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Revisiting the Long-Term Income Elasticity of Social Security Collections in Brazil: Empirical Evidence from 1997 to 2023

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ABSTRACT

This study aims to measure the elasticity of pension revenues from the General Social Security Regime (RGPS) in relation to the Gross Domestic Product (GDP) in Brazil, considering monthly data from January 1997 to July 2023. Based on a rigorous analysis of stationarity, co-integration and temporal causality, considering the presence of structural breaks, the results obtained from the estimation of an autoregressive vector model with an error correction mechanism (VECM) indicate that social security revenues are elastic in relation to GDP in the long term.

Keywords: social security, GDP, elasticity, cointegration, VECM.
JEL Code: H5, H55, C2, C22.

SUMMARY

1. INTRODUCTION

The search for social security sustainability is a challenge that has been faced by several countries, including Brazil. It is a complex discussion involving economic, social, legal and political aspects. In Brazil, social security can be classified into three distinct regimes: the General Social Security Regime (RGPS), the Private Social Security Regime (RPPS) and the Complementary Social Security Regimes (RPC). Among the public schemes, the RGPS is the one that covers the majority of Brazilian workers, while the RPPS are the schemes for civil servants and the military. RPCs are private schemes offered by companies and financial institutions.

The RGPS is a simple pay-as-you-go system, in which benefits are paid out of the contributions of active workers. This means that workers who are currently contributing are paying for the benefits of pensioners. However, the RGPS has been showing financial deficits due to various factors. On the expenditure side we have factors such as the increase in life expectancy concomitant with the reduction in the birth rate. On the revenue side, we have factors such as historical fluctuations in the level of employment, formality in the labor market and economic growth.

Some social, economic and political groups have high hopes that continued national economic growth will partially or fully eliminate the growing deficit projections for the RGPS. In this context, understanding the empirical essence of social security collection is crucial for planning public policies, anchoring expectations and referencing fiscal projections about the future of Brazilian social security.

One way of analyzing how feasible the expectations of economic growth are for the sustainability of the RGPS is to measure the elasticity between the two. Elasticity shows the economic relationship between two variables. In other words, it quantifies these relationships and looks at the impact that one variable can have on the other. Given the importance of understanding how the level of economic activity affects social security collection, this study seeks to answer the following question: what is the elasticity of RGPS social security collection in relation to GDP? In this way, we can understand how much a variation in GDP affects social security collection for the RGPS.

The elasticity of social security collection in relation to Gross Domestic Product (GDP), also known as the income elasticity of social security collection, is a measure that assesses how social security collection in Brazil varies in response to changes in GDP. This measure is useful for understanding the sensitivity of social security collection in relation to the level of economic activity. When measuring the elasticity between two variables, the proportional variation of one in relation to the other over a period is calculated. Thus, as a certain percentage change in GDP occurs, social security collection can vary by a smaller, equal or greater proportion. This study will therefore address the following research hypotheses.

The first hypothesis is based on the argument that social security revenues are elastic in relation to GDP. An elasticity greater than 1 would indicate that social security revenues are elastic in relation to GDP, which means that social security revenues grow at a proportionally higher rate than GDP. The second hypothesis is based on the argument that social security revenues are inelastic in relation to GDP. An elasticity of less than 1 would indicate inelasticity, suggesting that social security revenues grow at a proportionally lower rate than GDP. Finally, a third hypothesis is based on the argument that social security revenues have unitary elasticity in relation to GDP. Under this argument, social security revenues grow at a rate proportionally equal to GDP.

The general objective of this study is to analyze the relationship between social security revenues and the level of economic activity by estimating the elasticity of RGPS social security revenues in relation to GDP. Elasticity studies are important for understanding market structures and their effects on economic agents. Elasticities can be used to assess the impact of price changes on consumer and producer surplus, as well as for government planning. Calculating the elasticity of social security revenues in relation to GDP is essential for understanding the empirical essence of social security revenues, formulating public policies, anchoring expectations and referencing fiscal projections about the future of Brazilian social security. The specific objectives are: (i) to consolidate official GDP and RGPS collection data as of July 2023; (ii) to verify the stationarity of the seasonally adjusted time series; (iii) if the time series analyzed show stationarity properties in first differences, then the cointegration between the time series should be evaluated in order to avoid the problem of spurious regression; (iv) use the vector autoregressive model approach with an error correction mechanism to measure the elasticity of RGPS social security collection in relation to GDP .

This study is delimited as follows. Firstly, the elasticity of social security collection in relation to GDP can be influenced by various factors, including demographic changes, social security policies, contribution rates, employment levels and other economic and social variables. However, in this study the focus will be on analyzing the sensitivity of social security collection in relation to the level of economic activity. Secondly, this study did not look at the revenues of the federal entities' RPPS or the RPCs of any institution.

Studies on social security are important because they make it possible to understand how it works and to identify its strengths and weaknesses. This is essential for developing public policies that contribute to improving the system. They also analyze the effectiveness of the system by checking whether it is fulfilling its social protection objectives and meeting the needs of the population. They also identify challenges and opportunities and help develop strategies for the future of social security. Considering the economic contingencies faced by the social security sector in Brazil, analyzing the elasticity of social security collection in relation to GDP is extremely important for understanding the impact of macroeconomic factors on the social security sector and vice versa.

2. THEORETICAL FRAMEWORK

The elasticity of social security collection in relation to Gross Domestic Product (GDP) is a measure that expresses the sensitivity or responsiveness of social security collection to changes in GDP. In other words, the income elasticity of social security collection measures the sensitivity of social security collection to changes in GDP. In mathematical terms, this elasticity is defined as the ratio between the percentage change in social security collection and the percentage change in GDP. In other words, it indicates how much social security collection varies in response to a percentage change in GDP. The general formula for calculating this elasticity is given by:

$$\varepsilon_{PIB}^{RP} = \frac{\Delta\%Receita\ Previdenciária}{\Delta\%PIB} \tag{1}$$

If the elasticity is greater than 1 $\left(\varepsilon_{PIB}^{RP} > 1\right)$ this indicates that social security collection is elastic in relation to GDP, i.e. it varies by a greater proportion than the percentage change in GDP. If the elasticity is less than 1 $\left(\varepsilon_{PIB}^{RP} < 1\right)$ then social security collection is inelastic in relation to GDP, indicating that it varies to a lesser extent than the percentage change in GDP. Finally, if the elasticity is equal to one $\left(\varepsilon_{PIB}^{RP} = 1\right)$ i.e. unitary elasticity, this means that social security collection varies in a proportion equal to the percentage change in GDP.

The elasticity of social security collection in relation to GDP can be influenced by various factors, such as changes in the demographic structure of the population, the employment rate, wages, social security policies, among others. For example, if the population ages, the demand for social security benefits may increase, affecting the elasticity of social security collection in relation to GDP. It is important to note that elasticity can vary over time and in different economic contexts.

In recent studies on social security systems, experts have emphasized the importance of dynamic adjustments in social security policies to deal with constantly evolving demographic and economic challenges (AFONSO, CARVALHO; 2021). In addition, they also highlight the importance of the parameter of sensitivity of tax collection to GDP for analyzing the effects of fiscal policy on the economic cycle (BLANCHARD and PEROTTI, 2020). Studies such as these highlight the need for an adaptive approach to ensure the effectiveness and sustainability of the RGPS in the current Brazilian context.

Marquezini (2018) presents an empirical analysis of the elasticities of Brazil's Gross Domestic Product (GDP) and social security collection, using the Engle-Granger cointegration methodology. The results show that variations in GDP have a significant effect on social security collection, with a short-term elasticity of 0.8 and a long-term elasticity of 1.2. In addition, changes in the composition of GDP, such as the fall in the share of industry and the increase in the services sector, affected the long-term elasticities. The Engle-Granger cointegration methodology proved useful for analyzing the relationship between variables and can be applied in other economic analyses.

Casalecchi and Barros (2018) prepared Technical Note No. 19 of the Independent Fiscal Institute (IFI). The aim was to analyze the sensitivity of government revenue to economic growth, using data from 1997 to 2019. The methodology used was estimation by cointegration and error correction methods, which made it possible to identify a drop in the elasticity of revenue in relation to GDP from 2008 onwards. In addition, the results show that the sensitivity of tax revenues to economic growth is relatively high, with coefficients between 1.1 and 1.3. The conclusion is that the fall in revenue elasticity poses notable challenges for the speed of the fiscal consolidation process and that many refinements to the analysis are possible and desirable.

