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DATING, CHRONOLOGY AND DYNAMICS OF BRAZILIAN HIGH DIMENSIONAL FISCAL CYCLES

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ABSTRACT

The study proposes the dating of Brazilian fiscal cycles. The cycles of Gross Revenue of Primary Expenditures of the Central Government were dated. The study also analyzes the dynamics of synchronization of the fiscal cycles and the economic and electoral cycles. Multivariate dating makes it clear that the duration of the phase of expenditure expansions was, in the study period, increasing. The multivariate model also shows that revenue recessions are extending their duration. The results show that there is synchronization between the spending cycle and the business cycle, as well as with the election cycle, in these cases with greater synchronization than with the revenue cycle. The tax collection cycle showed synchronization with the economic cycle, but no synchronization with the electoral cycle.

Keywords: Fiscal Cycles; Dating; Chronology; Cycle Synchronization; Dynamic Factorial Model with Regime Shifting.

JEL Classification: C55; G32.

SUMMARY

1. INTRODUCTION

The study of business cycles has been a key area of research for economics. In this regard, it should be mentioned that there are two types of business cycles in the economic literature: the classical business cycle and the growth cycle. Classical cycles refer to alternating periods of contraction and expansion in the level series, while growth cycles refer to alternating periods of acceleration and deceleration in economic activity and focus on the variation in the series, the more modern line of research has focused on growth cycles. In general, before the economy enters a recession, there is a deceleration of activity, and it usually accelerates before reaching an expansionary phase. In addition, there can be decelerations that do not translate into recessions or accelerations that do not correspond to expansionary phases. Therefore, the timing of the turning points do not necessarily coincide between the two types of cycles.

While the first concept is based on the level of economic activity, the second is based on deviations from a long-term trend. From a practical point of view, the former is more treatable, because the latter implies a decomposition into the trend and the cycle, which are unobservable components. Thus, the analysis of growth cycles depends on the method chosen to reduce the macroeconomic time trend. The study of cycles is subdivided into a few parts: dating, estimation of the cycle itself, synchronization analysis in the time and frequency domain, among others. In general, dating and chronology are the first steps because their results are used for the other topics. Although the literature focuses more on the estimation of the cycle itself, the dating literature closely follows such literature. For the Brazilian case the literature has focused on the estimation of the cycle, in particular the business cycle, opening space for the pioneering study of other cycles, such as the fiscal cycles presented in this study. The purpose of this study, therefore, is to establish a benchmark fiscal cycle chronology for Brazil over the last twenty years (1997 to 2017), describe in detail the chronology of Brazilian revenue and expenditure cycles, and present the dynamics of synchronization of fiscal cycles with economic and electoral cycles.

Initially we present a review of the business cycle literature in Brazil and worldwide. In the sequence we present the main methodologies to be used: the univariate dating models, the treatment of irregular interval-time data, given the characteristics of the Brazilian economic series; the multivariate models for dating in the context of high dimensionality and for this we present the dynamic factor model with regime change and the exploratory measures and inferences of synchronization of cycles in the time domain.

The results of seasonal adjustment of expenditures and collections are presented, as well as the quadratic trend analysis. The research considers a database with 2571 time series, in this context, usual smoothing methods such as LASSO and ADA-LASSO, although efficiently capturing linear dynamics, do not perform as well for the non-linear issues that are present in the study of cycles. To deal with this issue we estimated the probabilities of recession and expansion for all variables, and using cluster analysis we delimited the set of variables to be used in the multivariate dating model: Markov-Dynamic Factorial Markov-Switching Model (MS-DFM). Prior to the estimation of the MS-DFM models a cross-correlation analysis of the lag of the time series selected by cluster analysis was implemented in order to determine the temporal order to be estimated in the MS-DFM model. The study is concluded in the synchronization analysis between fiscal cycles, economic cycles and electoral cycles.

2. REVIEW OF THE LITERATURE

The topic related to business cycles has an old theoretical economic literature, which comes from the regular and periodic cycles proposed by Kalecki and Keynes (Schumpeter [1927], Keynes [1940], Kalecki [1937], Schumpeter and Fels [1939], Kalecki [1968]), through the criticism of many authors (Kuznets [1930], Friedman and Schwartz [1965], Lucas [1973, 1972]) to the real cycles of real business (Hodrick and Prescott [1997], Kydland and Prescott [1982], Kydland and Prescott [1982, 1990, 1996], Prescott [1986]), which in turn has already been tested by Mankiw [1989],and finally are guiding the more recent lines of research, which question to what extent distortions should be put into the basic real cycles model in order to be able to accurately reproduce the business cycle of a given economy, as we have in Chari et al. [2007], or even use the real cycle models to evaluate historical episodes of a given economy Cole and Ohanian [2004].

The econometrics of cycle studies have also evolved over time. From the pioneering works of Mitchell [1927], Mitchell [1930] and Burns et al. [1946] called the classical business cycle school, incorporating more powerful statistical ferra- mental (Tinbergen [1939] Tinbergen [1940b] Tinbergen [1940a]), which enabled international comparisons of cycle behavior (Backus and Kehoe [1992], Backus and Kehoe [1992]) through the cyclic component extraction filter literature (Hodrick and Prescott [1997], Beveridge and Nelson [1981], Neftici [1982], Baxter and King [1999], Proietti and Harvey [2000], Pollock [2000], Pedersen [2001], Christiano and Fitzgerald [2003] Pollock [2007]). Kaldor [1961] The summary of some evidence on the statistical behavior of business cycles became widely known as Kaldor's stylized facts. The econometric evolution reaches a turning point using dynamic factor models (DFM) (Jungbacker et al. [2008], Stock and Watson [1999, 2011, 2002]) and Markov Switching (MS) models (Hamilton [1989],Chauvet and Hamilton [2006]) and their more complex versions such as MS-DFM (Chauvet [1998], Chauvet and Su [2014], Leiva-Leon [2014], Kim [1994], Kim and Nelson [1999], Kim et al. [2008], Kim and Yoo [1995]).

The more recent econometric literature of business cycles deals with several issues beyond series co-movement (using DFM), and with stochastic regimes (with MS-DFM). Chauvet and Popli [2003] found several structural breaks for volatility for several industrialized countries, as well as changes in the US business cycle, Chauvet and Potter [2001]. As the structural breaks literature has gained ground in econometrics, business cycle models have incorporated their effects, as in the work of Chauvet and Potter [2002], Chauvet and Potter [2005] and more recently Chauvet and Su [2014] and Chauvet et al. [2016]. To address the challenge of high-dimensional data, a new alternative business cycle model has been proposed: Generalized Dynamic Factorial Model (GDFM) proposed by Forni et al. [2000]; Forni and Lippi [2001]; Forni et al. [2001, 2004, 2005].

Considering the Brazilian literature on business cycles, we have a more restricted set of publications. In terms of real business cycles Ellery Jr et al. [2002] used the HP filter (Hodrick and Prescott [1997]) in addition to the bandpass filter (Baxter and King [1999]) to obtain cyclical components used in a general equilibrium model, seeking to reproduce the close behavior of the economy, three years later Ellery-Jr and Gomes [2005] evaluated the properties of the Brazilian cycle. Val and Ferreira [2001] presented a real business cycle model for Brazil, analyzing some stylized facts, and the model parameters in this study were estimated by the GMM method. Kanczuk [2002] presented a dynamic general equilibrium model between interest rates and the economy for specific Brazilian facts. One of the first works applied in Brazil was the study by Cribari-Neto [1993], which estimated the cyclical component of the Brazilian GDP. The first work that dealt with the Brazilian business cycle using Markov Switching models was presented by Chauvet [2002] who differentiated the classical business cycle from the growth cycle, using the annual GDP from 1900 to 1999. Céspedes et al. [2006] evaluate, in the view of business cycles, the Brazilian quarterly real GDP from 1975Q1 to 2002Q2, suggesting that non-linear models are superior to linear models in predictive terms. More recently, Araújo et al. [2008] studied the business cycle using annual GDP per capita from 1850 to 2000 without finding changes in the volatility of GDP per capita, considering however, that volatility became more persistent after World War II. The most recent study, presented by Pereira and Vieira [2013], analyzed the Brazilian business cycle using quarterly real GDP from 1900 to 2012, with the period from 1900 to 1979 being obtained from temporal disaggregation using the structural time series model with state space representation, building the chronology of the cycle with the Markov Switching model.

The cycle dating literature has a smaller set of articles compared to cycle estimation studies per se. Even cycle estimation, by and large, assumes some established reference dating, which in most studies focuses on (economic) business cycle studies. Layton [1996] evaluates the performance of MS models for US coincident indicators to establish business cycle inflection points. Layton uses a "pseudo" real time analysis in which data are fully revised and are used in recursive estimates to evaluate in "pseudo" real time to evaluate business cycle dating. Recursive estimates are used to evaluate the real-time performance of the business cycle dating algorithm. Chauvet and Piger [2008] uses a real-time dataset that provided a more realistic evaluation of how the dating rules would have performed because it does not assume knowledge of data revisions that were not available at the time the rule would have been used. Chauvet and Potter [2002] use real-time data to evaluate the business cycle performance of univariate employment models and use MS models for the real GDP case, while Chauvet and Hamilton [2006] perform a similar exercise for multivariate MS models. Harding and Pagan [2003] also provide a comparison of univariate versions of dating rules. However, this comparison does not consider multivariate methods or the real-time performance of the methods. Stock and Watson [2014] use a high-dimensional database to estimate inflection points and US business cycle dating, they use both parametric and non-parametric methods for this purpose. The literature presented ensures that there is room for a novel study using high-dimensional, irregular interval-time series and dating the Brazilian fiscal cycles.

3. METHODOLOGY

3.1 Univariate Dating Models

Originally, cycle dating procedures and models focused on obtaining dating exclusively from the time series of the cycle under study. All dynamics and information were extracted from its own series. In this section we present the two most commonly used methods for this purpose: non-parametric (Bry-Brousch algorithm) and parametric (Markov-Switching model).

