold all the capital in the economy. That said, this study aims to estimate the tax rates on factors and consumption in the Brazilian economy in the period from 2000Q1 to 2022Q4. Nevertheless, two rates will be derived for labor and capital income. One of them reproduces the methodology of Mendonza et al. (1994), while the other estimates taxes on labor and capital income derived from an alternative functional income distribution. In particular, the

1 And, after all, neither of them is wrong. Imagine an agent who used their overdraft limit to consume. In this case, the IOF does indeed apply to consumption. Now consider an agent who redeemed the capital invested in x to invest in y. In this case, the tax applies to the agent's capital gains.

study incorporates the contributions of Gollin (2002), adjusting effective taxation in order to define a tax burden on the income of self-employed workers and not only on wages and salaries.

In addition to this introduction, the article is divided as follows. The next section reports on the theory of functional income distribution according to Gollin's (2002) adjustments. The third section is dedicated to describing the data and the methodology used to calculate the aforementioned tax rates. The fourth section presents and analyzes the results obtained. The fifth section concludes the study and highlights the implications and policy suggestions. The sixth and final section presents the bibliographical references.

2. FUNCTIONAL INCOME DISTRIBUTION AND THE EFFECTIVE TAX RATE ON LABOR INCOME

Before formally defining the effective tax rate on labor income, here is some evidence that justifies a different concept of labor income from the conventional one used in the National Accounts

As is well known in national accounting, the operating surplus of the economy, as its name suggests, is accounted for as the difference between the product generated by the factors and the remuneration of employees and self-employed workers. In the seminal work by Mendonza et al. (1994), the authors systematized a methodology to obtain very consistent tax rates for OECD countries, where economies are formalized and the income of self-employed workers has a much lower relative weight on output. When attempting to reproduce this methodology for emerging countries such as Brazil, the result is high taxation on formal wage earners in contrast to low taxation on capital, as demonstrated by Araújo Neto and Souza (2003).

The functional distribution of income, however, seeks to capture more structural characteristics of an economy. The type of income received by each individual will depend on how they fit into the production process and how the total income will be divided between surplus (interest, rent, profits) and the worker's salary. Nothing prevents, however, the coexistence within the same economic space of indeterminate forms such as subsistence production and precarious service activities, typically performed by self-employed workers. In addition, the portion of "labor" income attributed to employers, which among all subgroups is by far the best paid according to the PNADC, could very well represent a share of human capital as described by Lucas (1988). In fact, senior public officials and senior corporate executives would also be better classified in this category. Given the binary nature of the functional distribution of income

in most DSGE models, however, it is necessary to characterize this range of income also in a binary manner, between capital or labor income.

In addition to wages paid and operating surplus, the income of self-employed workers is the only dubious portion in determining factor cost output according to the System of National Accounts (SNA). Gollin (2002) proposed adjustments to this amount of income so that the factor share would remain constant over time. According to Gollin (2002): "economic models assume that the shares of labor and capital in income should be identical over time and space. But while the time series for industrialized countries appear to be consistent with this hypothesis, cross-sectional data seem to contradict it." In his article, Gollin (2002) suggests that better treatment of the information would point to similar factor shares between high- and low-income countries. In particular, the article focuses on the differences in the share of self-employed workers in income generation observed between countries. For Gollin (2002), the income from self-employment is incorrectly treated as capital income. When, therefore, income shares are corrected to reflect this aspect, the enormous differences in the share of labor in income observed between poor and rich countries become much smaller.

As Considera and Pessôa (2013) observe, the agreed income distribution crystallizes a positive relationship between the share of labor compensation in income and the per capita income of various countries. This also suggests that the share of labor income in total income would rise with economic growth. This observation, however, contradicts one of Kaldor's (1961) "stylized facts," rendering inconsistent models in which the share of factors in income remains constant.

The fact to be highlighted is that, when statistics are corrected, taking into account the differences between the participation rates of self-employed workers in the production process, there is relative stability in the share of capital in income between countries, whether rich or poor. Nevertheless, using Gollin's (2002) adjustments, it is possible to obtain the taxation of income-generating factors with the adjusted distribution[2] . Furthermore, when identifying labor income with the income received by formal workers, taxation on labor income is overestimated to the same extent that taxation on capital income is underestimated.

Gollin (2002) proposes three methodologies to correct the functional distribution of income: i) treat all self-employed income as labor remuneration; ii) treat self-employed income as proportionally divided between labor and capital; and iii) attribute to the self-employed a labor income similar to the average income of an employee in the economy. Evidently, all three alternatives increase the share of labor in income relative to the conventional calculation, where

2 Taxation on consumption and general income remains unchanged.

the income of the self-employed is attributed entirely to capital. However, contrary to the conventional calculation, the three results also show significant stability in the share of capital in income between countries, whether they are richer or poorer. In other words, there is no longer a systematic relationship between the share of capital in income and the level of economic development between countries.

Gollin's (2002) adjustments, therefore, have a direct impact on labor tax rates. Although formal workers and employers face high labor costs, these costs do not reflect the average taxes on formal and informal workers in the economy. Using PNADC data, formal work has an average share of 54% of the total number of employed persons during 2012-2022. In other words, it is unreasonable to assume that the income of 46% of those employed comes from capital gains. However, since multiplying the number of employed persons by the average income produces a wage bill that exceeds the sum of wages and gross mixed income (RMB), the best way to reconcile the average tax rate for the factors is to use one of the adjustments.

Table 1[3] below has four columns: the first shows the year of the survey, the second shows the conventional adjustment, which consists of dividing Workers' Remuneration by PIBCF (product at factor cost, excluding the surplus generated by Public Administrations APU and Non-Profit Institutions IFSFL), the third shows adjustment 1, which consists of the ratio between the sum of workers' remuneration and the RMB divided by the PIBCF, and the fourth shows adjustment 2, which consists of the ratio defined by dividing workers' remuneration by the sum of this remuneration and the operating surplus:

3 The source of all data and their respective links can be found in 6.1 reference for data.

Table 1 - Functional Distribution of Income			
Years	Conv	Aj. 1	Aj.2
2000	46.99	61.83	55.18%
2001	47.96	62.18	55.91
2002	47.15	60.83	54.62
2003	46.52	60.81%	54.28
2004	46.74	59.85	53.79
2005	47.82%	60.54	54.79
2006	48.55	60.64	55.23
2007	48.73	60.56	55.27
2008	49.64	61.13	56.08
2009	51.31	61.91	57.39
2010	50.48	60.79%	56.28
2011	51.04	61.08	56.73
2012	51.70	62.00	57.64
2013	52.06	62.46%	58.10
2014	52.04	62.15	57.89
2015	53.39	63.37	59.31
2016	53.32	63.38	59.28
2017	53.16	63.12	59.04
2018	52.55	62.58	58.41
2019	52.37	62.42	58.22%
2020	49.99	59.91	55.50
2021	47.35	56.74	52.26
2022	46.91	56.21	51.72
Average	**49.90**	**61.15**	**56.21**
DP	**2.41**	**1.80**	**2.13**

Finally, the adjustments calculated above show that when wages are added to the RMB, the trend of growth in the share of labor in income disappears. This trend was quite evident in the conventional adjustment prior to the 2014-2016 recession. The volatility of the labor share also decreases with the adjustment, and the behavior of the adjusted functional income distribution appears to be consistent with the stylized facts described by Kaldor (1961).

2.1. *The Adjustment In Factor Income And The Construction Of Series For The Tax Base*

The construction of a robust database for both labor factor income and capital factor income necessarily depends on obtaining a high-frequency series for wages paid between 2000Q1-2022Q4. Thus, the series made available by the IBGE for the period were used, so that the database consists mainly of: PMEA (Monthly Employment Survey, old methodology), PME (Monthly Employment Survey), PNAD (National Household Sample Survey), and PNADC (Continuous National Household Sample Survey).

The first step in the analysis was to observe how the annual growth rate of the PNAD

wage bill evolved in relation to the growth rate of the PME wage bill. To this end, the nominal growth rate of wages paid observed in the PNAD between September (when the survey is conducted) throughout the years 2000 and 2012 was calculated. Next, the growth rate of wages paid to workers (with formal employment contracts) was observed, added to the wages of employees and civil servants in the PME. The database is monthly, but the variation in this formalized wage bill between September 2000 and 2012 is very close to the annual PNAD growth rate, qualifying it as a consistent indicator for the breakdown of the low-frequency series.