Casalecchi and Bacciotti (2021) signed the IFI's Technical Note No. 16. The aim of this note is to update estimates of the variation in public revenue in response to variations in GDP. The methodology used was cointegration analysis and error correction, based on quarterly data from 2000 to 2020. The long-term elasticity found for RGPS revenues was 1.06. This is the highest among the measures analyzed. This means that RGPS revenues are more sensitive to variations in GDP than the other revenue measures analyzed in the study. One hypothesis for this behaviour is that GDP growth may be accompanied by a more intense increase in the formalization rate in the labour market.

In 2020, the Secretariat for Economic Policy (SPE) published a study entitled Long--Term Income Elasticity of Gross Federal Tax Collection. It aimed to calculate the long-term income elasticity of taxes, individually and grouped, for 23 taxes. By estimating autoregressive and distributed lag (ARDL) models, the results obtained showed that the long-term income elasticity found for social security revenues was 1.14 for the period from May 2019 to August 2020, which included these revenues among the taxes that prove to be most sensitive to the level of economic activity.

This study contributes to the literature on the subject by carrying out a rigorous procedure to identify the order of stationarity and co-integration of the series analyzed, even taking into account the presence of structural breaks.

3. METHODOLOGY

3.1 Stationarity Analysis

The modified Dickey-Fuller (ADF^{GLS}) and Phillips-Perron $\left(\overline{MZ}_{\alpha}^{GLS}\right)$ The modified Dickey-Fuller and Phillips-Perron tests proposed by Elliot, Rottemberg and Stock (1996) and Ng and Perron (2001) are used to verify the stationarity of time series. These tests overcome the problems of low statistical power and size distortions of the traditional tests of Dickey and Fuller (1979, 1981), Said and Dickey (1984) and Phillips and Perron (1988). The modifications to the standard unit root test of Dickey and Fuller (1979, 1981) and Said and Dickey (1984) are based on two central aspects: the extraction of trend in time series using ordinary least squares (OLS) is inefficient and the importance of an appropriate selection for the lag order of the augmented term in order to obtain a better approximation to the true data generating process.

In the first case, Elliot, Rottemberg and Stock (1996) propose using generalized least squares (GLS) to extract the stochastic trend of the series. The standard procedure for estimating the ADF^{GLS} is used to test the null hypothesis of the presence of a unit root against the alternative hypothesis that the series is stationary. With regard to the second aspect, Ng and Perron (2001) show that the Akaike (AIC) and Schwarz (SIC) information criteria tend to select low values for the lag when there is a large negative root (close to -1) in the moving average polynomial of the series. This generates distortions and prompted the development of the modified Akaike information criterion (MAIC) for selecting the autoregressive lag, in order to minimize the distortions caused by inadequate lag selection.

Ng and Perron (2001) propose that the same modifications should also be applied to the traditional Phillips and Perron (1988) test, giving rise to the following test $\overline{MZ}_{\alpha}^{GLS}$. By means of simulations, Ng and Perron (2001) show that the joint application of GLS to extract the deterministic trend and the MAIC lag selection criterion produces tests with greater power, but smaller distortions of statistical size when compared to the traditional ADF and PP tests. The critical values of the ADF^{GLS} e $\overline{MZ}_{\alpha}^{GLS}$ statistics are reported in Ng and Perron (2001).

However, even the modified ADF^{GLS} e $\overline{MZ}_{\alpha}^{GLS}$ have low power in the presence of structural breaks, becoming biased in the sense of not rejecting the null hypothesis of a unit root, even when the series is stationary. The pioneering work of Perron (1989) illustrates the importance of including a structural break in traditional unit root tests. Three structural break models were considered. Model A, which is known as the *crash model*, allows for a one-period change in level. Model B, which allows for a break in the trend of the time series. Model C, which is known as the *changing growth path*, includes a one-period change in both level and trend.

Later research adopted an endogenous procedure to determine the breakpoint from the data. In this context, Vogelsang and Perron (1998) developed a unit root test with endogenous breakpoint estimation, based on Perron's (1989) A, B and C models and the *Innovation Outlier* (IO) and *Additive Outlier* (AO) methods. The AO model allows for a sudden change in the average (crash model), while the IO model allows for more gradual changes. Thus, the two models are used to check the stationarity hypothesis: break in the intercept, break in the intercept and trend, both at level and first difference.

In turn, Saikkonen and Lütkepohl (2002) and Lanne, Saikkonen and Lütkepohl (2002, 2003) propose that structural breaks can occur over a number of periods and expose a smooth transition to a new level. Therefore, a level shift function is added to the deterministic term of the data generating process. The deterministic terms are extracted by generalized least squares (GLS) and then an ADF test is applied to the fitted series. Critical values for the test are tabulated by Lanne, Saikkonen and Lütkepohl (2002).

3.2 Co-integration Analysis

The co-integration analysis of the series analyzed in this study will be based on the implementation of three different tests. Firstly, the Engle-Granger test and the augmented Engle-Granger test (ENGLE; GRANGER, 1987) is a three-step procedure carried out for just a single econometric equation: (i) unit root tests are applied to the time series individually in a bivariate system (i.e. a system with just two variables), to check whether they are really I(1); (ii) if the variables are I(1), the equilibrium equation is estimated via ordinary least squares at level. When estimating the long-term relationship, obtain the residual term $\hat{\varepsilon}_t$ (iii) at the end, the Dickey-Fuller test or augmented Dickey-Fuller test is applied to the residuals of the equilibrium equation estimated via MQO at level. If the null hypothesis of a unit root of the residuals is not rejected, the variables are not co-integrated. On the other hand, if this null hypothesis is rejected, the series of residuals is stationary in level - I(0) - implying that the variables are co-integrated, because the residuals are stationary. Therefore, perform the unit root test on the estimated residuals using the ADF procedure:

$$\Delta\hat{\varepsilon}_t = \alpha\hat{\varepsilon}_{t-1} + \sum_{i=1}^{p-1} \lambda_{i+1}\Delta\hat{\varepsilon}_{t-i} + u_t \tag{1}$$

Failure to reject H_0:α=0 implies that the residuals have a unit root, so the variables do not co-integrate. If the null hypothesis is rejected, these residuals can be used to estimate the error correction model.

The Engle-Granger test and the augmented Engle-Granger test are applied as follows. First, a static econometric regression is estimated and the residual error terms are obtained. Then the Dickey-Fuller unit root test is applied to the time series of the residual error term. However, a word of caution is in order. As the $\hat{\varepsilon}_t$ estimates are based on the estimated co-integrating parameter $\hat{\beta}_2$ the critical significance values of DF and ADF are not appropriate. Engle and Granger (1987) calculated these critical values. For this reason, the DF and ADF tests in this context are known as the Engle-Granger test or the augmented Engle-Granger test. Another alternative is to use the MacKinnon procedure (1991) *apud* Bueno (2011, p. 246-247), as it covers all sample sizes, and the table depends on the number of observations and endogenous variables, as well as the existence or not of a constant and linear trend.

The limitation of the Engle-Granger test is clear: the true model must have only one co-integrating relationship (SANTOS, 2017, p. 184). In view of this, two other econometric approaches are used to analyze the co-integration relationship between the series analyzed, but this time using a multivariate approach.

Secondly, according to Enders (2015, p. 378), the Johansen Cointegration Test is a multivariate generalization of the Dickey-Fuller test. Consider the following model:

$$\Delta x_t = \pi x_{t-1} + \sum_{i=1}^{p-1} \pi_i \Delta x_{t-i} + \varepsilon_t \tag{2}$$

where : $\pi = -\left(I - \sum_{i=1}^{p} A_i\right)$, $\pi_i = -\sum_{j=i+1}^{p} A_j$; x_{t-1} e ε_t are vectors (n x 1); π is defined as (A_1-I), where A_1 is a parameter matrix (n x n) and I is an identity matrix (n x n); and p is the number of lags.

The rank of (A_1-I) equals the number of co-integrating vectors. If (A_1-I) consists of zeros, i.e. the rank of π is zero, this means that all sequences $\{x_{it}\}$ are unit root processes, in other words, I(1) processes. Since there is no linear combination of the processes $\{x_{it}\}$ which is stationary, the variables are not co-integrated.

Co-integration is determined by the rank of the matrix which is equal to the number of independent co-integrating vectors, posto(π)=r. If posto(π)=0, the matrix is null and equation (10) represents a VAR model in first differences. If posto(π)=r=n, the VAR is stationary. If posto(π)=1, there is a single co-integrating vector and the expression πx_{t-1} is the error correction term. For other cases where 1<posto(π)=r<n, there are multiple co-integrating vectors. It is known that the number of distinct co-integrating vectors can be obtained by checking the significance of the characteristic roots of π. To test the significance of the characteristic roots, the following procedure should be followed: (i) obtain the matrix π, calculate its n characteristic roots, $|\pi-\lambda I| = 0$, and order them in such a way that $\lambda_1>\lambda_2>...>\lambda_n$.