3.1.1 *Bry-Brousch Algorithm*

Business cycle dating methods have been studied in particular by the NBER, which has been responsible for dating the American cycle since the 1940s, Burns et al. The studies by Bry and Boschan [1971] and Boschan and Ebanks [1978] set a milestone among cycle dating methods by proposing nonparametric interpolation between peaks and valleys. More recently Harding and Pagan [2003] compare parametric and nonparametric methods of cycle dating. The most widely used procedure, proposes initially a dating with non-parametric methods and then a comparison with the dating generated from a parametric model. The non-parametric method presented by Harding and Pagan [2003] considers that the cycle of a series can be expressed in terms of its inflection points, which are local maxima and minima within a time sample. In general, $Yt = \ln y_t$ is used without loss of generality. The peak at period t in Y_t occurs when the value of Yt exceeds the values of Y_s for **s > t** and **s < t**. For this purpose a time window is used (t - k, t + k), in studies with monthly data, Bry and Boschan [1971] used a value of k = 5 while in studies with quarterly data, Harding and Pagan [2003] used a value of k = 2, they thus define the peaks (P_t) and valleys (V_t) as being:

$$P_t = ([Y_{t-2}, Y_{t-1}] < Y_t > [Y_{t+1}, Y_{t+2}]);$$

$$Vt = ([Y_{t-2}, Y_{t-1}] > Y_t < [Y_{t+1}, Y_{t+2}]).$$

In the pioneering work on cycle dating it was intended to ensure that cycle phases had a minimum duration of 6 months and that completed cycles had a minimum duration of 15 months. The method of Bry and Boschan [1971] was refined by Haywood [1973] to include a criterion for cycle length. Chauvet and Hamilton [2006] presents parameter methods with multiple indicators for identifying inflection points. In the pioneering work on cycle dating, it was intended to ensure that cycle phases had a minimum duration of 6 months and that complete cycles had a minimum duration of 15 months. The **?** method was refined by Haywood [1973] to include a criterion for cycle length. Chauvet and Tierney [2009] present parametric methods with various indicators for identifying inflection points. **?** presents a comparison between different dating methods and, more recently, proposes Stock and Watson [2014] high dimensional dating process using a combination of BB procedures and panel data models. Chauvet and Piger [2008] presented an approach with Markov Switching models for real-time dating.

3.1.2 *Markov-Switching Model*

A common approach for parametric estimation of cycles, are markovian regime shift models. Originally presented by Quandt [72], a simple model with two regimes:

$$Y_t = X_{1t}\beta_1 + u_{1t}, \text{ Regime 1}$$

$$Y_t = X_{2t}\beta_2 + u_{2t}, \text{ Regime 2}$$

Where X_{1t} and X_{2t} are independent variables, with Y_t is generated by regime 1 or regime 2, but not both.

Define an indicator variable, such as:

$$I_t = 1, \text{ Se } Y_t \text{ para o regime 1}$$

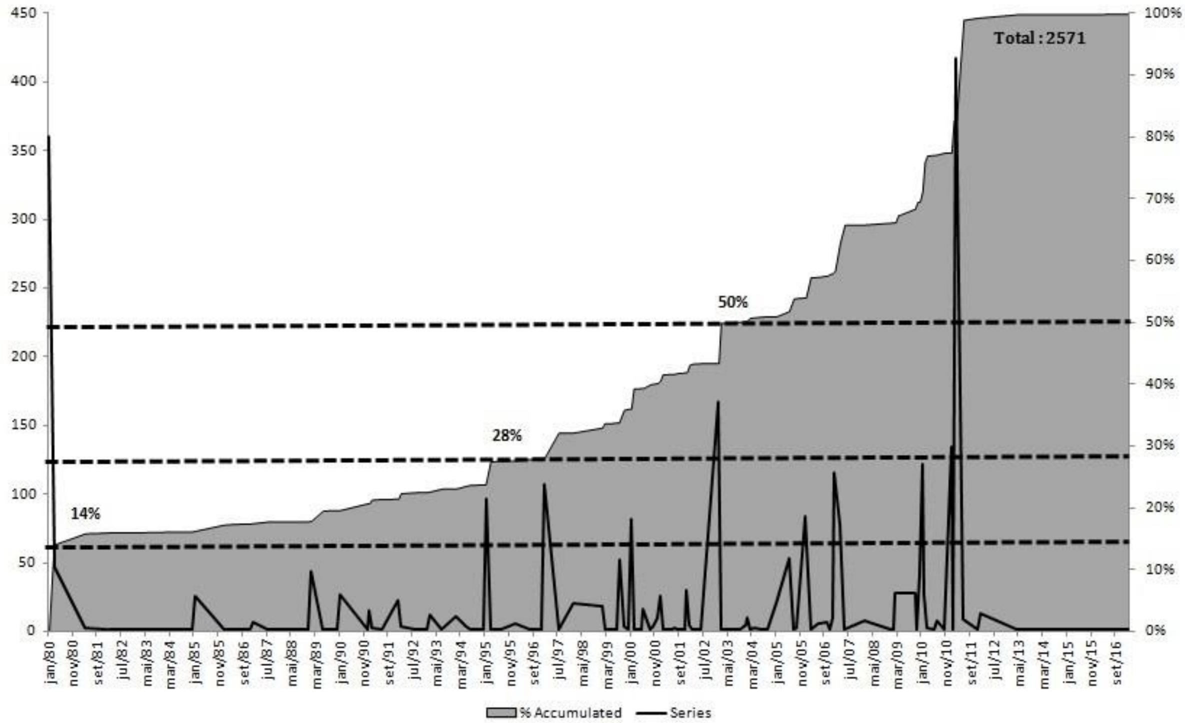
$$I_t = 0, \text{ Se } Y_t \text{ para o regime 2}$$

If I_t is observable, equations (3.1) can be estimated via MQO using the observations contained in the respective regimes. When the regime change is endogenous, even with known sample separation, equation (3.1) cannot be estimated by MQO, this model will result from a mixture of distributions, in these cases even the maximum likelihood procedure fails, because for some values of the variance of the residual, the likelihood function explodes. Therefore, iterative maximum likelihood is used, which considers at each step the probability estimation of each regime until the convergence of a threshold.

3.2 Irregular Time Interval Treatment

Time series related to the Brazilian economy have grown rapidly over time, a period of greater economic stability, allowed for the consolidation and creation of data research methodologies that allowed for this expansion in available time series. This growth can be seen in Figure 1, where we can see that only 14% of all series listed for this study[1] (2571) begin in 1980. The first big jump in new series occurs in 1995, reaching 28%. The second big jump occurs in 2002, reaching 50%. However, the biggest jump occurs between 2010 and 2011.

Figure 1 - Time evolution of the Brazilian series available for economic studies



An important fact is that, in quarterly terms (the time frequency that will be used in this study), we have 152 quarters since 1980, 84 since 1997 and only 28 since 2010, which was the maximum limit allowed to compose the database. The categories with the largest number of series beginning in 1980 are related to agriculture and livestock, climate data, National Accounts, financial series, international and sectoral series. Among the most recent series, most are concentrated on services, credit, confidence series, and general surveys. The data sources with older series are concentrated in IBGE, Brazilian Central Bank, sectorial (such as ELETROBRAS, ANFAVEA, ANP) and international series like Bureau of Labor Statistics, US Department of Commerce, and US Census Bureau. The distribution of the series by information source is detailed in Table 1.

[1] Given the high dimensionality of the data used in this study, a complete description of these data would exceed the total number of pages proposed by the rules of the contest, so these descriptions can be requested from the author.

Table 1 - Description - Database Source

Nr.	Fonte	Séries	Percentual
1	BACEN - Central Bank of Brazil	624	0,24
2	CNI - National Confederation of Industry (Brazil)	437	0,17
3	IBGE - Brazilian Institute of Geography and Statistics	210	0,08
4	U. S. Census Bureau	117	0,05
5	FUNCEX -Foundation Center for Foreign Trade Studies (Brazil)	105	0,04
6	CAGED - General Register of Employees and Unemployed (Brazil)	104	0,04
7	Bureau of Economic Geology - The University of Texas at Austin	96	0,04
8	BLOOMBERG - Technology and data company	89	0,03
9	STN - National Treasury Secretariat (Brazil)	84	0,03
10	FIESP - Federation of Industries of the State of São Paulo (Brazil)	73	0,03
11	SERASA - Brazilian analysis and information company for credit decisions (Brazil)	57	0,02
16	FGV - Getulio Vargas Foundation (Brazil)	46	0,02
12	CNC - National Confederation of Commerce (Brazil)	45	0,02
13	MDIC - Ministry of Industry, Foreign Trade and Services (Brazil)	45	0,02
14	BNDES - National Bank for Economic and Social Development (Brazil)	44	0,02
15	CPB Netherlands Bureau for Economics	44	0,02
17	CEPEA - Center for Advanced Studies in Applied Economics (Brazil)	41	0,02
18	JP Morgan - Holding company	40	0,02
19	ELETROBRAS - Brazilian Electric Power Plants	30	0,01
20	U. S. Bureau Of Labor Statistics	26	0,01
21	U.S. Census Bureau	22	0,01
22	ANFAVEA - National Association of Motor Vehicle Manufacturers (Brazil)	20	0,01
23	FENABRAVE - National Federation of Motor Vehicle Distribution (Brazil)	18	0,01
24	CBOT - Chicago Board of Trade	17	0,01
25	SABESP - Brazilian company that holds the concession of public basic sanitation services in the State of São Paulo	14	0,01
26	ANP - National Agency of Petroleum, Natural Gas and Biofuels (Brazil)	12	0,00
27	CONAB - National Supply Company (Brazil)	10	0,00
28	O.N.S - National Electrical System Operator (Brazil)	10	0,00
29	ABRAS - Brazilian Association of Supermarkets	9	0,00
30	The Conference Board	9	0,00
31	FIPE - Institute of Economic Research Foundation (Brazil)	8	0,00
32	ANEFAC - National Association of Finance Executives (Brazil)	7	0,00
33	ABECS - Brazilian Association of Credit Card Companies and Services	6	0,00
34	ABIMAQ - Brazilian Association of Machinery and Equipment Industry	6	0,00
35	Eurostat - European Commission statistics	6	0,00
36	FAO - Food and Agriculture Organization of the United Nations	6	0,00
37	US Department of Commerce	5	0,00
38	BMF - Brazilian Board of trade	4	0,00
39	ABRACICLO - Brazilian Association of Manufacturers of motorcycles, mopeds, scooters, mopeds and similar	3	0,00
40	ANDA - National Association for Fertilizer Diffusion (Brazil)	3	0,00
41	DIEESE - Department of Statistics and Socioeconomic Studies (Brazil)	3	0,00
42	FECOMERCIO - Federation of Commerce (Brazil)	3	0,00
44	University of Michigan	3	0,00
45	ABCR - Brazilian Association of Highway Concessionaires	2	0,00
46	ABPO - Brazilian Association of Corrugated Paper	1	0,00
47	Chicago Board Options Exchange	1	0,00
48	Deutsche Borse e Goldman Sachs	1	0,00
49	IAB - Brazil Steel Institute	1	0,00
50	National Association of Realtors (Brazil)	1	0,00
51	PETROBRAS - Brazilian oil company	1	0,00
52	SPC - Credit protection service, credit bureau (Brazil)	1	0,00
53	ACSP - Commercial Association of São Paulo (Brazil)	1	0,00
	Total	2571	1,00

Fifty-three open data sources were used, with IBGE (8%), Brazil's Central Bank (24%), and CNI (17%) accounting for almost 50% of all series used. Approximately 15% of the series are from international information sources. 31% of the information sources are from research institutes, associations and confederations.