Regressions were performed using the methodologies described in Appendix A, and although the results obtained were satisfactory, the best fit was chosen as a *proxy* for the PNAD wage series. Finally, to complement the remaining years of the *proxy* series, the wage growth rate in the PNADC was used. As before, the PNADC data refer to workers with formal employment contracts and civil servants. The series were chained in March 2012 and evolved until December 2022. Both the growth and the level of the series were so close to the figures observed in the national accounts that, initially, it was intended to use the chained series. However, in order to refine the adjustment, temporal disaggregation techniques were used and the series were calibrated so that the average of the indicator in that year corresponded to the variable observed in the national accounts.

Table 2 – Wage Income Data

Series	Period	Description	Source
Total payroll (registered employees)	2000:01-2002:3	SME (Former)	IBGE
Total payroll (registered employees/civil servants)	2002:03-2012:9	SME (New)	IBGE
Total payroll	2000-2012	PNAD	IBGE
Growth Total Wage Bill (formal)	2012:09-2022:12	PNAC (Continuous)	IBGE
Salaries Paid	2000-2022	CEI	IBGE

Source: Prepared by the Author

After obtaining the high-frequency wages, it was not difficult to accumulate them quarterly in order to obtain the quarterly series. It could be said that this series would be a consistent statistic for the formal labor market. But what about informal work? What would be the appropriate adjustment? For mixed income, Jones (2002) and Leeper Plante and Traum (2010) suggested simply dividing it in half, assigning 50% to each factor of production. This adjustment is very close to the second adjustment by Gollin (2002). Therefore, even though it is a natural adjustment given the variation in the composition of the product each year, the second adjustment was chosen instead of the average or the total incorporation of mixed income.

The second adjustment signals when there have been marginal increases in wages, and although the behavior of one market does not necessarily depend on the other, the weighted mixed income tends not to compromise the process that generates the wage series. Finally, the

Labor Income for each year becomes:

$$YL_i = SP_i + \frac{RFT_i}{RFT_i + EO_i} \times RMB_i \quad (1)$$

Equation (1) states that Labor Income in year *i*, YL_i is equal to the sum of wages paid in year *i* plus the product resulting from the adjustment to gross mixed income. Where RFT_i is the income from the labor factor (Wages Paid (SP_i)+ Employer Social Contributions (CSE_i)) and in the denominator EO_i is the operating surplus of households added to the surplus of companies (Financial and Non-Financial). Similarly, capital income can be obtained according to the following equation:

$$YK_i = EO_i + \frac{EO_i}{RFT_i + EO_i} \times RMB_i \quad (2)$$

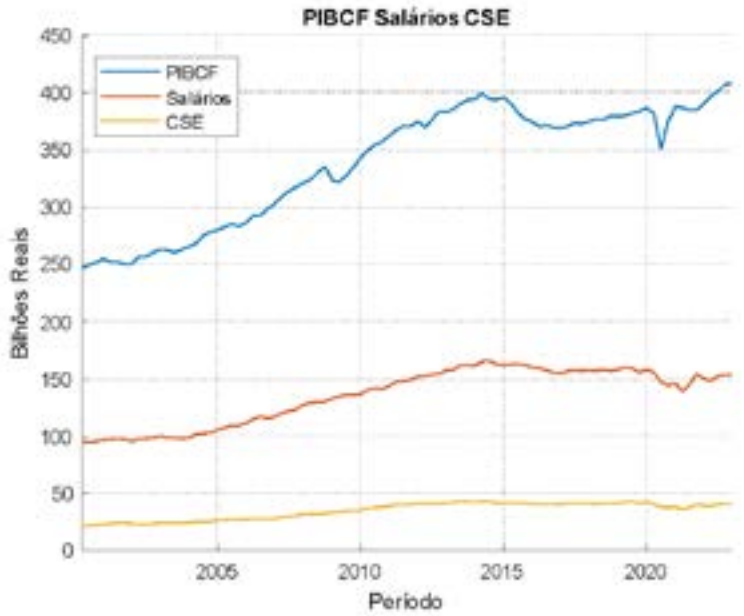
In equation (2), YK_i is the income from labor in year i and the other variables are already categorized

Finally, with the variables qualified and the income breakdown methodology exposed. Using the derived wages as proposed above, the CSE_i and the RMB obtained in the integrated economic accounts *CEI, SCN ref.2010*, it was possible to disaggregate the variables quarterly in order to compose labor income.

The matrix for the decomposition of capital income is the product at basic prices, the index already deflated and seasonally adjusted. This variable was chosen because wages, quarterly moving average, have already been automatically adjusted. The difference between GDP at basic prices and GDP at factor cost is OILS (other taxes net of subsidies), which, added to the other *EO* adjustments described in the text, constitutes a negligible amount in relation to GDP, with little capacity to interfere in the process generating the time series. Thus, after disaggregating the product at factor cost defined in the text, it was sufficient to reduce this amount by labor income, which is the sum of wages and employers' contributions, to obtain surplus income, which is capital income.

In accordance with the discussion developed throughout the text, Figure 1 shows wages and employers' social contributions which, together with a small portion relating to mixed income, constitute labor income. Nevertheless, capital income arises from the difference between PIBCF and labor income.

Graph 1 Primary Income

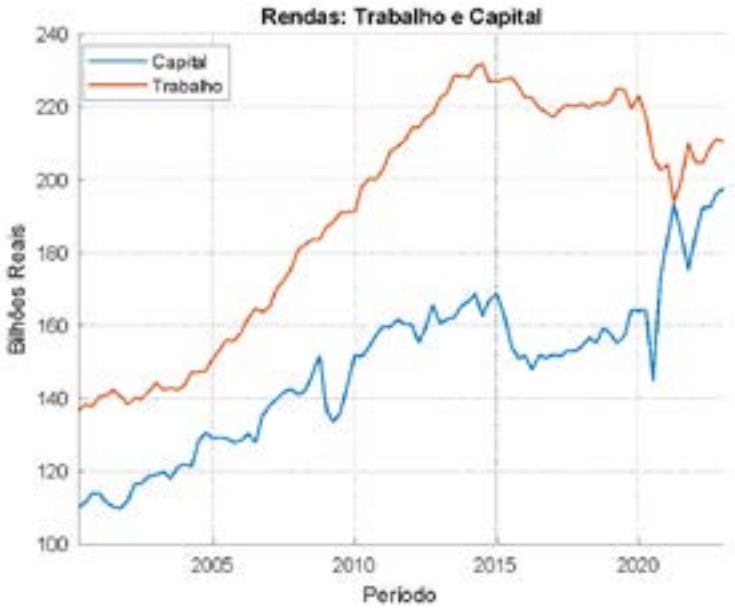


Source: Prepared by the author

Figure 2 below shows the evolution of capital and labor income during the period. From 2000 to 2005, labor income grew less than capital income, reflecting the period of adjustment and consolidation of the real economy. From 2005 to 2014, due to product expansion and the reduction in the unemployment rate, the wage bill grew at a faster pace, increasing the gap between labor and capital income. Finally, the great recession in the period 2014-2016 caused an increase in factor unemployment, and both incomes fell and were unable to recover until the end of the health crisis in 2021.

Some points should be highlighted: capital income reflects the impact of the American crisis, but the labor market does not. This is due to the characteristics of the labor market, which, due to labor legislation, is extremely rigid. This lack of flexibility also prolongs the economic cycle; after the layoffs during the 2014-2016 recession, the country stagnated, and entrepreneurs only began hiring again after the pandemic, when labor income began to grow again after more than six years of stagnation.

Graph 2 – Derived Income



Source: Prepared by the author

3. DEFINITION OF EFFECTIVE TAX RATES

Following Mendonza et al. (1994), Araújo Neto and Souza (2003), and Almeida et al. (2017), the OECD's standardized four-digit coding was used to simplify the empirical definition of effective tax rates in the Brazilian economy [4]. Tax collection data were aggregated using the following OECD four-digit code standard:

1100 – Taxation on the income, profits, and capital gains of individuals;

1200 – Taxation on the income, profits, and capital gains of corporations;

2000 – Social contributions;

2200 – Employer contributions to social security;

3000 – Payroll and labor force taxation;

4100 – Taxation on real estate property;

4400 – Taxation of financial and capital transactions;

5110 – Taxation on goods and services in general;

5121 – Specific taxation.