The Johansen Co-integration Test (JOHANSEN; JUSELIUS, 1990; JOHANSEN, 1995) includes two ways of obtaining the eigenvalues, which are called the Trace Test and the Maximum Eigenvalue Test, whose statistics are described below:

$$\lambda_{traço}(r) = -T \sum_{i=r+1}^{n} \ln ln\left(1 - \hat{\lambda}_i\right),\ H_0: r \le r_0 \tag{3}$$

$$\lambda_{max}(r, r+1) = -T \ln ln\left(1 - \hat{\lambda}_{i=r_0+1}\right),\ H_0: r = r_0 \tag{4}$$

where $\hat{\lambda}_i$ = the estimated values of the characteristic roots (the eigenvalues) obtained from the estimated matrix matrix, arranged in descending order: $\hat{\lambda}_1 > \hat{\lambda}_2 > ... > \hat{\lambda}_k$. The basic rule is that large values of the test statistic lead to rejection of the null hypothesis. T = the number of usable observations. When the appropriate values of r are clear, these statistics are simply referred to as $\lambda_{traço}$ e λ_{max}.

In the **Trace Test**, the null hypothesis of the existence of r* cointegrating vectors (r = r*), is tested against the alternative hypothesis that r > r* cointegrating vectors. When there is no cointegration, the eigenvalues found will be close to zero, indicating the non-stationarity and instability of the matrix π, The null hypothesis cannot be rejected. There is evidence of more than one cointegration vector when the null hypothesis tha r = r* is rejected. In other words, this first statistic tests the null hypothesis that the number of distinct co-integrating vectors is less than or equal to r (H_0:*posto*(π)≤r) against the general alternative hypothesis (H_a:*pos-*

to(π)>r).

The second way of obtaining the eigenvalues of π is through the **Maximum Eigenvalue Test**, in which the null hypothesis implies the existence of $r = r^*$ cointegrating vectors, against the alternative hypothesis that $r^* + 1 = r$ cointegrating vectors. The main function of the Maximum Eigenvalue Test is to verify the maximum significant eigenvalue that a cointegration vector produces. As with the Trace Test, when the null hypothesis is rejected, the interpretation is that there is more than one cointegration vector. Therefore, the second statistic tests the null hypothesis that the number of co-integrating vectors is r $(H_0{:}posto(\pi)=r)$ against the alternative hypothesis of $(r + 1)$ co-integrating vectors $(H_a{:}posto(\pi)=r+1)$.

The critical values of $\lambda_{traço}$ and λ_{max} are obtained using the Monte Carlo approach. In short, the importance of the Cointegration Test for the time series model is to determine whether or not there is long-term equilibrium between the chosen variables, so that the econometric model can then be chosen to achieve the general and specific objectives of this study.

Thirdly, Johansen *et al*. (2000) demonstrate how traditional cointegration analysis can be used to identify possible types of structural breaks. The author proposed a generalization of Perron's (1989) trend-intercept break model, model C, in the context of multivariate time series. The author shows how traditional cointegration analysis can be used to identify some types of structural break, although there are some conceptual differences such as the need to generate a new table of asymptotic results. It is shown that using this theoretical apparatus it is possible to identify and test changes in the trend present in the cointegration vectors. However, in order to use this type of traditional analysis, according to the author, it is necessary to exclude observations after the (previously known) break, using impulse *dummies*. The number of *dummies* corresponds to the number of lags in the system and the inclusion of these *dummies* implies a reduction in the sample.

3.3 Multivariate Analysis

The Vector Autoregressive Model with Error Correction Mechanism (VECM) corrects the problem of omitting relevant variables that arises when using the Vector Autoregressive Model (VAR) with variables in first differences, although it does incur the problem of omitting relevant lags by imposing the same number of lags for all the variables in the system being modeled. In short, the VECM model is a re-parameterization of the VAR model.

The VECM is expressed by equation (10), where $\pi=\alpha\beta'$. The speed of the adjustment parameters is given by the term $\alpha = \alpha_1, \alpha_2, \ldots, \alpha_n$, which represents a matrix with r adjustment vectors (i.e. the elements of are adjustment coefficients). The co-integrating vector is given by $\beta=1, \beta_2, \beta_3, \ldots, \beta_n$', i.e, β is a matrix with r co-integration vectors (i.e. the columns of β form the co-integration space). Therefore, in equation (10), Δx_t is explained by the short-term factors $\left(\sum_{i=1}^{p-1} \pi_i \Delta x_{t-i}\right)$,as well as by the long-term relationship between the coordinates of the vector of endogenous variables πx_{t-1}. Furthermore, since $x_t \sim CI(1,1)$, then x_t has a representation in the form of a VECM. This is the Granger Representation Theorem, which states that it is always possible to associate error correction to the VAR in the presence of co-integration.

The main advantage of writing the system of equations using the VECM model is that it incorporates both short- and long-term information. Another advantage of working with co--integrated variables is the possibility of estimating the long-term elasticities between them. Specifically, in the co-integrating equation, as the vector is normalized, the first coefficient will always be equal to 1. The other coefficients represent the elasticity of these variables in relation to the first, remembering that the **negative sign** indicates a **positive elasticity**.

In addition, the estimation of a VECM model allows three additional techniques to be explored. Firstly, the concept of Granger causality is associated with the idea of temporal precedence between variables. Thus, if y_t contains past information that helps predict x_t, and if this information is not contained in other series used in the model, then y_t Granger-cause x_t (Granger, 1969). The Granger causality of the variable x_t to the variable y_t is assessed by testing the null hypothesis that the coefficients of the variable x_t in all its lags are simultaneously statistically equal to zero in the equation in which y_t is the dependent variable. If the null hypothesis is rejected, it is concluded that the variable x_t variable Granger-causes the y_t. Consider the following bivariate VECM with one lag:

$$\Delta y_t = \alpha_y \left(y_{t-1} - \beta x_{t-1}{}^{\varepsilon_{t-1}}\right) + \phi_y \Delta y_{t-1} + \phi_x \Delta x_{t-1} + \varepsilon_{yt} \tag{5}$$

$$\Delta x_t = \alpha_x \left(y_{t-1} - \beta x_{t-1}{}^{\varepsilon_{t-1}}\right) + \phi_y \Delta y_{t-1} + \phi_x \Delta x_{t-1} + \varepsilon_{xt} \tag{6}$$

In an Error Correction Mechanism (ECM)$\left(y_{t-1} - \beta x_{t-1}{}^{\varepsilon_{t-1}}\right)$, If one of the adjustment speeds (alphas) is zero, there will be two cases. Firstly, if α_y=0, then the dynamic behavior of y_t is not affected by the deviation from equilibrium in the previous period (i.e. the ECM), and y_t is said to be weakly exogenous. Secondly, if α_x=0, then the dynamic behavior of x_t is not affected by the deviation from equilibrium in the previous period (i.e. the MCE), and x_t is weakly exogenous.

In an Error Correction Model, Granger causality needs to be reinterpreted. For example, in equation (6), note that y_t does not Granger-Cause x_t if: (i) the lagged values of Δy_t do not enter the equation for Δx_t, i.e, $(\phi_y = 0)$ and x_t does not respond to deviations from the previous equilibrium (α_x=0), i.e, x_t is weakly exogenous. Remember the Granger Representation Theorem here.

Secondly, the generalized impulse-response function (FIRG) is used. It should be noted that the main argument for this procedure is that the generalized impulse-response does not vary if there is a reordering of variables in the VAR. Pesaran and Shin (1998) developed the generalized impulse-response function as a way of eliminating the problem of variable ordering in the VAR model. There are two potential advantages in applying this method (Ewing, 2003): (i) the generalized impulse-response function provides more robust results than the orthogonalized method, and (ii) because orthogonality is not imposed, the generalized impulse-response function allows for a more accurate interpretation of the response of the initial impact resulting from each shock caused by one variable on the others.