3.2.1 EM Algorithm for Irregular Interval-Time

Bańbura and Modugno [2014] proposed general treatment for time series with arbitrary missing data pattern, with flexibility to allow correlation of the idiosyncratic component; the essential idea of the algorithm is to write the maximum likelihood as if the data were complete (initially replacing the missing data with random values of $N(0,1)$) and iteratively use the two-step EM algorithm: in the expectation step, we "fill" the missing data with probabilities with characteristics of the residue distribution, while in the maximization step, we again optimize this expectation. Under some regularity conditions, the EM algorithm converges to a local maximum of the maximum likelihood. Direct maximization of the likelihood is computationally not feasible for large N. However as proposed by Doz et al. [2012], the computational complexity can be circumvented in some cases in the Expectation-Maximization (EM) algorithm. Bańbura and Modugno [2014] offer a solution to problems for which incomplete or latent data produce intractable likelihood. The essential idea is to write the maximum likelihood of the complete data and back-dating (re-estimating) "fabricating" the missing data in the conditional hope step of the algorithm. In particular in dynamic factor models this approach allows to estimate from a large set of time series, common F_t factors (cyclic), with high degree of co-movements. Thus, estimating both F_t and θ.

To derive the steps of the EM algorithm for the model described above, we will define the joint Log-likelihood of Xt and Ft , by $l(X_t, F_t, \theta)$ where θ is a parameter to be estimated, given the available data $\Omega_T \subseteq X_t$, given that some data in xit may be missing. According to Bańbura and Modugno [2014] the EM algorithm follows two steps that alternate:

1.To obtain initial values for the parameters, θ(0) replace the missing observations in x_{it} (Note that x_{it} already normalized for cycle and dating studies) with random samples from the $N(0, 1)$ distribution;

2.E-step - the Log-likelihood hope of the data is calculated using the estimates obtained in the previous step iteration (or from an initial step), θ_j:

$$L(\theta,\theta_j) = E_{\theta(j)}[l(X_t, F_t, \theta)|\Omega_T]$$

3.M-step - the parameters are re-estimated in order to maximize the log-likelihood with respect to θ:

$$\theta(j + 1) = \underset{\theta}{\mathrm{argmax}} L(\theta, \theta_j)$$

4.The interactions stop when the increment in likelihood between two consecutive steps is very small from an assumed truncation. When x_{it} contains no missing data, we will have:

$$E_{\theta(j)} [X_t X'_t \mid \Omega_T] = X_t X'_t \tag{2}$$

$$E_{\theta(j)}\,[X_t F'_t \mid \Omega_T] = X_t\, E_{\theta(j)}\,[F'_t \mid \Omega_T] \tag{3}$$

Finally, the conditional estimation of the moments of the latent factors,

$$E_{\theta(j)}\,[F_t \mid \Omega_T] \tag{4}$$

$$E_{\theta(j)}\,[F_t F'_t \mid \Omega_T] \tag{5}$$

$$E_{\theta(j)}\,[X_{t-1} F'_{t-1} \mid \Omega_T] \tag{6}$$

$$E_{\theta(j)}\,[X_t F'_{t-1} \mid \Omega_T] \tag{7}$$

can be obtained using smoothed Kalman filter in a state-space representation. However, when X_t contains missing values we can no longer use 2 and 3, the solution proposed by Bańbura and Modugno [2014] uses W_t a diagonal matrix *N* with ith diagonal element equal to 0 if x_{it} is missing and 1 otherwise, consider for this purpose $E\theta(j)\ [.|\Omega T\ ] \otimes Wt\ ]$. Intuitively, W_t works as a selection matrix, so only the available data is used at each iteration in a joint manner. Again, only the data available and updated at each step are used for estimation and re-estimation. I - W_t in terms of the last "selected" data input $E\theta(j)$ $[.|\Omega T\ ]$ will correspond to the last missing value. When we apply the Kalman filter with state-space representation for the cases where some observations in xit are missing, the corresponding row in x_{it} and $\theta(j)$ are ignored.

Given the parameter estimates θ and the data set Ω_T we can obtain the conditional expectations for the missing observations of:

$$E_{\hat{\theta}}[x_{it} \mid \Omega_T] = \hat{B}_i E_{\hat{\theta}}[F_t \mid \Omega_T] + E_{\hat{\theta}}[\xi_t \mid \Omega_T]\text{, for } x_{it} \notin \Omega_T \tag{8}$$

Where $\hat{B}_i$ denotes ith line in $\hat{B}$. $E_{\hat{\theta}}[\,F_t \mid \Omega_T]$ e $E_{\hat{\theta}}[\xi_t \mid \Omega_T]$ are obtained by applying the Kalman filter with smoothed estimates of the state-space representation. If, for example, series *i* is available only in period $t_i > t$ Kalman smoothing can be obtained for back-dating past estimates of the series: $E_{\hat{\theta}}[x_{it} \mid \Omega_T]$ for $t < t_i$ conditional the information of the other series and their correlation structure in the period of the data that are observed.

3.3 Multivariate Dating Model

By modeling the behavior of several different variables simultaneously, we can capture widespread cyclical fluctuations in several different characteristics of the phenomenon under study. Since recessions and expansions are caused by different shocks over time, the inclusion of different variables increases the model's ability to represent and signal the phases of the business cycle. In addition, the combination of variables reduces measurement errors in the individual series and thus the likelihood of false turn signals, which is particularly important when data from a single series.

3.3.1 *Markov-Switching Dynamic Factorial Model*

Consider a time series vector that includes the series of interest, as being:

$$y_{t-1} = (y_{1,t-1}, y_{2,t-1,\dots,} y_{k,t-1,\dots,} y_{1,1,} y_{2,1,\dots,} y_{k,1}$$

The interference in this process will be given by the latent states of regimes to be estimated, as we saw in 3.1.2:

$$P_r(S_t = i, S_{t-1} = j \mid Y_t)$$

However since we now have a k-series set, we follow Chauvet [1998] and Kim and Nelson [1999] in specifying how a recession can affect different economic indicators at the same time, they propose:

$$\Delta Y_t = \lambda \Delta F_t + \Delta v_t$$

$$\Delta F_t = \mu s_t + \emptyset \Delta F_{t-1} + \eta s_t$$

$$\Delta v_t = d(L)\Delta_{v_t-1} + \in_t \;\; \text{com} \;\; \Delta = 1 - L$$

$$\eta_{St} \sim N(0, \sigma^2_{St}) \qquad\qquad \epsilon_t \overset{i.i.d}{\sim} N(0, \Sigma)$$

Where η_{St} is the common shock that generates changes in the phases of the business cycle, ϵ_t is the error of the measure. In order to capture the potential asymmetries in different states of the business cycle, the factor intercept and variance change according to a Markov process by means of the variance S_t:

$$\mu_{St} = \alpha_0 + \alpha_1 S_t$$

with

$$s_t = \begin{cases} 1, for\ economic\ expansion\ regime \\ 0, for\ economic\ expansion\ regime \end{cases}$$

The factor volatility measured by σ^2_{St} can also take on different values according to the regime. The regime change is determined by the transition probabilities of the Markov process of two first-order states:

$$p_{i,j} = Prob\,[S_t = j \mid S_{t-1} = i], onde\ \textstyle\sum_{j=0}^{1} pi, j = 1,\ \forall i, j = 0{,}1.$$

The model separates the common noise underlying the observed variables from individual variations in each of the economic activities represented by the set of covariates, from the true signals of business cycle change. The dynamic factor captures broad simultaneous unfolding and catch-up movements of various sectors of the economy, which is in line with the definitions of business cycles presented by Burns et al. [1946]. In the case where only one of the variables suffers a downturn, it will not be able to characterize a recession in the model, and would be captured as a variation of the idiosyncratic term. A recession (expansion) will occur when all n variables decrease (increase) at approximately the same time (coincident movements), or in advance increase or decrease together (antecedent movements). The model also predicts important issues such as the non-linear combination of coincident and antecedent variables, this represents an advantage over models that do not use regime change.

3.4 Cycle Synchronization Analysis

A relevant question when analyzing the results of dating cycles is their degree of synchronization with other cycles. In the time domain, this synchronization can reveal a dynamic between expansion and contraction movements, which reveal whether two cycles are pro-cyclic or counter-cyclic, that is, whether the expansion/expansion phases are concordant, or whether they are antagonistic to each other. We present for this purpose an exploratory (IC) and inferential measure of cycle synchronization.

3.4.1 *Harding-Pagan Concordance Index*

Harding and Pagan [2006] proposed an index of cycle synchronization: Concordance Index (CI), The CI measures, in an exploratory way, the degree of co-movement between two cycles. It counts the fraction of time when both series are simultaneously in the same state $S_t = 1$ of expansion and $S_t = 0$ of contraction so that:

$$CI_{i,j} = \frac{\sum_{k=1}^{n} I_{(S_{i,t}=1;\, S_{j,t}=1)} + I_{(S_{i,t}=0;\, S_{j,t}=0)}}{n} \tag{9}$$

Where $I(.)$ represents the indicator function of the event in question. If two cycles are exactly pro-cyclical, the index will be 1, while a value of zero will indicate as being exactly counter-cyclical.