4 When necessary, the coding used by the National Treasury Secretary in the annual CTB (Tax Burden of Brazil) publication was also used.

Of the items described above, item 1100 consists of taxes on income that cannot be distinguished between capital and labor, such as Personal Income Tax (IRPF), Income Tax withheld at source – Remittances abroad (IRRF Remittances abroad) and Income Tax withheld at source – Other Income (IRRF – Other Income). Item 1200 refers to taxation on capital income and contains information on the following taxes: Corporate Income Tax (IRPJ), Social Contribution on Net Income (CSLL), and Withholding Income Tax – Capital (IRRF – Capital).

Employers' contributions, item 2200, consist of contributions to the RGPS (general social security regime) and private pension schemes, the Civil Servant Asset Formation Program (PASEP), the length-of-service guarantee fund (FGTS) and Imputed Social Contributions, which consist of the difference between social benefits paid by the government directly to its employees (beneficiaries of the Civil Servant Social Security Plan - PSS) in the form of retirement pensions, pensions, etc., and contributions received in the form of PSS, military pensions, civil mutual aid funds, etc.

Code 3000 comprises payroll taxes such as PIS charged on payroll[5] and education salary contributions and the S system (Sesi, Sesc, Senai, and Senac), in addition to other deductions related to union dues; contributions to maritime and aviation education; rural contributions; and contributions to the PMDF/BMDF health fund.

Code 2000 comprises all social contributions from employees and families, OECD heading 2100, employer contributions (2200), self-employed contributions (2300), and social contribution supplements (2400)[6].

Heading 4100 (Property taxes) comprises the following taxes: Urban Property Tax (IPTU), Motor Vehicle Property Tax (IPVA), Rural Property Tax (ITR), Real Estate Transfer Tax (ITBI), and Tax on Transfer of Property Causa Mortis and Donation (ITCMD).

Item 4400 (Taxation of financial and capital transactions) includes the following taxes: Provisional Contribution on Financial Transactions (CPMF) and Tax on Financial Operations (IOF).

Item 5110 (Taxation on goods and services in general) includes the following taxes: Import and Export of Goods and Services, Contribution on Intervention in the Economic Domain (CIDE), Contribution for Social Security Financing (COFINS); Tax on Industrialized Products (IPI), Import Taxes (II), Social Integration Program (PIS)[7] , as well as the Tax on the Circula-

5 Que difere do PIS sobre o faturamento conforme definido pela Receita Federal plan INC03, onde são separados a incidência dos tributos sobre a Renda, Salários, Consumo, Movimentação Financeira e Bens de capital, ver apêndice.

6 As rubricas 2100, 2300, 2400 equivalem aos códigos D.613/614 nas CEI, contas econômicas integradas.

tion of Goods and Services (ICMS) and Services Tax (ISS). Code 5121, specific taxation, refers to the IPI levied on automobiles, beverages, and tobacco; the ICMS on fuels and electricity, the CIDE on fuels, in addition to federal taxes, other state and municipal taxes.

Finally, the national account data used in this research were:

C – Household consumption;

G – Government Consumption (Government Spending) at the federal, state, and municipal levels;

GW – Compensation of employees paid by government service providers;

Y_L – Labor Income[8];

Y_K – Capital Income;

PEI – Household Property Income[9]

$OSPUE$ – Operating Surplus of Producing Households

OS – Gross Operating Surplus

Where household consumption (C) corresponds to the actual final consumption of households, described as household consumption expenditure plus consumption through social transfers in kind from public administration units or non-profit institutions serving households. In other words, it is the actual consumption of households recorded in the Integrated Economic Accounts (CEI). Government consumption or government spending (G) corresponds to the final effective consumption of public administrations.

The heading GW corresponds to compensation of employees paid by government service providers, while labor income Y_L and capital income Y_k have already been defined above.

PEI is the property income of households and companies, while $OSPUE$ is the income of producing households, also defined as RMB gross mixed income[10]. Furthermore, OS is the surplus of households, financial and non-financial companies, excluding the surplus of the Government and non-profit institutions IFSL.

7 In SCN 2010, the calculation of PIS no longer includes employer contributions. The tax is now levied on company revenues, code 4100.04. See tax burden in Brazil, STN.

8 Labor income YL includes employers' social contributions, as the CSE consists of social security contributions and FGTS, which is nothing more than today's work performed in exchange for a payment or a stream of payments in the future.

9 As in General Equilibrium models, households own all capital and property in the economy; this means that the property income of financial and non-financial companies is considered household income. The property income of the Government and IFSLs is excluded.

10 It should be noted that the operating surplus of households is the rental income imputed to resident tenants. In other words, although the nomenclature is similar, the concepts are distinct.

3.1. Alíquota Tributária Efetiva Média sobre a Renda Geral

The average effective tax rate on total income is the first step in obtaining the tax rate on labor income and capital income. The tax rate must be constructed because tax collection data generally do not differentiate between individuals' labor income and capital income. Therefore, it is necessary to divide these sources of income for taxation purposes. Thus, the tax rate on general income is given by:

$$\tau_{RG} = \frac{1100}{Y_K + W_A} \times 100 \tag{3}$$

$$\tau_{RG} = \frac{1100}{PEI + OSPUE + W} \times 100 \tag{4}$$

The tax rate levied on the representative agent's general income is given by the ratio between the tax revenue actually collected from income tax (which corresponds to the difference between individuals' income before and after taxation) and taxable income[11]. In equation (3), the denominator presents wages adjusted to incorporate RMB[12] added to capital income (Y_K). In equation (4), capital income defined by Mendonza et al. (1994) is added to wages.

The table below describes the data[13], frequency, and source of the series used to construct the tax rate on general income:

Table 3 - Taxes on General Income

Series	Period	Description	Source
IRPF	2000:01-2022:4	Tax Revenue Withholdings	STN
IRRF - Non-Residents	2000:01-2022:4	Tax Revenue Extractions	STN
IRRF – Others	2000:01-2022:4	Tax Revenue Extractions	STN
Salaries Paid	2000-2022	Integrated Economic Accounts	IBGE
PEI	2000-2022	Integrated Economic Accounts	IBGE
OSPUE	2000-2022	Integrated Economic Accounts	IBGE

Source: Prepared by the Author

3.2. Average Effective Tax Rate on Labor Income

According to what has been proposed so far, we have a clear definition of income that results in the product at factor cost. Nevertheless, these magnitudes provide a proportional di-

11 Most self-employed workers do not have their income taxed or do not contribute to social security, which is one of the main causes of the imbalance in public accounts.

12 Adjusted wages should not be confused with labor income, which includes employer contributions.

13 The data for 2022 from the national accounts are projections.

vision of general income, and the tax rate on labor income can be written as:

$$\tau_L = \frac{\tau_{RG}(W_A) + 2000 + 3000}{W_A + 2200} \times 100 \quad (5)$$

$$\tau_L = \frac{\tau_{RG}(W) + 2000 + 3000}{W + 2200} \times 100 \quad (6)$$

It should be noted that, in equation (6), labor income ignores the income of self-employed workers. Nevertheless, as Almeida et al. (2017) point out, this calculation should incorporate all social security contributions, as well as other payroll deductions as part of the revenue from labor income, in addition to correcting the tax base by including the social security contribution paid by employers in the denominator of the equation.