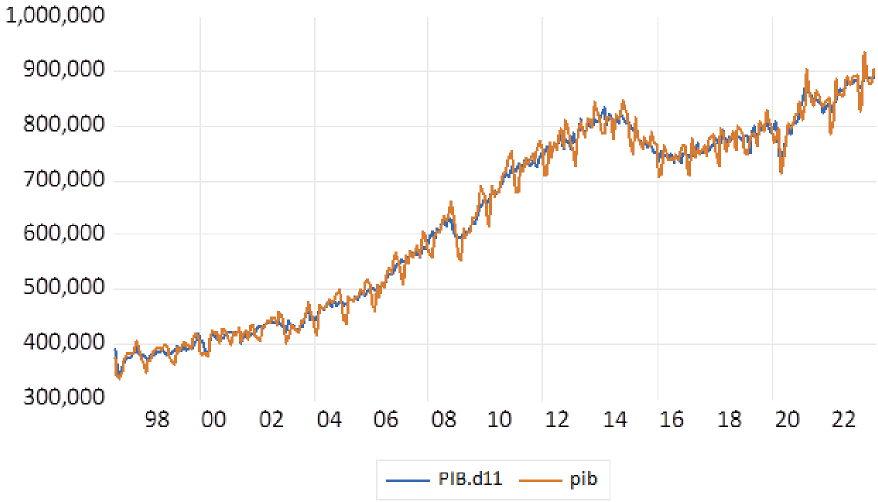
In general terms, it should be noted that in a VECM model, the impulse-response functions tend to cancel out over time, i.e. this means that the endogenous variables tend towards a long-term equilibrium path. Contrary to what would happen in a VAR model, in which the variables were not stationary in level, but in first differences, in the VECM model the shocks would not have permanent effects. Therefore, if the VECM model is used, the impulse-response functions can be analyzed as a succession of consequences or responses on a given variable, caused by deviations from its initial equilibrium in relation to another variable.

Thirdly, variance decomposition analysis (VDA) is a tool used to describe the dynamics of the system in the VAR approach. Using this method, it is possible to identify the proportion of a variable's total variation due to each individual shock in the model's k component variables. It should be noted that the ADV provides information on the relative importance of each innovation on the system's variables (ENDERS, 2015, p. 302). Variance decomposition is a tool used to show the variance of the forecast error that results specifically from each endogenous variable in a given period. Thus, through this analysis, it is possible to observe, in percentage terms, how a given variable behaves when it suffers an unanticipated shock from the other variables belonging to the same system of equations (MARGARIDO, 2000).

4. DESCRIPTION OF VARIABLES AND DATA TREATMENT

This study used monthly data on Brazil's Gross Domestic Product (GDP) and net revenue for the General Social Security System (RGPS), covering the period from January 1997 to July 2023.

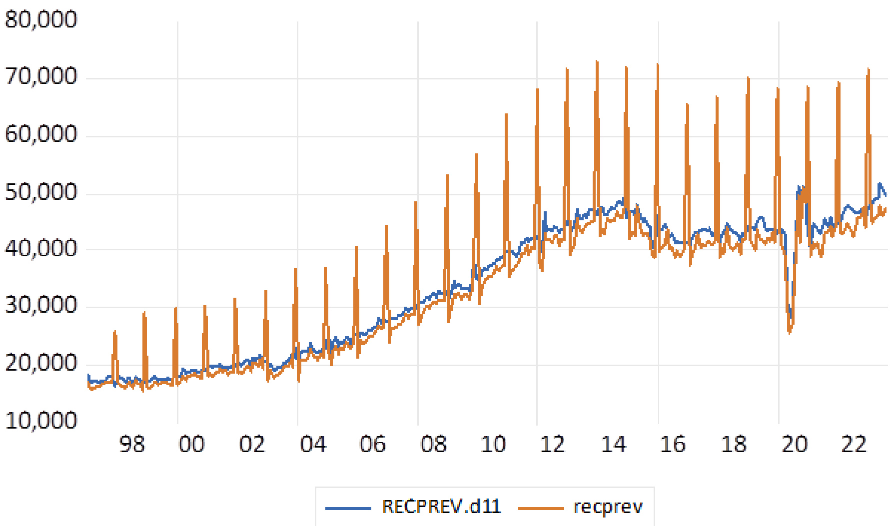
Graph 1: GDP as of July 2023 and seasonally adjusted GDP (GDP.d11).



R$ in millions of reais. Period from 01/1997 to 07/2023.

Source: Prepared by the author based on data from the Central Bank of Brazil.

Graph 2: Social Security Collection at July/2023 values and seasonally adjusted.



R$ in millions of reais. Period from 01/1997 to 07/2023.

Source: Prepared by the author based on data from the National Treasury Secretariat.

The time series of the monthly GDP estimate was extracted from the Time Series Management System (SGS), managed by the Central Bank of Brazil (BC), and was obtained from the official GDP data, at quarterly and annual frequencies, published by the Brazilian Institute of Geography and Statistics (IBGE).

The data on net revenue for the RGPS was taken from the National Treasury Results Bulletin (RTN) for July 2023. This is prepared and published by the National Treasury Secretariat (STN), an agency of the Ministry of Finance.

Both time series were converted into real terms using the IPCA historical series and then converted into natural logarithm terms so that the estimated coefficients can be interpreted as elasticities.

The data was seasonally adjusted using the Census X-13 method. This is a seasonal adjustment algorithm developed by the *US Bureau of the Census*. It is used to adjust time series of economic, demographic and other data to remove seasonality and reveal underlying trends. It is based on the ARIMA (*Autoregressive Integrated Moving Average*) model, which is a statistical model that describes the relationship between the values of a time series and its previous values. The X-13 model extends the ARIMA model to include seasonal components. The Census X-13 method is a non-parametric method, which means that it does not assume any specific format for the time series. This makes it a versatile method that can be used to adjust a wide variety of time series for seasonality.

5. ANALYSIS OF RESULTS

5.1 Stationarity Analysis

Unit root tests were carried out on the natural logarithm of the time series under study. The aim of unit root tests is to determine whether a time series is stationary or not. A time series is said to be stationary if its mean, variance and autocorrelation do not change over time. The presence of a unit root can cause problems in statistical inference, as it can lead to biased and inefficient estimates.

Stationarity tests were then applied, both linear and with structural breaks. The results of the unit root tests are consolidated in Table 1 below. In this table, the variable y_t^{sa} is the natural logarithm of monthly GDP, in real terms and seasonally adjusted. The variable τ_t^{sa} is the natural logarithm of social security collection in real, seasonally adjusted terms.

Analysis of the results of the tests without structural breaks shows that both the Brazilian GDP and social security collection time series are non-stationary in level and first differences. The values found in the test statistics are less negative than the corresponding critical values. Therefore, we do not reject the null hypothesis (H_0) that $\delta = 0$ and conclude that the time series tested are non-stationary.

The results of the structural break tests show that both the Brazilian GDP and social security collection time series are non-stationary in level, but are stationary in first differences. Note that for GDP in first difference (Δy_t^{sa}), both tests indicate that the estimated coefficients are statistically significant or reject the null hypothesis at the 1% statistical significance level.

As for the first-difference social security collection ($\Delta\tau_t^{sa}$), the Vogelsang and Perron (1998) test indicates that the estimated coefficients are statistically significant or reject the null hypothesis at a statistical significance level of 1%. The Saikkonen-Lütkepohl test (2002) indicates that the estimated coefficients are statistically significant or reject the null hypothesis at a 5% statistical significance level for the model with a constant and deterministic trend.

The dates of the selected structural breaks refer to the following periods: (i) 1997: The "Asian economic crisis" occurred, with the devaluation of the Thai *baht*. The crisis spread to other Asian countries such as Indonesia, South Korea and Malaysia; (ii) 1998: devaluation of the Real and *commodity* prices influenced by a strong financial crisis in Russia; (iii) 2009: The "*subprime* crisis" or housing bubble crisis in the US began in 2008 and spread globally in 2009; (iv) 2014 and 2015: period of recession in the Brazilian economy; (vi) 2020: health crisis caused by the Covid-19 pandemic.

Table 1 – Results of the unit root tests (1997 to 2023).

Variables	Model	No structural breakage			With endogenous structural breakage (date of breakage is unknown)					
					Vogelsang and Perron (1998)			Saikkonen and Lütkepohl (2002)		
		ADF^{GLS}	$\overline{MZ}_t^{GLS}$	*Lags*	Type of Model	Date of Breakin g	Statistics Test	Type of Model	Date of Breakdow n	Statistics Test
y_t^{sa}	C	1,83	2,01	6	*Innovational Outlier*	2014:12	-4.04 (12 *lags*)	*Rational shift*	2020:04	-1.66 (2 *lags*)
y_t^{sa}	C,T	-1,29	-1,21	6	*Innovational Outlier*	2009:05	-3.60 (6 *lags*)	*Rational shift*	2020:04	-1.17 (2 *lags*)
Δy_t^{sa}	C	-0,16	0,89	16	*Innovational Outlier*	1998:02	-21.68 (0 *lags*)(a)	*Rational shift*	2020:05	-4.15 (2 *lags*)(a)
Δy_t^{sa}	C,T	-2,23	0,07	16	*Innovational Outlier*	1997:06	-22.77 (0 *lags*)(a)	*Rational shift*	2020:05	-4.60 (2 *lags*)(a)
τ_t^{sa}	C	1,56	1,64	6	*Innovational Outlier*	2015:04	-3.58 (12 *lags*)	*Rational shift*	2020:07	-1.67 (2 *lags*)
τ_t^{sa}	C,T	-1,23	-1,18	6	*Innovational Outlier*	2009:10	-3.58 (6 *lags*)	*Rational shift*	2020:07	-0.96 (2 *lags*)
$\Delta\tau_t^{sa}$	C	-0,42	-0,34	16	*Innovational Outlier*	1997:12	-21.99 (0 *lags*)(a)	*Rational shift*	2020:06	-2.34 (2 *lags*)
$\Delta\tau_t^{sa}$	C,T	-1,64	-0,24	16	*Innovational Outlier*	1997:12	-21.98 (0 *lags*)(a)	*Rational shift*	2020:06	-3.50 (2 *lags*)(b)

Source: own elaboration. Using Eviews and JMULTI econometric software.