3.4.2 *Meller-Metiu Synchronization Test*

Meller and Metiu [2017] proposed a bivariate test for phase synchronization in the time domain. Once the phases of the cycle Cit have been estimated, these authors propose a way to synchronize the expansion and contraction phases in order to be able to test their statistical significance. Initially they map the iteration of the cycle phases by means of a binary variable Fit that assumes values 1 if both cycles are in the expansion phase and -1 if both cycles are in the contraction phase. Defining:

$$F_{it} = \begin{cases} 1 \; se \; C_{it} > \overline{C_{it}} \; if \; cycle \; i \; is \; expanding \; in \; time \; t. \\ -1 \; se \; C_{it} \leq \overline{C_{it}} \; if \; cycle \; i \; is \; expanding \; in \; time \; t. \end{cases} \quad (10)$$

This definition corresponds to the measure used to measure business cycle synchronization proposed by Mink et al. [2011]. We can use F_{it} to measure the synchronization between cycles i and j by means of its expected value:

$$S_{ij,t} = F_{it} \, X \, F_{jt} = \begin{cases} 1 \; if \; cycle \; i \; and \; j \; are \; in \; the \; same \; phase \; in \; time \; t. \\ -1 \; if \; cycle \; i \; and \; j \; are \; in \; the \; same \; phase \; in \; time \; t. \end{cases} \quad (11)$$

This will give us the expected value[2] of $S_{ij,t}$:

$$E(S_{ij,t}) = E(F_{it}F_{jt}) = 1P(F_{it} = 1, F_{jt} = 1) + 1P(F_{it} = -1, F_{jt} = -1) - 1P(F_{it} = 1, F_{jt} = -1) + 1P\,(F_{it} = -1, F_{jt} = 1) \quad (12)$$

Consider three extreme cases of synchronization (see also Harding and Pagan [2006]). First, perfect phase synchronization (PPS) and perfect negative phase synchronization (PNS) which we will define as:

$$PPS: P\big(F_{it} = 1, F_{jt} = 1\big) + P\big(F_{it} = -1, F_{jt} = -1\big)$$
$$PPS: P\big(F_{it} = 1, F_{jt} = -1\big) + P\big(F_{it} = -1, F_{jt} = 1\big) = 1$$

Second, non-synchronous (NonS) occurs if the cycles are extremely in opposite phases or in the same phase as follows:

$$NonS\;(case\;um): P\big(F_{it} = 1, F_{jt} = 1\big) + P\big(F_{it} = -1, F_{jt} = -1\big) = \tfrac{1}{2}$$
$$NonS\;(case\;dois): P\big(F_{it} = 1, F_{jt} = -1\big) + P\big(F_{it} = -1, F_{jt} = 1\big) = \frac{1}{2}$$

Consider the expected value of PNS, PNS e NonS:

$$PPS: E\big(S_{ij,t}\big) = 1$$

$$PNS: E\big(S_{ij,t}\big) = -1$$

$$NonS: E\big(S_{ij,t}\big) = 0$$

Thus, we could, in principle, test the PPS. However, the rejection of these hypotheses would not allow us to discriminate between the alternatives of positive (but not perfect) synchronization, no synchronization, and negative synchronization. Therefore, we consider the following null hypothesis:

$$H_0: \mu_{ij,t} = E\big(S_{ij,t}\big) \leq 0;$$

$$H_1: \mu_{ij,t} = E\big(S_{ij,t}\big) > 0;$$

[2] Note that phase synchronization can be interpreted as phase correlation.

Thus, we can test whether cycles are unsynchronized or negatively synchronized with the one-sided alternative of positive, but not necessarily perfect, phase synchronization. A rejection of the null hypothesis is interpreted as evidence of positive phase synchronization between two cycles. For the null hypothesis, we estimate the following OLS regression:

$$S_{ij,t}: \mu_{ij,t} + \epsilon_{ij,t} \tag{13}$$

and we used the one-tailed t-test to test H0 . The synchronization variable Sij,t e $\epsilon ij,t$ can show serial correlation, as they inherit their serial dependence structure from the underlying time series. Thus we use the standard error proposed by ? which was also used by Harding and Pagan [2006] as a way to remedy this issue in the context of binary variable regressions. We present the steps of the bivariate test for phase synchronization in the time domain:

1. You create the phase dating variable F_{it};
2. The phase synchronization variable $S_{ij,t}$ is calculated;
3. We estimate and test the phase synchronization $\mu_{ij,t}$ using 13.

4. RESULTS

4.1 Database

Challenges related to the database of young economies are always present in any study. Studies with high time series increase these challenges, as these data present problems of TISN[3], time frequency data that require a disaggregation procedure that will also require a large set of covariates, seasonal adjustment procedures that must be applied on a large scale, but considering the particular country issues, the discontinuities of methodologies that generate old and new series of the same phenomenon.

We address these issues in this chapter. To obtain a database, suitable for the study of cycles, several steps were taken to make the models estimated. The series were made compatible with the aim of ensuring as much information as possible; the annual series were disaggregated in a high-dimensional context at quarterly frequency; the series whose seasonalized versions were not published were submitted to a sequential seasonal adjustment procedure, taking into account Brazilian stylized facts; as will be the study.

[3] Abreviation for Temporal Irregular Span.

Focusing on the concept of business cycle growth (unlike, for example, the NBER method, which measures cycles at the series level, see Burns and Mitchell [1947]), we defined quarterly variation as the frequency to be studied. The series available monthly were transformed to quarterly by taking averages, resulting in 84 observations between 1997Q1 to 2017Q4. In addition, all activity variables are expressed in real terms. These are obtained by deflating nominal variables[4]. For all other variables (e.g., interest rates and exchange rates), nominal and real concepts were included in the dataset.

The models that will be estimated require stationary time series. We chose to apply the same stationary procedure to all series. We applied first difference for the level series by calculating their changes from the previous quarter and applying simple difference for negative values. For price variables, we performed percentage changes (consumer prices, stock prices, ...) with respect to the previous quarter of the index. We also applied this procedure to stationary variables, the reason for is: calculating quarterly changes for all variables allows us to capture the concept of the business cycle growth cycle. Interest rate spreads, which were calculated in levels, are the only exception to this rule. Given their covariance amply illustrated with the concept of business cycle growth, **?** show that this is common practice. Subsequently, the series were normalized to zero the sample mean and unit variance. This standardization is necessary to avoid overweighting the series with large variation when estimating the dynamic factors. Subsequently, the common component is denormalized to match the real series.

The final step, after normalizing the data, the TISN series are back-dating using the Bańbura and Modugno procedure [2014]. Only then the MS-DFM models were estimated.

4.2 Fiscal Seasonality

Macroeconomic time series provide some useful information about overall economic strength, provided they can be correctly interpreted. This interpretation can be misled if significant factors such as seasonality are not taken into account. As a result, incorrect conclusions are derived.

Therefore, an important stage in preparing the database to be used in dynamic factorial models is the treatment of seasonal effects. Methodologies have evolved from

[4] CPI Each case was studied: IGP-DI used for general series, IPCA used for consumer related series, US CPI for the case of variables from this country and therefore respectively for each specific case.

the simple application of seasonal dummies, to automated adjustment methods. For this purpose, we use the X-13ARIMA-SEATS-TRAMO, developed by the US Census Bureau with the support of the Bank of Spain and is maintained by the former institution. X13 was developed by the US Census Bureau with the support of the Bank of Spain. The program, created in July 2012, is the combination of two other seasonal adjustment programs and, which in addition to seasonalizing the series, offer various diagnostics to assess the quality of seasonal adjustment. Recent research on the topic has been conducted that compares the performance of the methods depending on the sample length. They conclude that TRAMO / SEATS performs better even if the sample is significantly reduced.

The algorithm for seasonality identification, treatment and adjustment was built in the following steps:

1. Identification seasonality tests are applied to all series;
2. If the series shows seasonality, the seasonal adjustment of the industrial production specification (PIM) is tested;
3. Applies the QS tests to the seasonally adjusted series, if the seasonality has been eliminated, it terminates;
4. If the series shows seasonality, the seasonal adjustment of the Monthly Trade Survey (PMC) specification is tested;
5. Applies the QS tests to the seasonally adjusted series, if the seasonality has been eliminated, it terminates;
6. If the series presents seasonality, the seasonal adjustment of the IBC-Br (IBC-Br) specification is tested;
7. Applies the QS tests to the seasonally adjusted series, if the seasonality has been eliminated, it terminates;
8. Applies the automatic X-13ARIMA-SEATS-TRAMO adjustment (considering the number of working days and the Carnival and Easter holidays for the Brazilian series).

We present in Figure 2 the results for government spending. Although the seasonal effects are distributed throughout the months of the year, the values for December and February are highlights. In December with an average increase of 38% and in February with an average decrease of 16%.

Figure 2 – Average monthly seasonality of Brazilian Government Expenditure

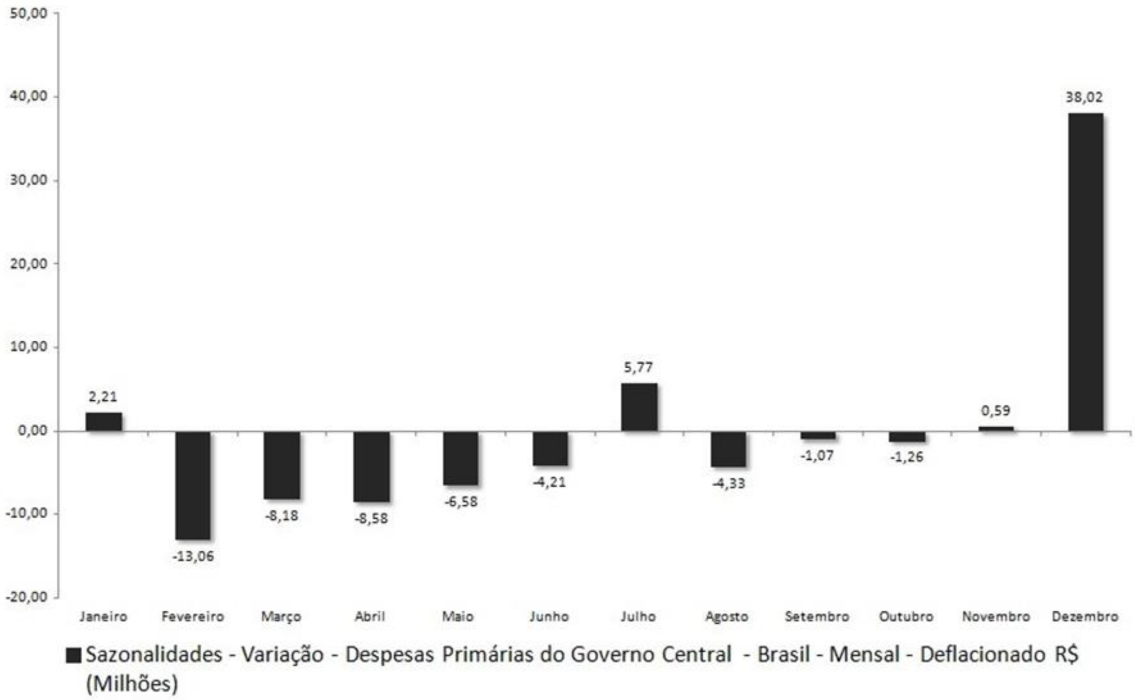
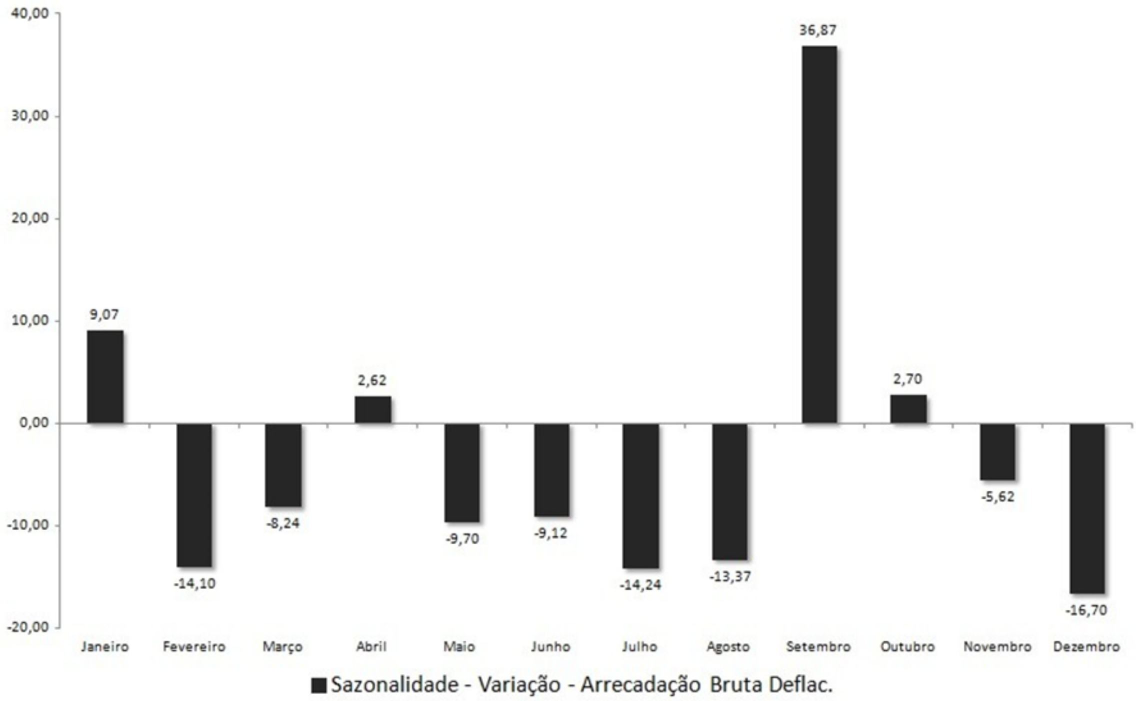