Table 4 below describes the data, frequency, and source of the series used to construct the tax rate on labor income:

Table 4 - Taxes on Labor

Series	Period	Description	Source
RGPS	2000:01-2022:4	Tax Revenue Extractions	STN
PIS/PASEP	2000:01-2022:4	Tax Revenue Extractions	STN
Education Salary	2000:01-2022:4	Tax Revenue Extractions	STN
IRRF-Work	2000:01-2022:4	Collection of Federal Revenue	SRF
FGTS	2000:01-2022:4	Gross Revenue Collection	IPEADATA
S System	2000-2022	Synoptic Tables	IBGE
Public Servant Contributions	2000-2022	Synoptic Tables	IBGE
Employer Contributions	2000-2022	Integrated Economic Accounts	IBGE
PSS	2000-2022	Tax Revenue Extractions	SRF

Source: Prepared by the Author

3.3. Average Effective Tax Rate on Capital Income

Like the labor tax rate, the average effective tax rate on capital income is calculated based on the portion of total income that is levied on individuals' capital: $\tau_{RG}Y_K$. Therefore, the average tax rate on capital income is given by:

$$\tau_K = \frac{\tau_{RG}Y_K + 1200 + 4100 + 4400}{Y_K} \times 100 \quad (7)$$

$$\tau_K = \frac{\tau_{RG}(PEI + OSPUE) + 1200 + 4100 + 4400}{OS} \times 100 \quad (8)$$

The denominator of equation (7) has already been derived above. The denominator of the

second equation (8), however, does not represent only the surplus of households and companies. It also includes the income of self-employed workers and excludes the surplus of public administrations and non-profit institutions.

Table 5 describes the data, frequency, and source of the series used to construct the tax rate on capital income:

Table 5 - Taxes on the Capital Factor

Series	Period	Description	Source
CPMF	2000:01-2022:4	Tax Revenue Withholdings	STN
IOF	2000:01-2022:4	Tax Revenue Extractions	STN
CSLL	2000:01-2022:4	Tax Revenue Extractions	STN
IRPJ	2000:01-2022:4	Tax Revenue Extractions	STN
IRRF-Capital	2000:01-2022:4	Tax Revenue Extractions	STN
IPVA	2000:01-2022:4	Federal Revenue Collection	SRF
ITR	2000:01-2022:4	Federal Revenue Collection	SRF
Export Tax	2000:01-2022:4	Federal Revenue Collection	SRF
ITBI	2000-2022	Synoptic Tables	IBGE
ITCMD	2000-2022	Synoptic Tables	IBGE
IPTU	2000-2022	Synoptic Tables	IBGE
Operating Surplus	2000-2022	CEI	IBGE

Source: Prepared by the Author

3.4. Average Effective Tax Rate on Consumption Income

The average tax rate on final consumption corresponds to the difference between the post--tax consumer price and the pre-tax price at which firms offer the good. Thus, the average effective tax rate levied on the representative agent's consumption in percentage terms is given by:

$$\tau_c = \frac{5110 + 5121}{C + G - G_W - 5110 - 5121} \times 100 \tag{9}$$

The numerator of equation 9 represents the revenue from indirect taxes and specific taxes levied on goods and services purchased by consumers. By definition, indirect taxation represents the difference between the nominal value of aggregate consumption at pre-tax and post--tax prices. As for the denominator, C represents the effective final consumption of households and $G - G_W$ the effective final consumption of the government (i.e., government consumption minus the remuneration of civil servants G_W) while items 5110 and 5121 remove the value of indirect taxation from the denominator. The denominator, therefore, is the basis for consumption taxation, which is a pre-tax value of consumption, measured as the value of post-tax consumption expenditure minus indirect consumption tax revenue.

It is worth noting here that although there is empirical evidence that the IOF and CPMF are passed on to consumers, according to the OECD, the tax on financial transactions,

code 4400[14], is levied on property. In addition, the STN has a specific account for taxes and contributions on current transactions: account 5000. Given the lack of definition, taxes on financial transactions were defined by the international reference standard; that is, accounted for as taxes levied on capital.

Table 6 describes the data, frequency, and source of the series used to construct the consumption tax rate:

Table 6 - Consumption Taxes

Series	Period	Description	Source
IPI	2000:01-2022:4	Tax Revenue Extractions	STN
Import Tax	2000:01-2022:4	Tax Revenue Extractions	STN
ICMS	2000:01-2022:4	Gross Revenue	SEFAZ
COFINS	2000:01-2022:4	Tax Revenue Extractions	STN
ISS	2000-2022	Synoptic Tables	IBGE
Household Consumption	2000:01-2022:4	SCNT	IBGE
Government Consumption	2000:01-2022:4	SCNT	IBGE
Government Worker Compensation (GW)	2000-2022	TRU	IBGE
CIDE	2000:01-2022:4	Tax Revenue Extractions	STN
PIS/PASEP	2000:01-2022:4	Tax Revenue Extractions	STN
PIS Consumption	2000-2022	CTB 2022	STN
Taxes - Services and Police Power	2000	CTB 2022	STN

Source: Prepared by the Author

4. RESULTS OF THE ANALYSIS

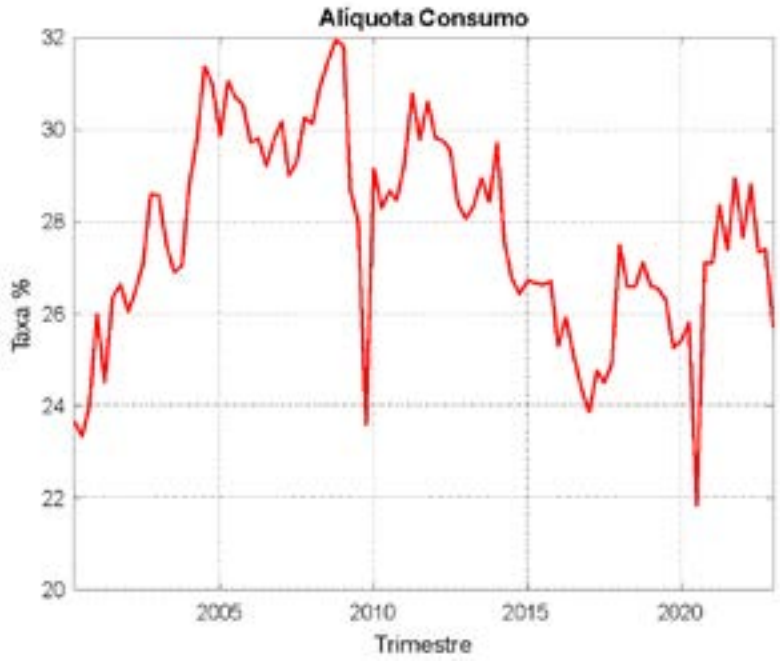
4.1. Tax Rate on Consumption

Taxation on consumption accounts for most of the tax revenue in Brazil. On average, the composition of taxes in the period 2000:1-2022:4 was 47.19% ICMS, 24.07% Cofins, 6.57% IPI, 5.74% ISS, 5.03% PIS, 3.90% Import Tax, and 3% Federal and State Fees. CIDE collection began in 2002, was interrupted in 2012, and then resumed in 2015. As a result, there were minor changes in the composition of tax revenue.

According to the graphical analysis, consumption taxes grew above GDP from 2000 to 2008. There was a significant reduction and a rapid recovery in the 2008-2009 recession, and from 2014 onwards, they fell and remained stagnant. The behavior of tax revenue during the two shocks allows us to conclude that consumption taxes are more sensitive to the economic cycle than GDP. This elasticity is evident in the pandemic, when tax revenue fell much more than GDP at the height of the shutdown. The graph below describes the effective tax rate on consumption between 2000:1 and 2022:4:

14 The OECD classification of taxes and interpretative guide

GRAPH 3 – Tax Rate on Consumption



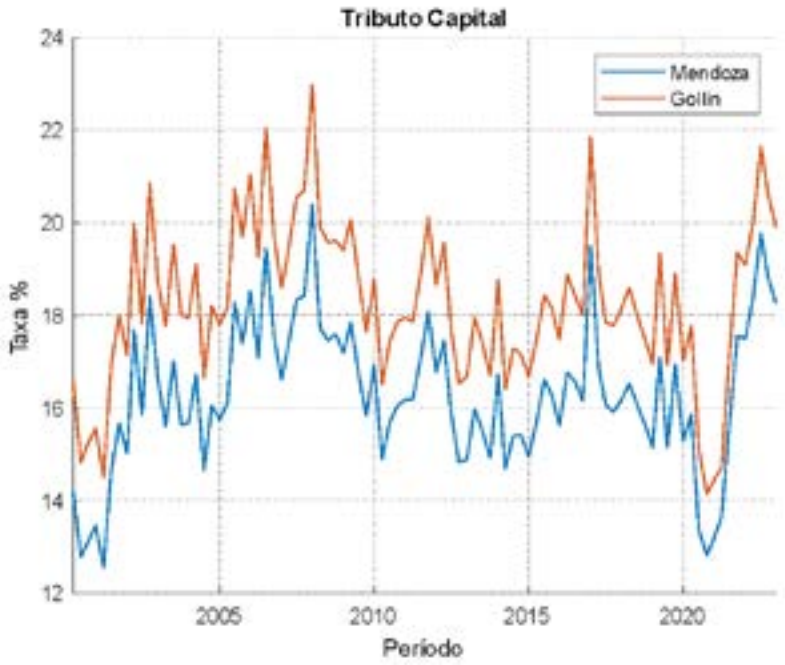
Source: Prepared by the author

4.2 Tax Rate on Capital

On average, the composition of taxes on capital in the period 2000:1-2022:4 was 32.64% IRPJ, 18.16% CSLL, 13.56% IRRF, 8.72% IPTU, 8.4% IPVA, 7.68% IOF, 3.67% ITBI-IT-CMD, 0.3% ITR, and 0.02% Export Tax. While it was in force, the CPMF accounted for more than 15% of capital taxes. With the abolition of the tax, part of the revenue was offset by a substantial increase in the IOF; however, income tax has always accounted for the largest share of revenue.