Note:

1 - "*Lags*" means *lags*. Model types: "C" means constant; "T" means deterministic trend. Maximum initial count of 16 lags. Δ is the first-difference operator. Note that (a), (b) and (c) indicate that the estimated coefficients are statistically significant or rejection of the null hypothesis at the statistical significance level of 1%, 5% and 10%, respectively. Monthly observations included: 319 (sample: 1997 to 2023).

2 - The critical values of the ADF^{GLS} are (El[illegible] Rothenberg and Stock, 1996): (i) model with constant: -2.57 (1%), -1.94 (5%) and -1.62 (10%). (ii) model with constant and determinis[illegible]end: -3.47 (1%), -2.91 (5%) and -2.60 (10%). Selection of the optimum number of lags using the modified Akaike information criterion.

3 - The asymptotic critical values of the test ZGL are (Ng and Perron, 2001, Table 1): (i) model with constant: -2.58 (1%), -1.98 (5%) and -1.62 (10%); (ii) model with constant and deterministic trend: -3.42 (1%), -2.91 (5%) and -2.62 (10%). Spectral estimation method: AR *GLS-detrented*. Selection of the optimum number of lags using the modified Akaike information criterion.

4 - The critical values of the Vogelsang and Perron test (1998) are: (i) model with constant and deterministic trend/intercept break: -5.35 (1%), -4.86 (5%), and -4.61 (10%); (ii) model with constant and deterministic trend/break of intercept and trend: -5.72 (1%), -5.17 (5%), and -4.89 (10%). (iii) model with constant and deterministic trend/break in trend: -5.06 (1%), -4.52 (5%), and -4.26 (10%). Types of break: *innovational outlier and additive outlier*. Selection of the structural break: minimized Dickey-Fuller *t-statistic*. Selection of the optimum number of lags: Schwarz Information Criterion.

5 - The critical values of the Saikkonen-Lütkepohl test are (Lanne *et al*., 2002): (i) model with constant: -3.48 (1%), -2.88 (5%), and -2.58 (10%); (ii) model with constant and deterministic trend: - 3.55 (1%), -3.03 (5%), and -2.76 (10%). Types of structural break: *Rational Shift, Exponential Shift and Impulse dummy.*

5.2 Co-integration analysis

Cointegration tests were then carried out on the time series under study. These are used in econometrics to check whether there is a long-term equilibrium relationship between two or more variables. Cointegration is an important property of economic time series, as it allows more sophisticated econometric models to be estimated. For example, error correction models (ECM) are based on the premise that co-integrated time series tend to return to their long-term equilibrium after being disturbed. Thus, three cointegration tests were carried out: the Engle--Granger test; the Johansen test and the Johansen test with structural break.

The results of the cointegration tests are consolidated in Tables 2, 3 and 4 below. With regard to the Engle-Granger Cointegration Test, the results of which are reported in Table 2, there is no co-integration in the series analyzed here.

Table 2 – Resultado do teste de Cointegração de Engle-Granger.

Dependent	*tau-statistic*	Prob.*	*z-statistic*	Prob.*
y_t^{sa}	-1,991485	0,5333	-10,84692	0,3104
τ_t^{sa}	-2,049882	0,5028	-11,31108	0,2862

Note: (*) stands for the p-values defined in MacKinnon (1996).

However, given the limitations of the Engle-Granger test, and due to the intrinsic endogeneity of the series analyzed, which exposes them to a potential simultaneity bias, the Johansen Co-integration Test was applied, as described in Table 3. The results of the Trace Test and the Maximum Eigenvalue Test indicate rejection of the null hypothesis of the absence of co-integration between the variables analyzed at a statistical significance level of 1%, according to the reported p-value, since the values of these two test statistics are higher than their critical values.

Table 3 – Results of the Johansen Cointegration Test - without structural break

H_0	H_1	Eigenvalues	Test Statistics	Critical Values	p-value
	Trace test				
$r = 0$ None	$r \geq 0$	0,096948	33,88452[(a)]	15,49471	0,0000
$r \leq 1$ Maximum 1	$r \geq 1$	0,005920	1,864392	3,841465	0,1721
	Maximum Eigenvalue Test				
$r = 0$ None	$r = 0$	0,096948	32,02012[(a)]	14,26460	0,0000
$r \leq 1$ Maximum 1	$r = 1$	0,005920	1,864392	3,841465	0,1721

Note:

1 - Note that (a), (b) and (c) indicate that the estimated coefficients are statistically significant or rejection of the null hypothesis at the statistical significance level of 1%, 5% and 10%, respectively;

2 - The Trace Test and the Maximum Eigenvalue Test indicate a co-integration equation at a significance level of 1%.

3 - P-values obtained from MacKinnon-Haug-Michelis (1999).

But the co-integration analysis needs to take into account the structural breaks that were identified in the previous stationarity analysis procedure. In view of this, when applying the Johansen Co-integration Test, in the presence of structural breaks, the results again confirmed that the series co-integrate, as reported in Table 4. The test results indicate that the null hypothesis of no co-integration is rejected. The value of the Trace statistic is 122.13, with a p-value of 0.000. Therefore, based on the test results, it can be concluded that there is a cointegration relationship between the time series. This relationship means that, in the long term, the two variables tend to move together.

Table 4 – Result of the Johansen Cointegration test - with structural break.

R0	LR	p-value	90%	95%	99%
0	122,13[a]	0,0000	33,31	36,34	42,48
1	8,86	0,5553	16,13	18,46	23,38

Note:

1 - Note that (a), (b) and (c) indicate that the estimated coefficients are statistically significant or rejection of the null hypothesis at the statistical significance level of 1%, 5% and 10%, respectively.

2 - Breaks in level and trend together. 1st break: 1997M12; 2nd break: 2020M05.

In summary, the Johansen co-integration tests (without and with structural break) implemented jointly demonstrate the existence of co-integration between the time series analyzed.

5.3 *Estimation of the VECM Model*

In order to determine the income elasticity of social security collection, the VECM model was estimated with one lag, according to the results of the usual lag selection criteria reported in Table A.1 of the Appendix. Table 5 below reports the results of estimating the co-integration vector:

Table 5 – Co-integration vector

Co-integrating equation	Social Security Collection $\left(\tau_{t-1}^{sa}\right)$	GDP $\left(y_{t-1}^{sa}\right)$	Constant
Coefficient	1,000000	-1,278737	6,666028
Standard Error		(0,02039)	
T-statistics		[-62,7265][a]	

Note: (a), (b) and (c) indicate that the estimated coefficients are statistically significant or rejection of the null hypothesis at the statistical significance level of 1%, 5% and 10%, respectively.

The co-integration equation that deals with the relationship between social security collection and GDP can be written as follows in period *t*:

$$\tau_t^{sa} = -\ 6,666028\ +\ 1,278737 y_t^{sa} \quad (15)$$

Equation (15) shows that, in the long term, a 1% increase in GDP has an impact of 1.27% on the collection of social security revenue. In other words, the first hypothesis of the research, in which social security revenue is elastic in relation to GDP, was confirmed. This shows that, in the long term, social security revenue varies in greater proportion to the variation in GDP over the period analyzed. This achieves the fourth specific objective: to measure the elasticity of RGPS net collection in relation to GDP.

Table 6 below compares the results of the income elasticity of social security collection obtained in this study with those obtained in previous studies. It can be seen that, by applying a more rigorous and robust econometric procedure, there is evidence that social security collection is elastic in relation to the level of economic activity, compared to the other results obtained in the specialized literature, which pointed to a unitary income elasticity of social security collection.