Figure 3 shows the seasonal results for tax collection. These seasonal results are distributed throughout the year, however with emphasis on the month of September, where we have an average increase of 36% in collection.

Figure 3 – Average monthly seasonality of the Brazilian Gross Revenue



All other series were seasonally adjusted using the described algorithm, only the series where the data source already provides the seasonally adjusted series were not seasonally adjusted, as shown in Table 2.

Table 2 – Method used for Seasonal Adjustment

Sources/ Seasonal Adjustment	Série	\%
X13-ARIMA-SEATS-TRAMO	1581	0,61
PIM - Specification	227	0,09
PMC - Specification	152	0,06
IBGE	143	0,06
U. S. Census Bureau	117	0,05
Without Seasonal Adjustment	103	0,04
CPB Netherlands Bureau for Economics	44	0,02
U. S. Bureau Of Labor Statistics	26	0,01
FIESP	25	0,01
U.S. Census Bureau	22	0,01
FGV	21	0,01
FENABRAVE	18	0,01
IBC-Br - Specification	10	0
The Conference Board	9	0
Brazil Central Bank	7	0
SERASA	7	0
ABECS	6	0
ABIMAQ	6	0
CNI	6	0
Eurostat	6	0
O.N.S	5	0
US Department of Commerce	5	0
.	.	.
Total	2571	1

4.3 Treatment of *Outliers*

In this section we present the treatment for eventual extreme values that may appear in the expenditure or collection series. For this purpose we use the Hampel algorithm (Hampel et al. [2011]; Hampel [1971]) in the level series. This pre-estimation treatment of the models guarantees that the cycles' movements are not products of atypical events, or are artificially generated by an isolated behavior of the series, compromising its medium and long term trajectory.

4.3.1 *Extreme Values for Expenditure Data*

The results show only 11 outliers for the monthly data as shown in Figure 4 and Table 3.

Figure 4 – *Monthly Outliers of Brazilian Government Expenditures*

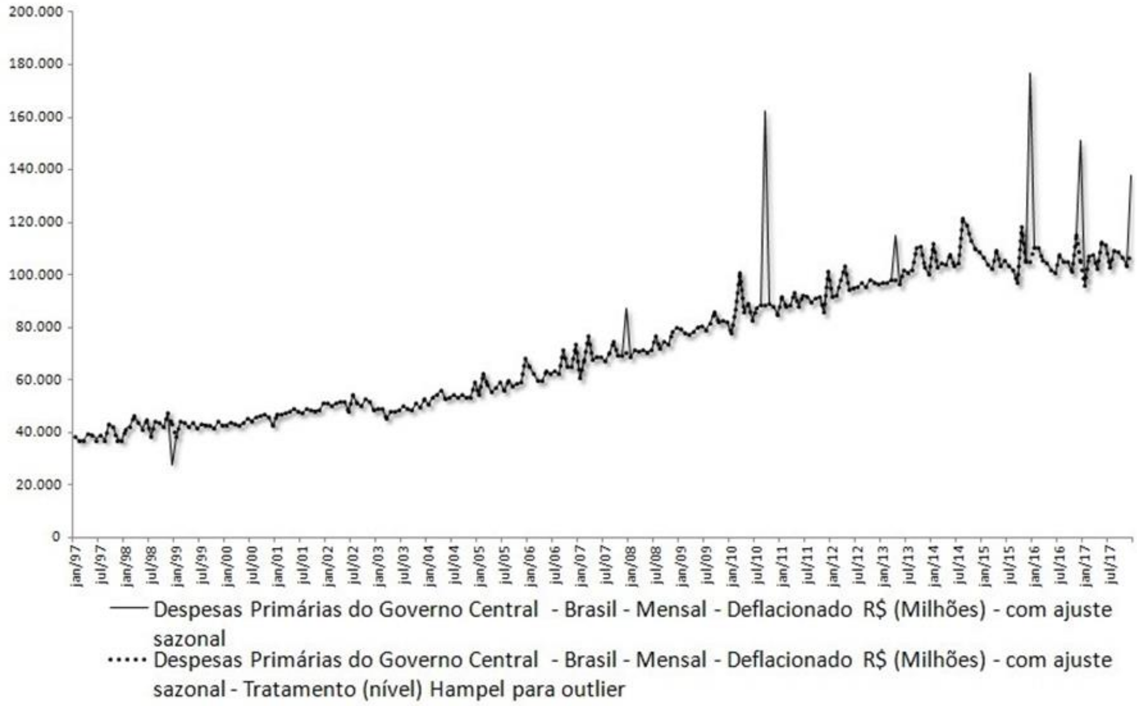
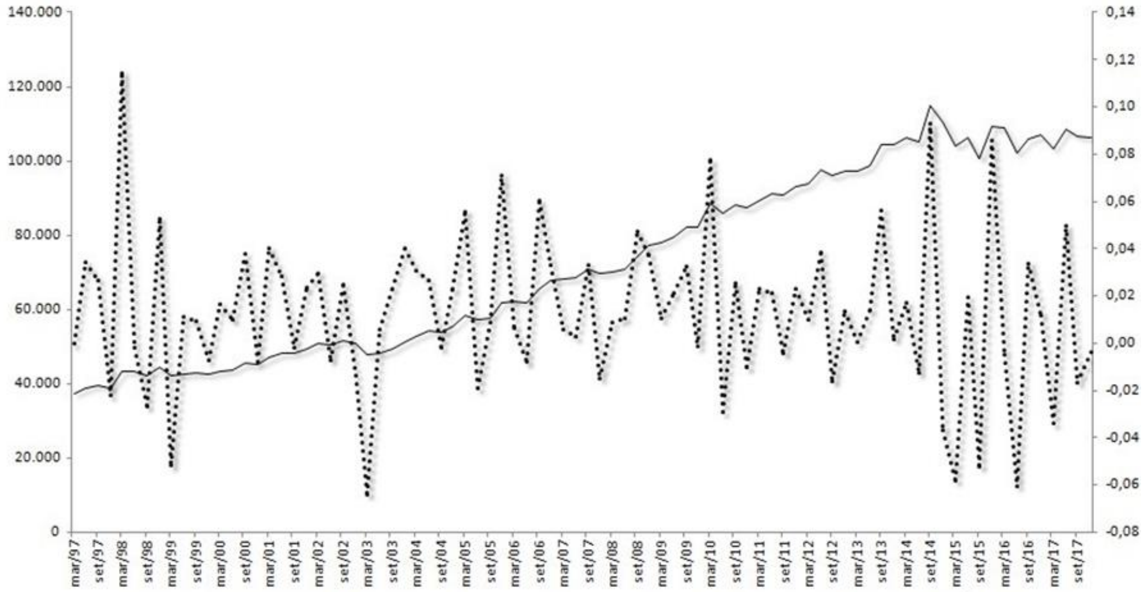


Table 3 – *Outliers for monthly Government Expenditure series*

***Outlier* identificados**
fev/98
ago/98
set/02
ago/04
nov/04
abr/06
out/09
nov/09
dez/09
out/16
nov/16

With the correction of the extreme values, the quarterly expenditure will be:

Figure 5 – Brazilian Government Expenditure on a quarterly basis



4.3.2 *Extreme Values for Revenue Data*

The results show only 10 outliers for the monthly data:

Figure 6 – Monthly *Outliers of* the Brazilian Gross Revenue

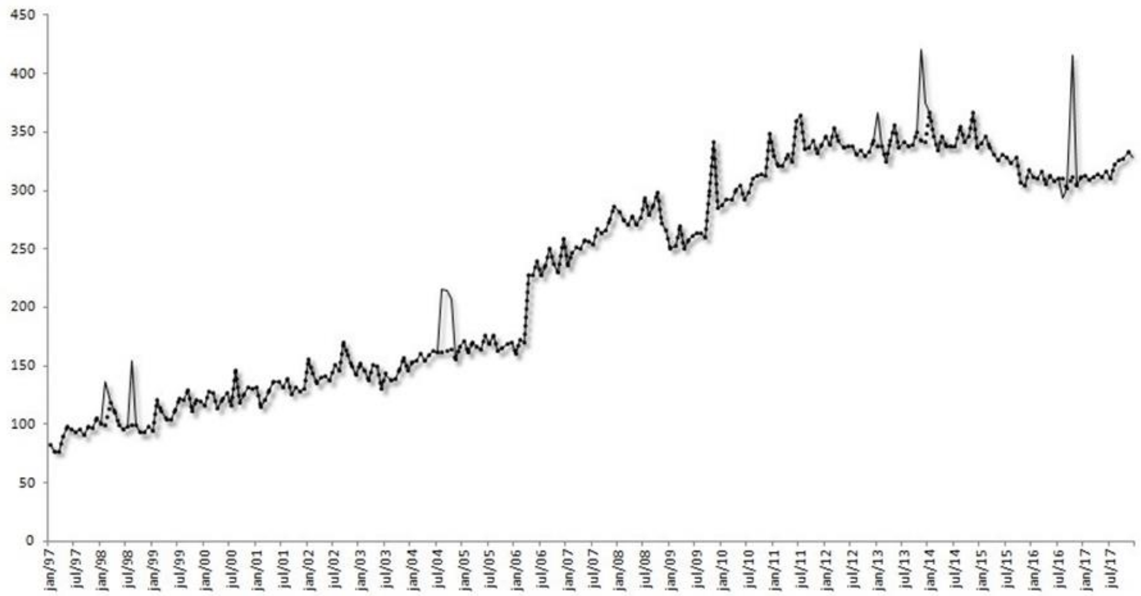
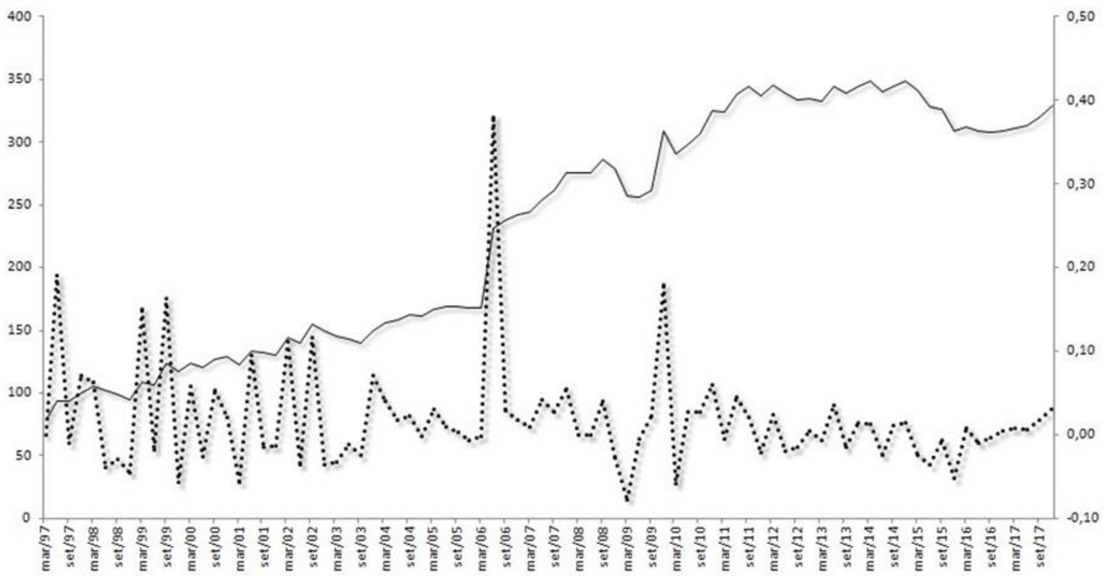


Table 4 – *Outliers* for the monthly Government Revenue series

Outlier identified
Feb/98
Aug/98
Aug/04
Sep/04
Oct/04
Jan/13
Nov/13
Dec/13
Aug/16
Oct/16

With the correction of the extreme values, the quarterly expenditure will be:

Figure 7 – Brazilian Gross Revenue quarterly



4.4 Fiscal Trend Models

In this section we present the quadratic trend models for the purpose of analyzing the tax-level growth rates that are the subject of our study.

4.4.1 *Fiscal Expenditure Trend Models*

In Table 6 the results show that government spending grew at increasing rates, with statistically significant t and t^2 parameters.

Table 5 – Tendência Quadrática - Despesas Governamentais Brasileiras

Estatística de regressão	
R-Multiplo	0,983
R-Quadrado	0,966
R-Quadrado Ajustado	0,965
Erro-Padrão	4579,11
Observações	84

ANOVA

	gl	*SQ*	*MQ*	*F*	*P-valor*
Regressão	2	48911618768,67	24455809384,48	1166,32	0,00
Resíduo	81	1698429157,72	20968261,20		
Total	84	50610047926,40			

	Coeficientes	*Erro padrão*	*t Stat*	*P-Valor*	*Inferior 95%*	*Superior 95%*	*Inferior 95%*	*Superior 95%*
Intercepto	34.022,544	1.535,275	22,161	0,000	30.967,828	37.077,260	30.967,828	37.077,260
t	731,652	83,365	8,777	0,000	565,783	897,522	565,783	897,522
t^2	3,074	0,950	3,235	0,002	1,183	4,965	1,183	4,965

4.4.2 *Tax Collection Trend Models*

The quadratic trend model for revenue shows a growth with decreasing rates over the period of analysis. The statistically significant parameters *t* and t^2 show this evidence.

Table 6 – Quadratic Tendency - Brazilian Government Gross Revenue

Estatística de regressão	
R-Multiplo	0,956
R-Quadrado	0,914
R-Quadrado Ajustado	0,912
Erro-Padrão	27,48
Observações	84

ANOVA

	gl	*SQ*	*MQ*	*F*	*P-valor*
Regressão	2	652178,79	326089,39	431,51	0,00
Resíduo	81	61210,98	755,69		
Total	84	713389,78			

	Coeficientes	*Erro padrão*	*t Stat*	*P-Valor*	*Inferior 95%*	*Superior 95%*	*Inferior 95%*	*Superior 95%*
Intercepto	48,254	9,217	5,235	0,000	29,915	66,592	29,915	66,592
t	5,584	0,500	11,157	0,000	4,588	6,579	4,588	6,579
t^2	-0,023	0,006	-4,093	0,000	-0,035	-0,012	-0,035	-0,012

The differences in trends between expenses and tax collection play an important role in diagnosing the current fiscal situation, but in terms of medium and long term behavior it is necessary to estimate the cyclical behavior of these variables so that we can implement a projection analysis, that is, combining linear forecasts (of trends, ARIMA, SARIMA, ARIMAx models, etc.) with non-linear forecasts (probability of recession, probability of turning points, etc.).

4.5 Univariate Dating Models

We present in this section the results of the univariate models[5] for dating the fiscal cycles. In this section, we present the results of two different dating procedures:

1. The modified BB algorithm proposed by Harding and Pagan [2002] that shares the same features as the original BB algorithm, but adapted to quarterly frequency, was applied to the level of the fiscal variables. We used the recession dates obtained to generate dummies that were used for a probit model with the log level of the fiscal variables as a variable response.

2. We estimate a univariate Markov-Switching (MS) model with growth rates of the fiscal variables, obtaining the probabilities of the recession state.

With these probabilities obtained (Pr_i), i = 1,2, we define a recession quarter:

$$D_{\substack{quarter \\ recession}} = \begin{cases} 1, Pr_i > \overline{Pr_i} + 0.5\ x\ \sigma Pr_i \\ \quad 0, \qquad otherwise \end{cases}$$

Dating was defined as the occurrence of these dummies in both procedures.

4.5.1 *Dating the Fiscal Expenditure Cycle*

We can see in Figure 8 that in the two univariate dating methods there is a large convergence of recessionary quarters. The recession probability estimated in the parametric model (MS) has higher values than the probability in the non-parametric model (BB). The parametric model has better identified recession probabilities. In both models we would have identified 5 recessions of the spending cycle, however the duration of each would be shorter in the non-parametric model. The model that showed the best fit had intercept and variance with regime shift, indicating that the uncertainty of government spending also has expansion and contraction regimes.

[5] Details of the estimated Univariate models are presented in the Appendix in Tables 16 and 17.

Figura 8 – Probability of Recession of Brazilian Government Expenditures

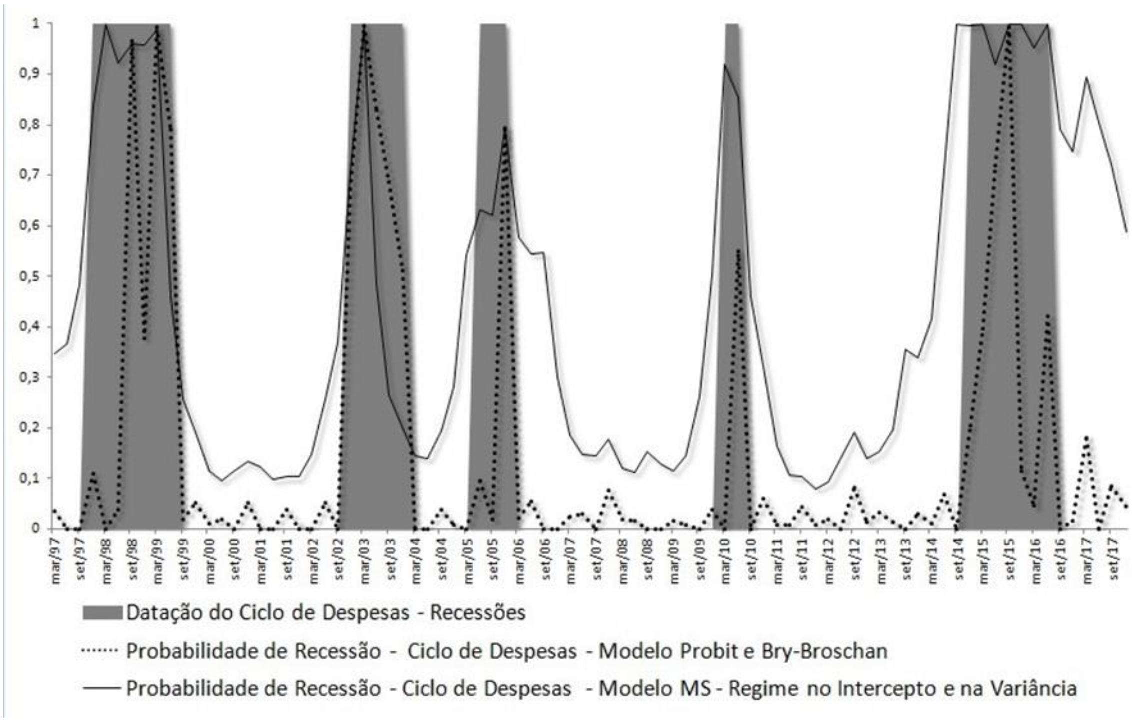
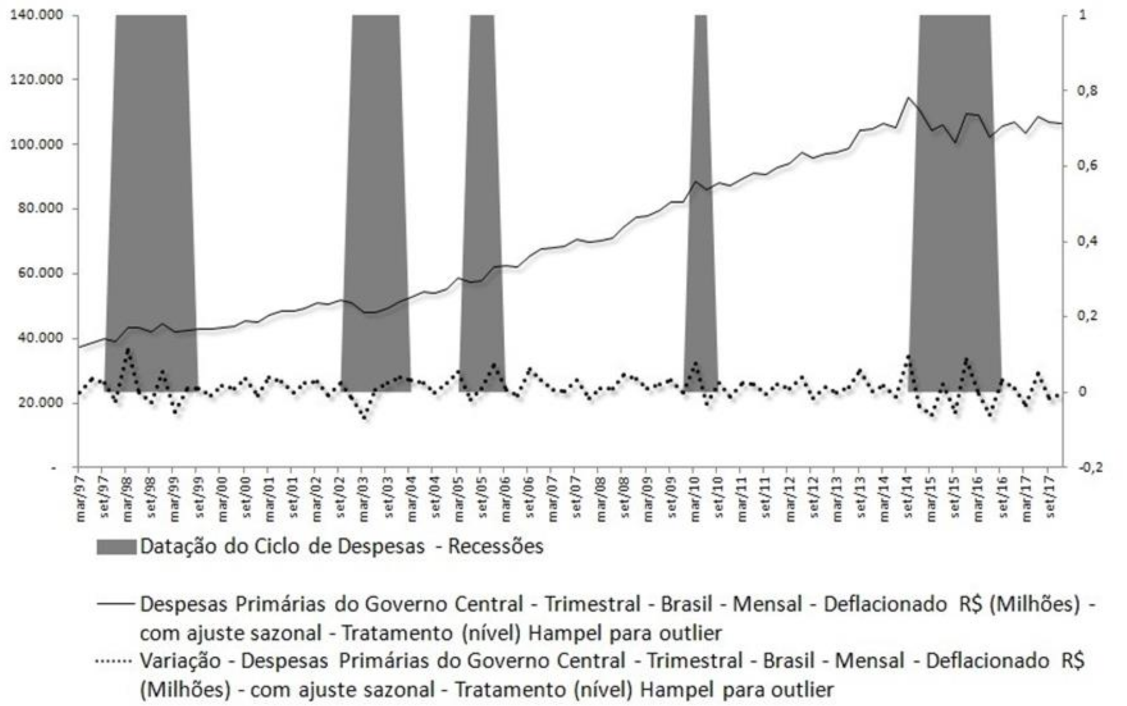