Graph 4 below shows rapid growth in capital taxes until the 2008 international crisis, when there was a significant drop. In the following years, the same level was maintained until a further decline during the pandemic.

GRAPH 4 – Capital Tax Rate[15]



Source: Prepared by the author

4.3 Tax Rate on Labor

Taxation on labor constitutes the second largest part of tax revenue in Brazil after consumption. On average, the composition of taxes in the period 2000:1-2022:4 was 17.38% IRRF, 13.71% Civil Servant Contributions[16], 47.63% contributions to the RGPS, 14.83% FGTS, 1.38% PASEP contributions, and 5.06% other payroll taxes.

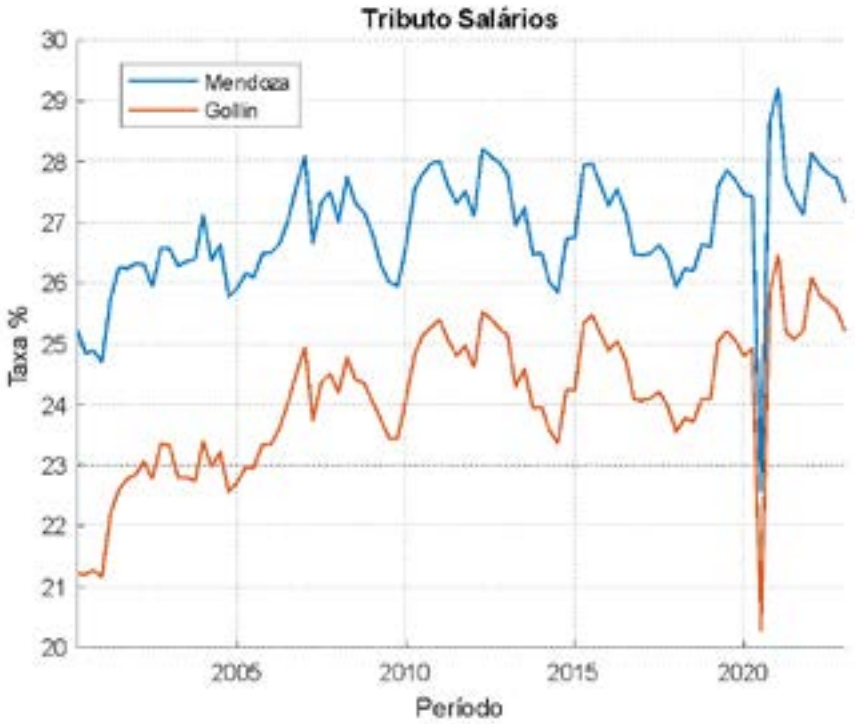
According to the graphical analysis, taxes on labor grew above GDP throughout the period. Compared to other factors, tax collection on labor was less affected by the 2008-2009 recession and, after peaking in 2014, declined during the 2014-16 recession. The reduction in taxes on labor accompanied the decline in employment, but at a lower level. A similar phenomenon was repeated during the pandemic, although the government helped to maintain formal jobs by reducing social security contributions for some sectors, as well as through programs such as PRONAMPE[17].

15 The data in the graph were seasonally adjusted using the Census X13 program.

16 There are three databases with statistics on tax collection: FGV, STN, and SCN. Starting in 2010, in particular, the contribution of civil servants in the SCN began to diverge from the other databases. In this study, the tax rate on labor follows the statistics observed in the SCN.

17 Informal workers, especially the self-employed, were the most affected by the great recession of 2014-2016 and during the pandemic. Hence the importance of Brazil Aid in amortizing the shock initiated by the health crisis.

CHART 5 – Labor Tax Rate



Source: Prepared by the author

4.4 Statistics

Table 7 below presents the descriptive statistics of the series estimated in this study. Column 1 presents the series of the tax burden on labor following the methodology of Mendoza et al. (1994). In this case, labor income is restricted to wages and salaries and, therefore, the denominator is smaller than the adjustment proposed by Gollin (2002). The percentage difference between the two methodologies, averages of columns 1 and 2, results in 3pp. The third row of Table 7 presents statistics on the tax burden on capital following the methodology of Mendonza et al. In this case, the rates differ by 4 pp, with the higher average resulting from the partial reduction of self-employed income in the denominator of the rate.

Table 7 - Statistics on the Tax Burden on Consumption and Factor Income							
Descriptive Statistics	*TLM*	*TLG*	*TKM*	*TKG*	*TC*	*RGG*	*RGM*
Average	26.8695	24.0769	16.2880	18.2656	27.7406	1.3410	1.1439
Standard error	0.1002	0.1247	0.1655	0.1812	0.2288	0.0350	0.0387
Median	26.7773	24.1548	16.1588	18.0734	27.6045	1.3063	1.0952
Standard deviation	0.9606	1.1964	1.5386	1.7381	2.1949	0.3356	0.3708
Curtosis	3.5224	0.6041	0.2168	0.2977	-0.4554	-0.4614	-0.2940
Asymmetry	-0.9917	-0.7403	-0.0598	-0.0182	-0.2478	0.4248	0.6026
Interval	6.6314	6.1716	7.8384	8.8425	10.1127	1.3621	1.5789
Minimum	22.5690	20.2804	12.5539	14.1367	21.8204	0.7772	0.5411
Maximum	29.2004	26.4520	20.3922	22,9793	31,9331	2,139	2.1200
Total	2471.99	2,215.07	1,498.49	1680.44	2552.14	123.37	105.24
Contagem	92.00	92.0000	92.0000	92.0000	92.0000	92.0000	92.00
Highest(1)	29.2004	26.4520	20.3922	22,9793	31,9331	2,139	2.1200
Minor(1)	22.5690	20.2804	12.5539	14.1367	21.8204	0.7772	0.5411
Confidence level (95.0%)	0.1989	0.2478	0.3288	0.3599	0.4546	0.0695	0.0768

Source: Prepared by the author

The consumption tax rate has the highest average among all tax rates and, to a lesser extent, the phenomenon of tax substitution is repeated as in Araújo Neto and Souza (2003). Regarding the volatility of the series, it is easier to understand the magnitude of this statistic, knowing that the volatility of the GDP growth rate at basic prices has a standard deviation of 1.65%. Consequently, only taxes on capital, Gollin's adjustment (2002), and taxes on consumption are more volatile th[18]. The other series are within expectations, with volatility around 1%.

Regarding the distribution of the series, the only one that approaches normality is the *TLM*, but the normality hypothesis was tested and rejected for all series. Furthermore, only the first series is mesocuritic, the others are platykuritic (flat), and none of them are symmetric. Nevertheless, in order to compare the series generation process, without seasonal adjustment, a standard routine developed by Victor Gomez (2019) was used, which, through a series of tests, *arimaeasy*, identifies both the regular pattern of the series and the seasonal pattern.

In the table below, (pr, dr, qr) defines a regular ARIMA process, where pr refers to the regular autoregressive process, qr to the moving average process, and dr to the degree of integration of the series, while (ps, ds, qs) are their seasonal counterparts in the categorization of the multiplicative process: *(pr ,dr, qr) (ps, ds, qs) _freq*.