Several factors can explain this result. But it is well known that social security collection depends crucially on the economy's wage bill. Between 2017 and 2022, however, there was an intensification of the process of structural and microeconomic reforms to stimulate sustainable economic growth. These reforms include labor reform and microeconomic reforms to improve the business environment. This positive elasticity may therefore be reflecting the effects of these reforms on the labor market.

Table 6 – Comparison of the long-term income elasticities of social security collection.

Studies	Marquezini (2018)	SPE (2020)	Casalecchi and Bacciotti (2021)	This Study (2024)
Long-term elasticity	1,13	1,14	1,06	1,28

The Granger causality test is a simple hypothesis test that checks whether including a variable in the regression equation of another variable improves the fit of the regression. In other words, the Granger causality test checks whether the past of one variable can help predict the future of another variable. There are two ways of measuring the sources of Granger causality in a VECM model: (i) the coefficients of the lagged and differentiated values of the series analyzed (short-term causality); (ii) the coefficient of the error correction term (long-term causality).

The results for Granger causality in the short term are consolidated in Table 7, which show that social security collection Granger-causes GDP unidirectionally at a significance level of 10%. This is a counterintuitive result and, for this reason, a more rigorous Granger causality analysis will be implemented next in order to better clarify the temporal precedence between the variables analyzed.

Table 7 – Results of the Granger Causality/Block Exogeneity Test in the VECM

Null Hypothesis	Test Statistics	Degrees of Freedom	Direction of Causality
y_{t-1}^{sa} no Granger-cause τ_{t-1}^{sa}	0,126180 (0,7224)	1	$y_{t-1}^{sa} \nrightarrow \tau_{t-1}^{sa}$
τ_{t-1}^{sa} no Granger-cause y_{t-1}^{sa}	2,820493$^{(c)}$ (0,0931)	1	$\tau_{t-1}^{sa} \rightarrow y_{t-1}^{sa}$

Note: (a), (b) and (c) indicate that the estimated coefficients are statistically significant or rejection of the null hypothesis at the statistical significance level of 1%, 5% and 10%, respectively.

With regard to the second source of Granger causality, which is related to the long term, in an error correction model, the most appropriate way to assess the degree of exogeneity between the dependent variables, and thus the causal relationship between them, is to work with the concepts of weak and strong exogeneity, defined in Engle, Hendry and Richard (1983) and Ericsson (1994). Weak exogeneity in co-integrated systems corresponds to certain "zero restrictions" on the matrix of adjustment parameters and can therefore be tested. For example, the hypothesis of weak exogeneity of a variable x_t for the matrix of cointegration vectors is not valid if one of the cointegration vectors β appears in both the conditional and marginal models. When the coefficients of the α are zero, the explained variable is said to be weakly exogenous. It is therefore necessary to reinterpret the causality condition in co-integrated systems. In a co-integrated system, $\{y_t\}$ does not Granger cause $\{x_t\}$ if the lagged values Δy_{t-i} do not explain Δx_t, and x_t does not respond to deviations from the long-term equilibrium. Therefore x_t is weakly exogenous.

The results reported in Tables 8 and 9 below confirm the existence of a long-term Granger bi-causality relationship in this bivariate system by verifying the statistical significance of the adjustment speeds of the equilibrium relationships.

Table 8 - Restriction test on the speed of adjustment of the r equilibrium relationships relating to the social security revenue equation.

Chi-Square statistic	21.63016[a]
p-value	0,000003

Note: (a), (b) and (c) indicate that the estimated coefficients are statistically significant or rejection of the null hypothesis at the statistical significance

Table 9 - Restriction test on the speed of adjustment of the r equilibrium relationships relating to the Gross Domestic Product equation.

Chi-Square statistic	5.477077[a]
p-value	0,019267

Note: (a), (b) and (c) indicate that the estimated coefficients are statistically significant or rejection of the null hypothesis at the statistical significance

Finally, as the Granger Causality Test is sensitive to the number of lags, in Table 10 below this test is implemented only on the pair of variables in first differences, considering lags 1 to 5, and the results obtained, which are more intuitive from an economic point of view, show that GDP Granger-causes social security income at a significance level of 1%, thus diverging from the results reported in Table 7.

Table 10 – Results of the Paired Causality Test

Null Hypothesis	Paired Granger Causality Test Statistics					Direction of Causality
	1 lag	*2 lags*	*3 lags*	*4 lags*	*5 lags*	
$\Delta\tau^{sa}_{t-1}$ no Granger-cause Δy^{sa}_{t-1}	1,89986 (0,1691)	0,58931 (0,5553)	0,89865 (0,4422)	0,80123 (0,5252)	0,73865 (0,5950)	$\Delta\tau^{sa}_{t-1} \nrightarrow \Delta y^{sa}_{t-1}$
Δy^{sa}_{t-1} no Granger-cause $\Delta\tau^{sa}_{t-1}$	20,5670[a] (8e)$^{-06}$	12,6991[a] (5e)$^{-06}$	9,29340[a] (7e)$^{-06}$	7,77562[a] (6e)$^{-06}$	8,03395[a] (4e)$^{-07}$	$\Delta y^{sa}_{t-1} \rightarrow \Delta\tau^{sa}_{t-1}$

Note: (a), (b) and (c) indicate that the estimated coefficients are statistically significant or rejection of the null hypothesis at the statistical significance level of 1%, 5% and 10%, respectively.

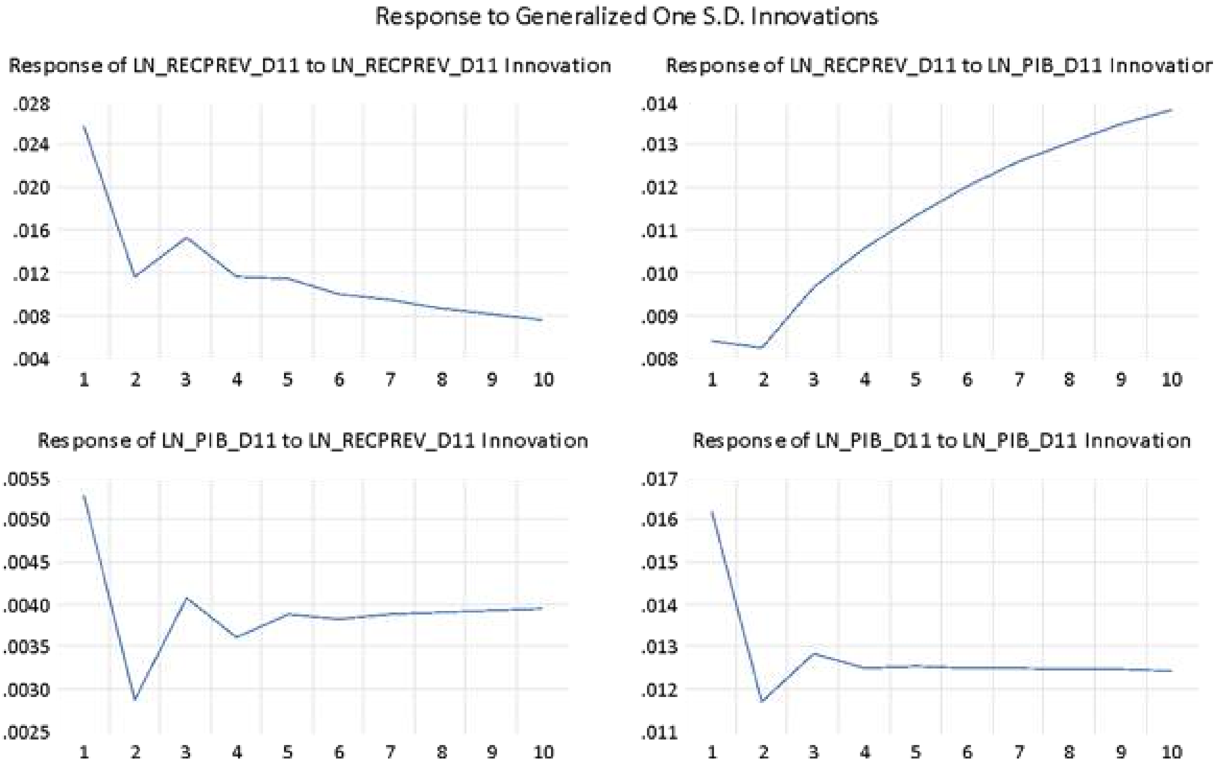
Therefore, while the results in Tables 8 and 9 point to a Granger bi-causality relationship between the variables in the long term, Table 10 points to a unidirectional Granger causality relationship between GDP and social security revenue in the short term.

The impulse-response function shows how a variable behaves after a shock to another variable in the system. This shock can be caused by any variable in the system, including the very variable being analyzed. The results of the impulse response function make it possible to assess how the shock will affect the variable in the short and long term.