Figura 9 – Dating of Brazilian Government Expenditure



The dating of the spending cycle can be seen in Figure 9. The dated expansions always showed longer duration than recessions.

4.5.2 *Timeline of the Government Spending Cycle*

Table 7 – Quarterly Chronology of the Brazilian Government Expenditure Cycle - Duration and Amplitude

Univariate Model

Período	**Recessões**			**Expansões**			**Pontos de Inflexão**	
	Duração Trimestral	Variação Acumulada na Fase	Variação média na Fase	Duração Trimestral	Variação Acumulada na Fase	Variação Média na Fase	Variação no Ponto de Inflexão no Vale	Variação no Ponto de Inflexão no Pico
1997T1 - 1997T3	-	-	-	3	0,06	0,02	-	0,03
1997T4 - 1999T2	7	0,08	0,01	-	-	-	-0,05	-
1999T3 - 2002T3	-	-	-	13	0,2	0,02	-	0,04
2002T4 - 2003T4	5	-0,01	-0,001	-	-	-	-0,06	-
2004T1 - 2005T1	-	-	-	5	0,13	0,03	-	0,06
2005T2 - 2005T4	3	0,06	0,02	-	-	-	-0,02	-
2006T1 - 2009T4	-	-	-	16	0,29	0,02	-	0,06
2010T1 - 2010T2	2	0,05	0,02	-	-	-	-0,03	-
2010T3 - 2014T3	-	-	-	17	0,3	0,02	-	0,09
2014T4 - 2016T2	7	-0,11	-0,02	-	-	-	-0,06	-
2016T3 - 2017T4	-	-	-	6	0,04	0,01	-	0,05

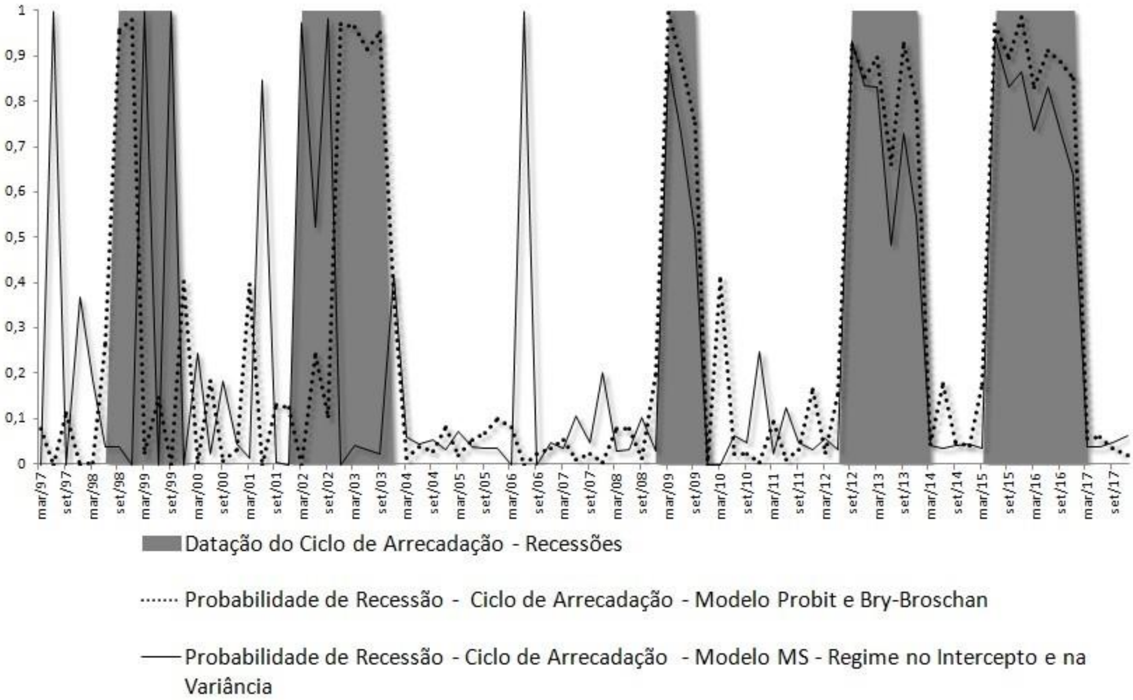
*All changes were calculated using the seasonally adjusted quarterly series.
*Univariate Model: Markov Switching in the intercept and variance.
*The criterion adopted for determining the inflection point considers the smoothed probability of state 1 (recession) being equal to or greater than the unconditional mean plus 0.5 of the unconditional standard deviation. The highest or lowest values in the range of this condition, was considered the inflection point.

The construction of the chronology is the most fundamental part of the process of dating a cycle. We present in Table 7 the dating of the government spending cycle considering the univariate models. The recessions of the spending cycle have an average duration of 5 quarters, with expenditures growing by an average of 1% in these recessions. The most severe recession dated in this model lasted 7 quarters between 2010Q3 - 2014Q3, with a cumulative decline of 11% and with an inflection point with a 6% decline in a single quarter. The most prolonged spending expansion lasted 17 quarters between 2010Q3 - 2014Q3, with a cumulative growth of 30%, and an inflection point with 9% growth in a single quarter.

4.6 Dating the Tax Revenue Cycle

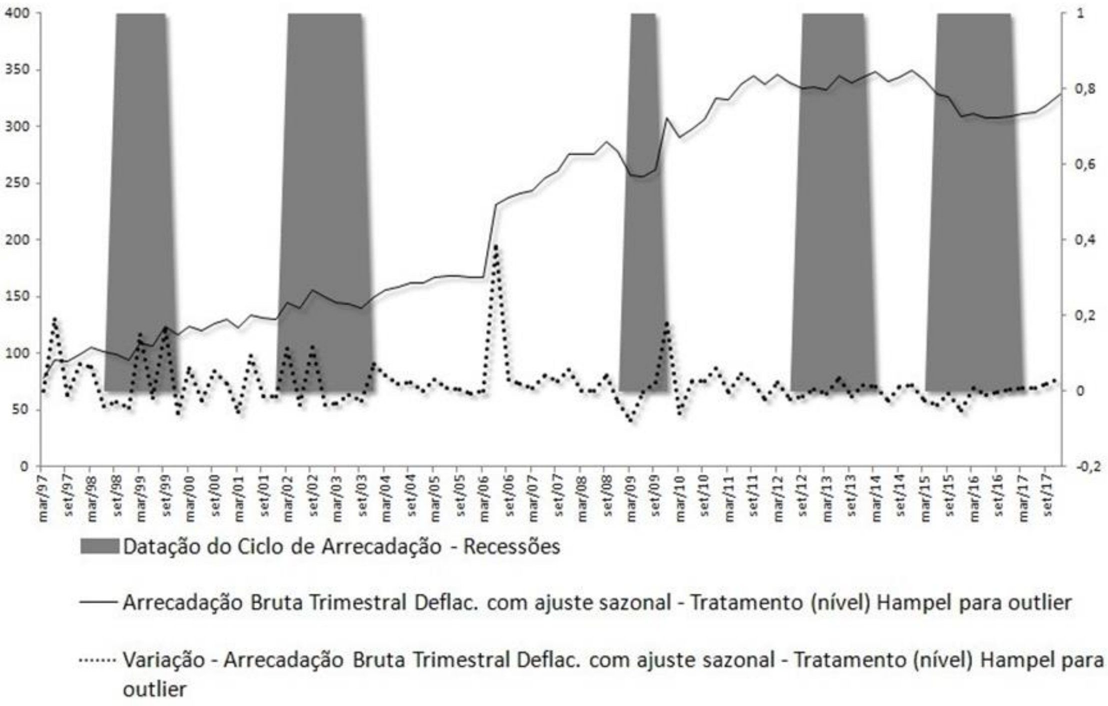
Figure 10 presents the dating results for the collection cycle.

Figure 10 – Probability of Recession of the Brazilian Gross Revenue



Although there is convergence in the dating of recessive quarters, some quarters were identified as recessive only by the parametric model, and according to the dating criterion adopted, only the quarters dated in both models were identified as recessive. Again, the parametric model with the best fit showed a regime shift in both the intercept and the variance, indicating that, as with expenditures, the uncertainties in collection also have moments of expansions and contractions. We identified 5 recessions in the revenue cycle of approximately the same duration in the parametric and non-parametric models. The dating of the revenue cycle can be seen in Figure 11. The duration and amplitude of the phases of this cycle are more periodic than that of the expenditure cycle.