That said, the labor, capital, and consumption series are not stationary. The consumption series differs from the others in that it does not have a seasonal component. The other series have strong seasonality, reflecting tax collection in the fourth quarter. It should also be noted that the adjustment in labor income with marginal increases from gross mixed income changes

18 The series on general income are not commented on, as they are part of labor income and capital, as proposed above.

the level, but not the process generating the series. The AR(1) statistics in the regular process and the MA(4) component in the seasonal process have very similar statistics, suggesting that there was no significant change in the process generating the rates on capital and labor, regardless of the methodology used.

Table 8 - Series Generating Process

		TLM		TLG		TKM		TKG		TC	
(pr, dr, qr)	(ps, ds, qs)_sa	(1,0,0)	(0,1,1)_4	(1,0,0)	(0,1,1)_4	(1,1,0)	(-0,1,1)_4	(1,1,0)	(-0,1,1)_4	(0,1,1)	(0,0,0)
AR(1)	t-ratio	-0.3483	-3.3935	-0.3388	-3.3191	0.4148	4.1572	0.4144	4.1429	X	X
MA(4)	t-ratio	-0.8224	-7.3196	-0.8272	-7.0386	-0.7809	-28.2074	-0.7896	-4.7409	X	X
MA(1)	t-ratio									-0.3263	-3.3621

Source: Prepared by the author

5. CONCLUSIONS AND POLICY IMPLICATIONS

Average macroeconomic effective tax rates have been widely used in international literature in empirical economic analyses to approximate taxes that distort fundamental economic decisions, mainly related to employment, savings, and investment. The Brazilian tax system, in turn, has several rates, taxes, and contributions; some with fixed rates, others that depend on income level and family characteristics, making it necessary to calculate the average tax rates faced by the agent (Almeida et al. (2017)).

The objective of this study was to calculate these tax rates related to consumption, labor income, and capital income, using tax collection and national accounts data for the period from 2000 to 2022, following the methodology proposed by Mendonza et al. (1994) and an alternative following one of the adjustments in the functional distribution of income proposed by Gollin (2002).

By comparing the two methodologies, we sought to distribute more equitably the burden arising from the distortion resulting from the taxation of capital and labor income. In terms of policy implications, the results indicated that the government has placed a heavier burden on formal labor income and consumption. Tax rates on formal labor income are increased by the growing volume of social contributions (and indirectly social security) on the gross tax burden. It should be noted that if the tax burden on payroll could be extended to the income of informal workers in the economy, the effective tax rate on labor would be lower. The inclusion of informal workers, therefore, automatically translates into a reduction in tax distortions.

Alternatively, it should be noted that, regardless of the methodology used, a higher tax rate on labor income can alter incentives and the leisure-work *trade-off*, as well as increase the cost to companies of hiring workers, encouraging informality in the labor market.

Regarding the tax rate on capital, there is a well-established macroeconomic interpretation:

Chamley (1986) *apud* Ljungqvist and Sargent (2004) demonstrates that if there is a steady-state equilibrium, the optimal fiscal policy is to set a tax rate on capital equal to zero. This stems from the fact that theoretical models and empirical analysis indicate that capital taxation has the greatest impact on social welfare by distorting companies' investment decisions. Therefore, when part of the income of self-employed workers is interpreted as labor income, the distortions caused by the reduced capital tax rate in the conventional adjustment decrease, allowing researchers to more accurately measure the loss resulting from this type of taxation.

As in Araújo Neto and Souza (2003), this study corroborates the tax substitution thesis insofar as the effective tax rate on household consumption is the highest. The consumption tax is regressive, harming the poorest households. It is noteworthy that the income tax withheld at source from wage earners is higher, in aggregate terms, than the tax actually collected from individuals. As the tax rate on the highest income bracket is significant, the study suggests that there is considerable tax avoidance. Some property taxes, such as the ITR (land and property tax) or inheritance tax (ITCMD), are also extremely low. In short, in addition to the distortions observed in the literature, the Brazilian tax burden is regressive, full of subsidies to interest groups, and, ultimately, those who earn less pay more taxes proportionally.

6. BIBLIOGRAPHICAL REFERENCES

ALMEIDA, Vinícius et al. Alíquotas tributárias efetivas médias para a economia brasileira: uma abordagem macroeconômica. **Revista Brasileira de Economia**, v. 71, p. 153-175, 2017. https://www.scielo.br/j/rbe/a/M6ZJ43KB8TMFBYXntNTGBVx/?format=html&lang=pt

AZEVEDO, Cyntia Freitas; FASOLO, Angelo Marsiglia. Effective tax rates on consumption and factor incomes: a quarterly frequency estimation for Brazil. **Brasília: BCB, 2015a.(Working Paper 398)**, 2015. https://liftchallenge.bcb.gov.br/content/publicacoes/WorkingPaperSeries/wps398.pdf

COOLEY, Thomas F. Frontiers of Business Cycle Research. 1995.

CONSIDERA, Claudio Monteiro; PESSOA, Samuel de Abreu. A distribuição funcional da renda no Brasil no período 1959-2009. 2013. https://www.researchgate.net/profile/Claudio-Considera/publication/260714366_A_DISTRIBUICAO_FUNCIONAL_DA_RENDA_NO_BRASIL_NO_PERIODO_1959-2009/links/0deec5320fd856f396000000/A-DISTRIBUICAO-FUNCIONAL-DA-RENDA-NO-BRASIL-NO-PERIODO-1959-2009.pdf

DE ARAÚJO NETO, Valter Borges; DE SOUSA, Maria da Conceição Sampaio. Tributação da renda e do consumo no Brasil: Uma abordagem macroeconômica. **Estudos Econômicos (São Paulo)**, v. 33, n. 1, p. 5-42, 2003. https://scholar.google.com.br/scholar?hl=pt-BR&as_sdt=0%2C5&q=ara%C3%BAjo+neto+e+souza+%282003%29+tax+rates&btnG=

GOLLIN, Douglas. Getting income shares right. **Journal of political Economy**, v. 110, n. 2, p. 458-474, 2002. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=bea62112279a43963444c6633834646e7b617b6f

GÓMEZ, Víctor et al. **Linear time series with MATLAB and OCTAVE**. Cham: Springer International Publishing, 2019.

JONES, John Bailey. Has fiscal policy helped stabilize the postwar US economy?. **Journal of Monetary Economics**, v. 49, n. 4, p. 709-746, 2002. https://www.sciencedirect.com/science/

article/abs/pii/S0304393202001137

LEEPER, Eric M.; PLANTE, Michael; TRAUM, Nora. Dynamics of fiscal financing in the United States. **Journal of Econometrics**, v. 156, n. 2, p. 304-321, 2010. https://www.nber.org/system/files/working_papers/w15160/w15160.pdf

LITTERMAN, Robert B. A random walk, Markov model for the distribution of time series. **Journal of Business & Economic Statistics**, v. 1, n. 2, p. 169-173, 1983.

LLEDO, Victor. Tax systems under fiscal adjustment: a dynamic CGE analysis of the Brazilian tax reform. 2005. https://papers.ssrn.com/Sol3/Delivery.cfm?abstractid=888011

LUCAS JR, Robert E. On the mechanics of economic development. **Journal of monetary economics**, v. 22, n. 1, p. 3-42, 1988. https://papers.ssrn.com/sol3/delivery.cfm/nber_R1176.pdf?abstractid=227120

MENDOZA, Enrique G.; RAZIN, Assaf; TESAR, Linda L. Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption. **Journal of Monetary Economics**, v. 34, n. 3, p. 297-323, 1994. https://www.nber.org/system/files/working_papers/w4864/w4864.pdf

MÖNCH, Emanuel; UHLIG, Harald. Towards a monthly business cycle chronology for the euro area. **Journal of business cycle measurement and analysis**, v. 2005, n. 1, p. 43-69, 2005. https://www.oecd-ilibrary.org/content/paper/jbcma=2005-5-k7m183v48tr?crawler=true&mimeType-application%2Fpdf

KALDOR, Nicholas. Capital accumulation and economic growth. In: **The Theory of capital: proceedings of a conference held by the International Economic Association**. London: Palgrave Macmillan UK, 1961. p. 177-222. http://gesd.free.fr/kaldor61.pdf