In an autoregressive vector modeling context, it is important to check how a variable responds to a shock in another variable, assuming that all other variables remain constant. This analysis makes it possible to check whether the shock has a positive or negative effect on the response variable.

From the graphs in Figure 1, it can be seen that shocks to social security revenues on GDP or on themselves show a decrease with a subsequent tendency towards stability. The same observation is made when there is a shock to GDP. However, the response of a shock to the GDP time series on social security revenues is striking. The graph in the top right-hand quadrant of Figure 1 shows a rise in the social security revenue variable that began in the second period after the shock and remains observable even after ten periods.

Figure 1 – Generalized Impulse-Response Functions



The forecast error variance decomposition is another tool that can be used to interpret the results of VAR models. It provides information on the extent to which the variation in a variable can be explained by its own past values and by other variables. This tool is useful for assessing the relative importance of internal and external shocks to a variable. Tables 11 and 12 show the variance decomposition values of the forecast error for the RGPS social security revenue and GDP variables.

The results show the proportion of the total variation of each variable due to an individual shock. This is why the sum of the values obtained for the (RecPrev) and (GDP) columns for a given period is 100%. The results are shown in the graphs in Figure 1.

According to Table 11, after ten periods (months) of the occurrence of a shock in social security revenues, around 69.33% of the behavior of this variable derives from itself and approximately 30.66% from GDP.

According to Table 12, after ten periods (months) of a shock to GDP, 90.52% of the behavior of this variable is due to the variable itself and, in addition, approximately 9.48% is due to RGPS social security revenues.

Table 11 – Variance decomposition of prediction errors for τ_t^{sa}

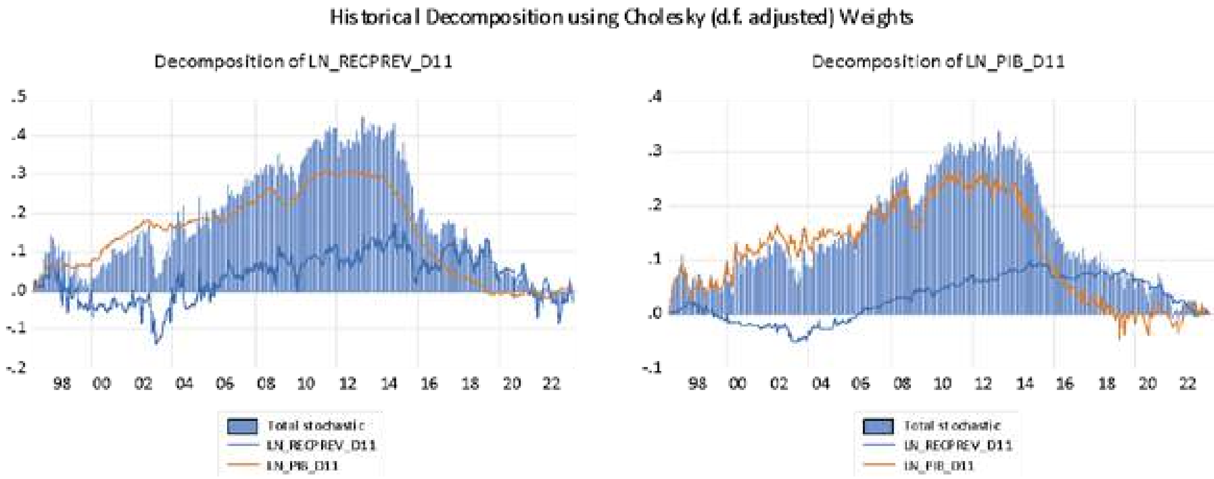
Period	S.E.	τ_t^{sa}	y_t^{sa}
1	0,025783	100,0000	0,000000
2	0,028649	97,30690	2,693101
3	0,032832	95,69850	4,301498
4	0,035548	92,23590	7,764104
5	0,038199	88,81557	11,18443
6	0,040567	84,84720	15,15280
7	0,042844	80,87402	19,12598
8	0,045032	76,87642	23,12358
9	0,047165	73,01841	26,98159
10	0,049253	69,33574	30,66426

Table 12 – Variance decomposition of prediction errors for y_t^{sa}.

Period	S.E.	τ_t^{sa}	y_t^{sa}
1	0,016181	10,65643	89,34357
2	0,019981	9,062574	90,93743
3	0,023754	9,351890	90,64811
4	0,026841	9,138954	90,86105
5	0,029627	9,218280	90,78172
6	0,032156	9,236977	90,76302
7	0,034493	9,301554	90,69845
8	0,036676	9,358271	90,64173
9	0,038731	9,420913	90,57909
10	0,040678	9,480430	90,51957

The interpretation of the data obtained from the decomposition of the forecast error variance can be complemented by the graphical analysis resulting from the use of the Cholesky method , as shown in Figure 2 below. The Cholesky method is an efficient method for decomposing symmetric positive definite matrices. It is often used in applications where it is necessary to solve linear systems, estimate econometric models or analyze time series.

Figure 2 – Historical decomposition using the weighted Cholesky method.



The graphs show the decomposition of the natural logarithm of the seasonally adjusted time series of the social security collection and GDP variables, respectively. The blue bars show the total stochastic (random) component, whose value for each period is the sum of the participation of each of the two variables analyzed. The decomposition graph for each of the variables under study makes it possible to understand the behavior of the stochastic component together with the trend and cyclical components, as well as to visualize the proportions/weights of each time series in the stochastic total.

6. FINAL THOUGHTS AND POLICY IMPLICATIONS

The article under review addresses the important relationship between Gross Domestic Product (GDP) and social security revenues in Brazil, covering an extended period from 1997 to 2023. Using a time series methodology, the study employs the Vector Error Correction Model (VECM) to investigate the long-term dynamics and causal relationships (temporal precedence) between these crucial variables. By applying unit root and co-integration tests, the article seeks to establish the existence of co-integration relationships, indicating a long-term equilibrium between GDP and social security revenues. This approach allows us not only to identify the long-term elasticities between the series, but also to examine the short-term influences through the model's adjustment dynamics, providing interesting insights into the underlying economic interactions between economic growth and social security fiscal sustainability.

The aim of this study was to measure the elasticity of social security revenue linked to the RGPS in relation to GDP. After duly calculating the time series and econometric tests, an elasticity of 1.278 was found. This result indicates an elastic relationship between social security collection and GDP. This means that, in the long term, an increase in GDP leads to a proportionally greater increase in revenues linked to the RGPS.

The analysis of the elasticity of RGPS collection in relation to GDP has important implications for Brazilian fiscal policy, as it indicates how social security collection varies in relation to economic growth. If the elasticity is high, it means that tax collection is growing more than proportionally to GDP. This would be positive for the government, as it would allow it to increase collection without increasing rates and/or contribution time. On the other hand, if the elasticity is low, it means that revenue is growing less than proportionally to GDP. This would be negative for the government, as it limits its capacity to collect and may require an increase in the tax burden in order to maintain fiscal balance.

In the context of seeking the financial sustainability of the social security system, long-term economic growth can contribute by increasing social security collection. By empirically understanding how these two fundamental economic variables (GDP and social security collection) are related, it opens the way for future studies to seek an interpretation of why they are so correlated.

Some hypotheses for social security collection being elastic to GDP are: (i) with economic growth, there is a tendency for wages to account for a greater share of total GDP; (ii) a reduction in informality in the labor market increases the number of taxpayers; (iii) economic growth resulting from an increase in productivity raises the amount paid by taxpayers.

Even taking into account the differences in methodology and period, one important aspect observed was that the elasticity found in this study was higher than those found in previous studies. Brazil underwent a labor reform (BRASIL, 2017) with the aim of reducing bureaucracy in hiring labor and thus encouraging the creation of formal jobs. In Brazil, membership of a public pension scheme is compulsory for formal workers. In this context, there is a legal relationship with economic implications between the level of employment and social security collection. This opens the way for empirical investigation of the relationship between levels of employment and economic activity. It is also suggested that such a study could ascertain the impact of the labor reform that took place in 2017 on social security revenues relating to the RGPS.

Finally, this study contributes to the literature by promoting a current assessment of social security collection, especially after the 2019 social security reform (BRASIL, 2019) and the Covid-19 Pandemic in 2020. For a long-term analysis, these two events are recent. Given that social security is a perennial public policy, studies, inferences and projections must be continuous in order to encompass new and relevant socioeconomic information.