QUILIS, Enrique M. **A Matlab library of temporal disaggregation and interpolation methods**. working paper, Ministry of Economy and Competitiveness, Spain, 2013. https://back.nber.org/appendix/w22279/analysis/code/TD/TD.pdf

SARGENT, Thomas J; LJUNGQVIST, Lars. Recursive macroeconomic theory. **Massachusetss Institute of Technology**, 2000. https://toc.library.ethz.ch/objects/pdf/e01_978-0-262-01874-6_01.pdf

6.1 Data

Extractions from Tax Revenues National Treasury – STN

https://www.tesourotransparente.gov.br/temas/estatisticas-fiscais-e-planejamento/estatisticas-fiscais-do-governo-geral

SCN – System of National Accounts

https://www.ibge.gov.br/estatisticas/economicas/industria/9052-sistema-de-contas-nacionais-brasil.html

IBGE – Statistics and Research

https://sidra.ibge.gov.br/acervo#/S/Q

SRF – Federal Revenue Collection (States, monthly)

https://www.gov.br/receitafederal/pt-br/centrais-de-conteudo/publicacoes/relatorios/arrecadacao-federal

IPEADATA – General Data

http://www.ipeadata.gov.br/Default.aspx

*Data from PMEA, monthly employment survey from 2000 to 2002, and gross ICMS were downloaded from IPEADATA

APPENDIX A

Quantitative economic analysis is based on data, and some of this data is only available annually. However, it is essential to obtain data from national accounts on a monthly or quarterly basis. The process of deriving high-frequency data (quarterly, monthly) from low-frequency data (annual) is known as temporal disaggregation.

There is a wide variety of methodologies that use temporal disaggregation. Among the univariate adjustment methods are those proposed by Denton (1971) and Ginsburgh (1973) *apud* Quilis (2013).

In addition to these, Wei and Stram (1990), Guerrero (1990), and Guerrero and Martinez *apud* Quilis (2013) combine ARIMA model techniques with the use of high-frequency series in order to overcome some arbitrariness in the choice of the process that generates errors in high--frequency variables. Nevertheless, in structural models such as that proposed by Mönch and Uhlig (2005), the components of the time series are obtained through the Kalman filter, while Proietti (2004) and Santos-Cardoso (2001) *apud* Quilis (2013) propose dynamic models based on the cointegration of variables.

In this study, we used methods derived from the methodology introduced by Chow-Lin (1971) and refined by Bournay and Laroque (1979), Fernandez (1981), and Litterman (1983) *apud* Quilis (2013). The choice of methodology was based on the following reasons: 1) The sample is quite large, minimizing arbitrariness; 2) Some of the disaggregated series are better adjusted with the addition of more than one indicator; 3) Disaggregation is based on raw data in order to preserve the seasonality of the series. 4) The three methodologies are easily applicable and the results are directly comparable.

HERE IS A BRIEF DIGRESSION ON THE MODELS USED.

$y_t = y_{A,t}$ is defined as high-frequency data at time t. Similarly $y_t = y_{B,t}$ is low-frequency data at time t. In order to relate high- and low-frequency data, a temporal aggregation matrix is used. For example, quarterly data converted to annual data:

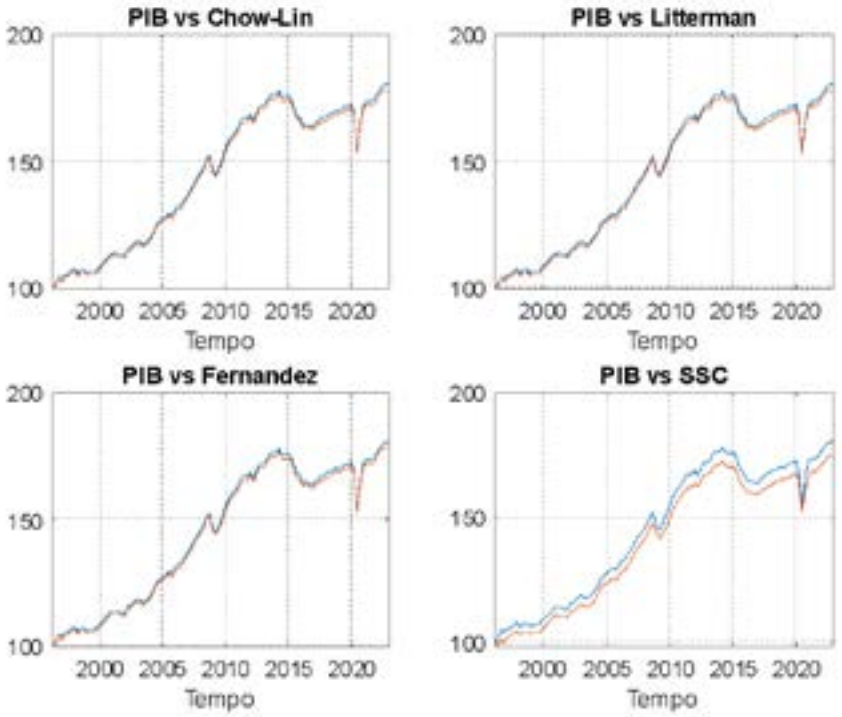
$$D_{t,4t} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & .. & .. & .. \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .. & .. & 1 \end{pmatrix}$$

Such that:

$$Y_{B,t} = D\left(X_{A,t}^{T}\beta + u_{A,t}\right)$$
$$\Delta u_{A,t} = \theta \Delta u_{A,t-1} + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$$

Litterman (1983) explicitly assumes that the process $u_{A,t}$ has an initial point $t = 0$ such that $\Delta u_{A,0} = u_{h,0} = 0$. Fernandez assumes that $\theta = 0$ and, therefore, $u_{A,t}$ would behave like a random walk. Chow and Lin postulate that the errors are stationary and random.

Example: Disaggregation of annual GDP at market prices as a function of quarterly GDP at basic prices in the period 1996Q1-2022Q4



In fact, as observed in the graphical analysis, the SSC method seems to present the greatest deviation from GDP. However, the *sigma* statistic, which consists of the root of the deviations of GDP observations from the observations of the extrapolated variables divided by the sample size n and the number of estimated parameters k; presents the lowest score, or the best fit.

$$\sigma = \frac{1}{n-k}\sqrt{\sum\left(g_{PIBi} - g_{Xi}\right)^2}$$

The SSC methodology also presents the lowest AIC and SIC statistics, suggesting a better fit of the model according to the table below:

Comparison of Time Series Disaggregation Methods			
Method	AIC	SBC	Sigma
Chow-Lin	-5.7094	-5.5654	0.00018
Fernandez	-5.9747	-5.8787	0.00021
Litterman	-5.9006	-5.7576	0.00021
SSC	-3.6495	-3.5055	0.00006

In the graphical analysis, however, all methodologies prove to be satisfactory. Finally, as the sample size of the analysis is close to 100 observations, it can be concluded, given the low scores presented in all methodologies, that with the correct indicators, the disaggregation methodology does not compromise the quality of the work.

DISAGGREGATED VARIABLES

Indicators: 1. GDP Basic Prices, 2. PME (Old methodology), 3. PME, 4. PNAD, 5. PNADC, 6. GDP Factor Costs, 7. Wages Paid 8. YL , 9. CSE , 10. YK, 11. RMB, 12. Property Income, 13. RGPS, 14. CPSS, 15. PASEP, 16. PIS/PASEP, 17. Education Salary, 18. GDP Services, 19 GDP Market Prices

Table 9 - Disaggregated Variables			
Variable	Indicator	Correlations[19]	Method
GDP-Factors	1	yoy rates: 0.9744	Litterman
Salaries Paid	2,3,4,5	yoy rates: 0.9707	Litterman
CSE	7	yoy rates: 0.8622	Litterman
YL	7.11	yoy rates: 0.9733	Litterman
Income Property	6, 10, 11	yoy rates: 0.4623	Litterman
OS	6, 10, 12	yoy rates: 0.9498	Litterman
PASEP	16	yoy rates: 0.8905	Litterman
Payroll	17	yoy rates: 0.9057	Litterman
ISS	1.18,19	yoy rates: 0.7951	Litterman
Taxes Other Taxes	1.18,19	-yoy rates: 0.4623	Litterman
CPSS (States Municipalities)	13, 14, 16	yoy rates: 0.4562	Litterman
Property Tax	1.18,19	yoy rates: 0.6951	Litterman