Appendix

In Table A.1 below, when estimating the VAR Model at level (or Long-Term VAR Model), 2 lags were selected in accordance with the usual lag selection criteria. However, given the existence of a long-term equilibrium relationship between the variables analyzed, according to the results of the co-integration tests implemented, one lag will be considered when estimating the VECM Model. In particular, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) play a crucial role in weighing up the complexity of the model and its empirical performance.

Table A.1 – Selection of VAR/VECM Models

VAR Lag Order Selection Criteria
Endogenous variables: LN_RECPREV_D11 LN_PIB_D11
Exogenous variables: C D1997M03 D1997M12 D2000M03 D2001M06 D2004M01 D2005M01 D2005M02 D2009M11 D2012M03 D2015M12 D2020M03 D2020M04 D2020M05 D2020M06 D2020M07 D2020M08 D2020M10 D2020M11 D2020M12 D2021M01
Date: 04/28/24 Time: 19:43
Sample: 1997M01 2023M07
Included observations: 317

Lag	LogL	LR	FPE	AIC	SC	HQ
0	542.2864	NA	0.000146	-3.156381	-2.658356	-2.957445
1	1559.455	1886.736	2.45e-07	-9.548614	-9.003158	-9.330731
2	1614.231	100.9113*	1.78e-07*	-9.868964*	-9.276077*	-9.632135*

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

In Table A.2, "CointEq1" is the error correction term which, in the equation in which social security revenue is the dependent variable, has a negative coefficient (-0.192210) and is statistically significant (t-statistic = -4.98394), indicating the speed of adjustment in the short term and leading to a long-term equilibrium relationship, i.e. a co-integration vector. Note that the result of the adjustment that the variations need to make in the short term in order to reach long-term equilibrium is given by the coefficient α=-0,192210where these variations are, on average, corrected by around 19.22% per month.

Tabela A.2 – Modelo VECM estimado

Vector Error Correction Estimates
Date: 11/25/23 Time: 15:45
Sample (adjusted): 1997M03 2023M07
Included observations: 317 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LN_RECPREV_D11(-1)	1.000000	
LN_PIB_D11(-1)	-1.278737	
	(0.02039)	
	[-62.7265]	
C	6.666028	
Error Correction:	**D(LN_RECPREV_D11)**	**D(LN_PIB_D11)**
CointEq1	-0.192210	0.059230
	(0.03857)	(0.02404)
	[-4.98394]	[2.46373]
D(LN_RECPREV_D11(-1))	-0.394052	-0.045092
	(0.04307)	(0.02685)
	[-9.14875]	[-1.67943]
D(LN_PIB_D11(-1))	0.031503	-0.250116
	(0.08868)	(0.05528)
	[0.35522]	[-4.52423]
C	0.004766	0.004264
	(0.00153)	(0.00095)
	[3.11736]	[4.47409]

R-squared	0.751607	0.325381
Adj. R-squared	0.732109	0.272425
Sum sq. resids	0.196186	0.076236
S.E. equation	0.025876	0.016130
F-statistic	38.54713	6.144319
Log likelihood	721.1301	870.9484
Akaike AIC	-4.398297	-5.343523
Schwarz SC	-4.113712	-5.058937
Mean dependent	0.003297	0.002791
S.D. dependent	0.049994	0.018911
Determinant resid covariance (dof adj.)		1.55E-07
Determinant resid covariance		1.32E-07
Log likelihood		1610.569
Akaike information criterion		-9.845861
Schwarz criterion		-9.252974
Number of coefficients		50

Table A.3 below reports the results of the normality test for the residuals. It can be seen that, at a statistical significance level of 5%, at the second lag, the null hypothesis of the absence of multivariate normality in the residuals of the estimated VECM model is not rejected. However, it can also be seen that excess kurtosis has affected the results of the Jarque-Bera test.

Table A.3 - Normality Analysis

VEC Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
Null Hypothesis: Residuals are multivariate normal
Date: 11/25/23 Time: 15:49
Sample: 1997M01 2023M07
Included observations: 317

Component	Skewness	Chi-sq	df	Prob.*
1	-0.223051	2.628554	1	0.1050
2	-0.105990	0.593520	1	0.4411
Joint		3.222075	2	0.1997

Component	Kurtosis	Chi-sq	df	Prob.
1	4.248178	20.57790	1	0.0000
2	3.595506	4.684041	1	0.0304
Joint		25.26194	2	0.0000

Component	Jarque-Bera	df	Prob.
1	23.20645	2	0.0000
2	**5.277561**	**2**	**0.0714**
Joint	28.48401	4	0.0000

*Approximate p-values do not account for coeficiente estimation

Table A.4 below reports the results of White's Heteroscedasticity Test, which shows that the null hypothesis of homoscedasticity of the residuals of the estimated VECM model cannot be rejected.

Table A.4 - Heteroscedasticity Analysis

VEC Residual Heteroskedasticity Tests (Levels and Squares)
Date: 11/25/23 Time: 15:50
Sample: 1997M01 2023M07
Included observations: 317

Joint test:

Chi-sq	df	Prob.
64.15576	78	0.8702

Individual components:

Dependent	R-squared	F(26,290)	Prob.	Chi-sq(26)	Prob.
res1*res1	0.082258	0.999729	0.4682	26.07580	0.4589
res2*res2	0.043977	0.513075	0.9782	13.94067	0.9738
res2*res1	0.036325	0.420433	0.9950	11.51495	0.9936

Table A.5 below reports the results of the LM Test for Residual Serial Correlation. It can be seen that the null hypothesis of no serial correlation cannot be rejected.

Table A.5 - Autocorrelation Analysis

VEC Residual Serial Correlation LM Tests
Date: 11/25/23 Time: 15:50
Sample: 1997M01 2023M07
Included observations: 317

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	7.617923	4	0.1066	1.913722	(4, 580.0)	0.1066
2	3.875174	4	0.4232	0.970358	(4, 580.0)	0.4232

Null hypothesis: No serial correlation at lags 1 to h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	7.617923	4	0.1066	1.913722	(4, 580.0)	0.1066
2	9.849464	8	0.2758	1.235281	(8, 576.0)	0.2758

*Edgeworth expansion corrected likelihood ratio statistic

Table A.6 below reports the results of the Stability Test. The stability of a VECM model can be verified by observing the behavior of the inverse roots of the characteristic polynomial. The result shows the presence of a stochastic trend or random walk, given that one of the roots is located on the unit circle. However, the other inverse roots of the characteristic polynomial are located inside the unit circle, indicating dynamic stability.

Table A.6 - Stability Analysis

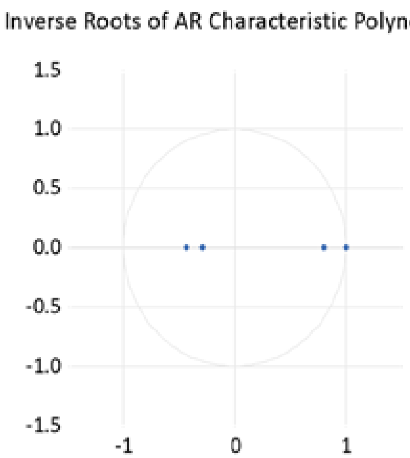


Table A.7 – Causality/Exogeneity Analysis. 1ST RESTRICTION - VECM.

Vector Error Correction Estimates
Date: 11/25/23 Time: 15:59
Sample (adjusted): 1997M03 2023M07
Included observations: 317 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
A(1,1)=0
Convergence achieved after 4 iterations.
Not all cointegrating vectors are identified
LR test for binding restrictions (rank = 1):

Chi-square(1)	21.63016
Probability	0.000003

Cointegrating Eq:	CointEq1	
LN_RECPREV_D11(-1)	-25.63904	
LN_PIB_D11(-1)	34.40113	
C	-192.4210	
Error Correction:	D(LN_RECPREV_D11)	D(LN_PIB_D11)
CointEq1	0.000000	-0.003730
	(0.00000)	(0.00078)
	[NA]	[-4.76633]

Table A.8 – Causality/Exogeneity Analysis. 2nd RESTRICTION - VECM.

Vector Error Correction Estimates
Date: 11/25/23 Time: 16:00
Sample (adjusted): 1997M03 2023M07
Included observations: 317 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegration Restrictions:
A(2,1)=0
Convergence achieved after 4 iterations.
Not all cointegrating vectors are identified
LR test for binding restrictions (rank = 1):

Chi-square(1)	5.477077
Probability	0.019267

Cointegrating Eq:	CointEq1	
LN_RECPREV_D11(-1)	-26.64546	
LN_PIB_D11(-1)	33.50008	
C	-169.9972	
Error Correction:	D(LN_RECPREV_D11)	D(LN_PIB_D11)
CointEq1	0.008432	0.000000
	(0.00136)	(0.00000)
	[6.21819]	[NA]

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