Source: Prepared by the author

APPENDIX B

Tabela 10 - Alíquotas Tributárias Efetivas (Mendoza e Gollin)

TRI	TLG	TLM	TKG	TKM	RG G	RGM	TC	TRI	TLG	TLM	TKG	TKM	RG G	RG M	TC
2000 T1	20.48	24.49	19.79	16.87	0.82	0.72	23.65	2011 T3	23.93	26.42	17.58	15.85	1.24	1.03	30.58
2000 T2	20.34	23.68	14.07	12.22	1.44	1.28	23.33	2011 T4	26.68	29.36	18.25	16.44	1.31	1.09	29.81
2000 T3	20.46	23.96	13.58	11.74	0.90	0.79	23.97	2012 T1	25.05	27.77	23.33	20.67	0.97	0.83	29.74
2000 T4	23.56	27.54	14.76	12.72	0.90	0.79	25.98	2012 T2	24.91	27.41	16.92	15.29	1.80	1.54	29.53
2001 T1	21.44	24.95	17.68	15.19	0.83	0.70	24.48	2012 T3	24.21	26.89	13.89	12.50	1.24	1.09	28.37
2001 T2	21.72	25.10	16.25	14.14	1.51	1.26	26.33	2012 T4	27.15	29.99	16.26	14.54	1.35	1.18	28.05
2001 T3	21.99	25.33	16.36	14.31	1.19	0.83	26.63	2013 T1	23.83	26.52	21.81	19.27	1.00	0.89	28.34
2001 T4	25.36	29.23	16.34	14.26	0.97	0.68	26.03	2013 T2	24.12	26.62	16.60	14.89	1.84	1.63	28.93
2002 T1	22.25	25.45	23.16	20.35	0.99	0.54	26.46	2013 T3	22.86	25.32	13.99	12.51	1.24	1.11	28.41
2002 T2	21.86	24.76	17.11	15.33	1.36	0.74	27.06	2013 T4	25.91	28.65	18.36	16.39	1.43	1.28	29.71
2002 T3	22.54	25.66	19.15	17.01	1.23	0.68	28.60	2014 T1	23.19	25.64	20.35	18.08	1.05	0.94	27.57
2002 T4	25.94	29.57	17.98	15.94	1.08	0.60	28.57	2014 T2	22.92	25.25	16.45	14.75	1.75	1.56	26.74
2003 T1	21.92	25.38	20.92	18.21	0.93	0.66	27.49	2014 T3	23.13	25.54	14.43	13.00	1.30	1.14	26.42
2003 T2	21.88	25.20	18.89	16.57	1.34	0.94	26.90	2014 T4	26.12	28.83	16.22	14.58	1.26	1.10	26.70
2003 T3	21.85	25.44	16.18	14.11	0.96	0.80	27.04	2015 T1	25.00	27.62	21.55	19.23	1.07	0.88	26.65
2003 T4	26.06	30.14	17.22	15.00	0.98	0.82	28.74	2015 T2	25.10	27.44	17.56	15.92	2.07	1.68	26.62
2004 T1	22.09	25.44	22.26	19.35	0.83	0.70	29.76	2015 T3	24.01	26.39	15.39	13.79	1.27	1.00	26.70
2004 T2	22.36	25.54	16.04	14.24	1.37	1.17	31.37	2015 T4	26.73	29.31	16.99	15.23	1.29	1.02	25.29
2004 T3	21.66	24.81	16.26	14.42	0.96	0.81	30.97	2016 T1	24.70	27.23	23.06	20.39	1.12	0.89	25.91
2004 T4	25.37	28.90	17.15	15.16	0.99	0.83	29.84	2016 T2	24.40	26.67	17.48	15.82	1.80	1.44	25.08
2005 T1	22.04	25.20	21.31	18.71	0.78	0.61	31.04	2016 T3	22.89	25.25	15.22	13.67	1.29	1.05	24.38
2005 T2	22.11	25.02	20.25	17.97	1.49	1.17	30.68	2016 T4	25.95	28.53	21.35	19.07	1.38	1.12	23.85
2005 T3	22.49	25.57	17.63	15.63	1.14	0.87	30.54	2017 T1	23.72	26.11	23.21	20.60	1.22	1.03	24.77
2005 T4	25.96	29.44	20.48	18.02	0.96	0.73	29.70	2017 T2	23.86	26.13	16.86	15.24	1.91	1.61	24.49

2006 T1	22.64	25.66	22.38	19.67	0.78	0.59	29.80	2017 T3	22.83	25.22	15.03	13.44	1.31	1.16	24.90
2006 T2	23.20	25.96	21.58	19.17	1.58	1.18	29.20	2017 T4	25.51	28.09	17.52	15.65	1.40	1.24	27.49
2006 T3	23.70	26.71	17.57	15.70	1.14	0.87	29.77	2018 T1	23.31	25.78	22.96	20.37	1.28	1.19	26.58
2006 T4	27.52	30.95	18.11	16.18	1.10	0.84	30.16	2018 T2	23.31	25.66	17.00	15.24	1.95	1.81	26.57
2007 T1	22.81	25.74	22.63	20.04	0.91	0.70	28.99	2018 T3	22.95	25.43	14.79	13.23	1.51	1.44	27.10
2007 T2	23.55	26.31	20.03	18.00	1.63	1.25	29.29	2018 T4	26.21	28.90	16.22	14.50	1.69	1.60	26.59
2007 T3	23.65	26.59	18.42	16.46	1.28	0.99	30.25	2019 T1	24.47	27.03	23.77	21.00	1.42	1.32	26.54
2007 T4	26.67	29.74	22.59	20.07	1.41	1.09	30.12	2019 T2	24.75	27.26	15.90	14.28	2.09	1.95	26.29
2008 T1	24.00	26.98	23.16	20.45	1.05	0.83	30.94	2019 T3	23.93	26.49	16.28	14.60	1.49	1.42	25.26
2008 T2	23.64	26.34	18.98	17.04	1.65	1.30	31.47	2019 T4	27.13	29.96	16.20	14.55	1.79	1.71	25.41
2008 T3	23.42	26.18	17.26	15.54	1.30	1.05	31.93	2020 T1	24.19	26.73	22.21	19.79	1.44	1.44	25.79
2008 T4	26.41	29.41	19.05	16.90	1.39	1.10	31.81	2020 T2	19.69	21.88	14.10	12.53	1.74	1.73	21.82
2009 T1	23.13	25.68	23.45	20.67	1.07	0.84	28.67	2020 T3	24.71	27.43	11.54	10.49	1.59	1.63	27.10
2009 T2	22.71	25.09	18.18	16.28	1.56	1.24	27.97	2020 T4	29.04	31.94	13.60	12.42	1.80	1.87	27.08
2009 T3	22.44	24.89	15.21	13.70	1.23	1.00	23.57	2021 T1	24.40	26.94	19.08	17.55	1.34	1.36	28.36
2009 T4	26.38	29.10	18.43	16.66	1.35	1.11	29.15	2021 T2	24.38	26.62	16.27	14.95	2.10	2.10	27.37
2010 T1	24.28	27.02	19.99	17.82	0.87	0.72	28.29	2021 T3	24.01	25.82	16.80	15.27	1.73	1.67	28.95
2010 T2	24.49	26.97	16.76	15.19	1.66	1.39	28.67	2021 T4	28.77	30.96	18.18	16.65	1.84	1.80	27.63
2010 T3	24.25	26.90	15.37	13.86	1.12	0.94	28.46	2022 T1	24.99	27.19	24.32	22.22	1.34	1.33	28.82
2010 T4	27.55	30.38	17.57	15.86	1.29	1.08	29.23	2022 T2	25.00	27.03	20.78	19.05	2.14	2.12	27.33
2011 T1	24.55	27.13	21.47	19.24	1.01	0.84	30.77	2022 T3	24.35	26.41	18.09	16.55	1.48	1.47	27.38
2011 T2	24.20	26.54	18.14	16.51	1.77	1.47	29.76	2022 T4	27.87	30.14	18.97	17.39	1.62	1.62	25.67

Fonte: Elaborado pelo autor