Figure 11 – Dated Brazilian Gross Revenue



4.7 Timeline of the Gross Revenue Cycle

Table 8 – Quarterly Chronology of the Brazilian Government Revenue Cycle - Duration and Amplitude

Univariate Model

Period	Recessões			Expansões			Pontos de Inflexão	
	Duração Trimestral	Variação Acumulada na Fase	Variação média na Fase	Duração Trimestral	Variação Acumulada na Fase	Variação Média na Fase	Variação no Ponto de Inflexão no Vale	Variação no Ponto de Inflexão no Pico
1997T1 - 1998T2	-	-	-	6	0,27	0,05	-	0,19
1998T3 -1999T3	5	0,22	0,04	-	-	-	-0,04	-
1999T4 - 2001T4	-	-	-	9	0,06	0,01	-	0,09
2002T1 - 2003T3	7	0,09	0,01	-	-	-	-0,04	-
2003T4 - 2008T4	-	-	-	21	0,76	0,04	-	0,38
2009T1 - 2009T3	3	-0,06	-0,02	-	-	-	-0,08	-
2009T4 - 2012T2	-	-	-	11	0,28	0,03	-	0,18
2012T3 - 2013T4	6	0,02	0	-	-	-	-0,02	-
2014T1 - 2015T1	-	-	-	5	-0,01	0	-	0,01
2015T2 - 2016T4	7	-0,1	-0,01	-	-	-	-0,05	-
2017T1 - 2017T4	-	-	-	4	0,07	0,02	-	0,03

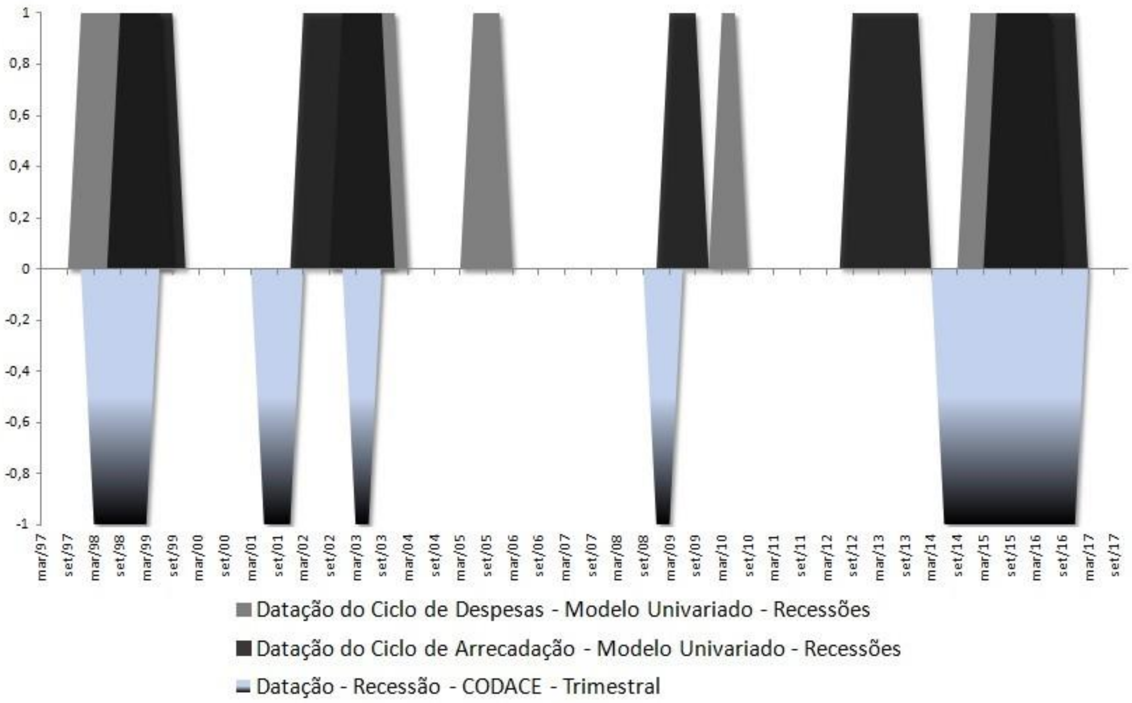
*All changes were calculated using the seasonally adjusted quarterly series.
*Univariate Model: Markov Switching in the intercept and variance.
*The criterion adopted for determining the inflection point considers the smoothed probability of state 1 (recession) being equal to or greater than the unconditional mean plus 0.5 of the unconditional standard deviation. The highest or lowest values in the range of this condition, was considered the inflection point.

The chronology of the revenue cycle is presented in Table 8. We can see that the average duration of recessions is 5.25 quarters, with an average growth in revenue of 0.5%, i.e. revenue recessions are larger than expenditure recessions, where the average growth in recession was 1%. The longest recession lasted 7 quarters from 2015Q2 - 2016Q4, with a cumulative drop of 10% and a turning point with a drop of 1% in a single quarter. The longest-lasting expansion had 21 quarters between 2003Q4 - 2008Q4, with cumulative growth of 76%, an inflection point with growth of 38% in a single quarter.

4.8 Fiscal Dates and the Brazilian Economic Cycle

We present in Figure 12 the relationship between the dates of the fiscal cycles and the CODACE business cycle. We can observe that even though the recessions of the fiscal cycles occur together with the economic recessions, some fiscal recessions are not related to the economic cycle. By the dating a close relationship between duration and amplitude of the phases of the fiscal and economic cycles, however, a more detailed analysis of the synchronization of these movements is necessary.

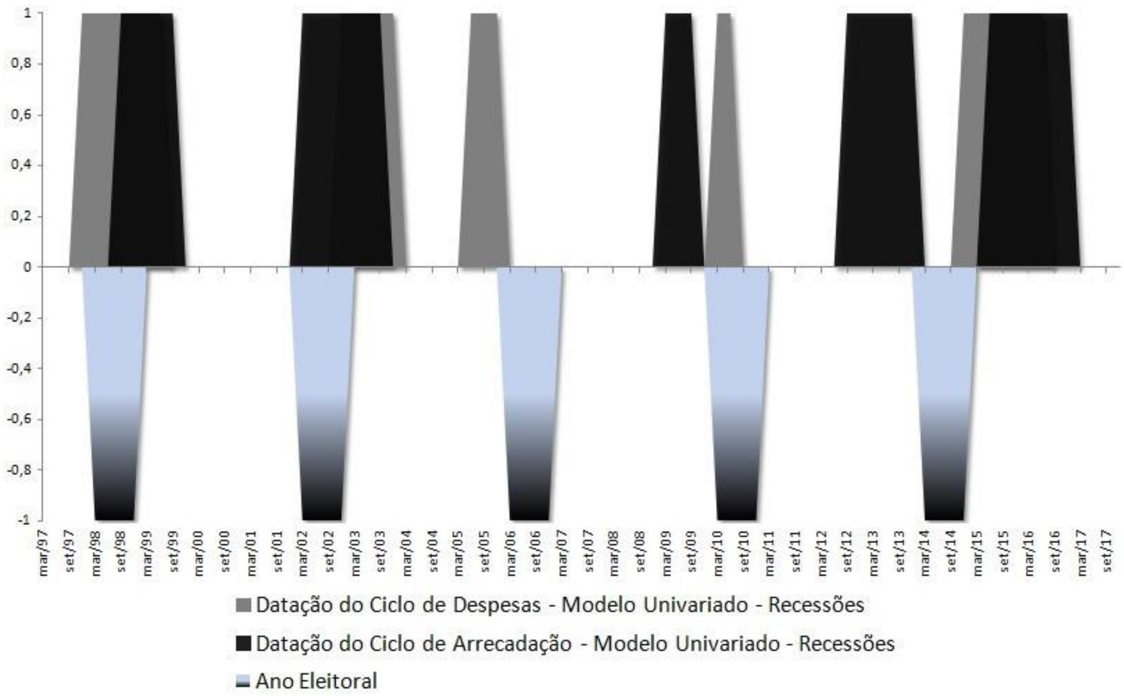
Figure 12 – Economic Cycle and Fiscal Cycles



4.9 Fiscal Dates and the Brazilian Political Cycle

We present in Figure 13 the relationship between the dating of the fiscal cycles and the electoral cycle. We can observe an expansion of expenses, in general, occurring before election years, without there being a defined pattern in relation to the behavior of the revenue cycle with election years.

Figure 13 – Political Cycle and Fiscal Cycles



4.10 Synchronization Analysis - Univariate Model

To make evident the dynamics of the fiscal, economic and electoral cycles, we have implemented a synchronization analysis of the dating of these cycles that is presented in Table 4.8. Considering the Harding-Pagan Concordance Index, there is a greater procyclical synchronism between the expenditure cycle and the economic and electoral cycles than with the revenue cycle. The revenue cycle shows procyclical synchronism with the economic cycle, but low synchronism with respect to the electoral cycle. Inferentially, the greatest procyclical synchronization is related to the expense cycles and the electoral cycles. The business cycle also presents high synchronism with the expenditure cycle. The revenue cycle presents higher synchronism with the economic cycle, no statistically significant synchronism with the electoral cycle and a low synchronism with the expenditure cycle.

Table 9 – Synchronization Dynamics between the Fiscal, Economic and Political Cycles

Dinâmica	CI	Sincronização Estimada	P-valor
Despesas - Arrecadação	0,31	0,270312	0,0511
Despesas - CODACE	0,55	0,648711	<.0001
Despesas - Eleições	0,56	0,721356	<.0001
Arrecadação - CODACE	0,42	0,393561	0,0199
Arrecadação - Eleições	0,17	0,152231	0,1239
CODACE - Eleições	0,38	0,309524	0,0037

* Note: Results used Newey-West standard error

4.11 Multivariate Dating Models

Univariate dating models can be useful in certain situations, such as non-linear prediction of the study variable. Dating considering only the study series brings relevant information of the cycle, however multivariate models bring a different characterization and can bring relevant information. In this sense we implement in this section a multivariate model for dating fiscal cycles considering a high-dimensional database (2571 series). In this context two methods are predominantly used when dealing with high-dimensional datasets:

1. Variable selection involves using techniques to select the best series that increase the predictive power of the model, using regularization measures as described by Tibshirani [1996], Zou [2006], Chernozhukov et al. [2015], Medeiros and Mendes [2017] among others;

2. Variable reduction involves generating new sets of latent variables that are derived from the original set of variables, such as Dynamic Factorial models, as for example used in the studies: **?**, Chauvet and Su [2014], Chauvet et al. [2016] among others.

Regularization measures use convergence properties that ensure the selection of highly informative variables for linear dynamics, however in dating and cycle studies, there is a need to treat the nonlinear dynamics of the series so dating and cycle studies focus on dimension reduction models, such as factorial dynamic models. High-dimensionality is also a recent topic when it comes to dating and cycle studies the variable selection or dimension reduction methods must consider the nonlinearity. In this study we deal with variable selection with nonlinear characteristics with methods suitable for this purpose.

4.12 *Cluster* of Probabilities of Fiscal Recessions and Expansions

To deal simultaneously with high dimensionality of the data, non-linearity and multivariate dating models we initially estimated using MS models[6] the probabilities of recession and expansion of the 2571 series. To deal with non-linear selection we constructed clusters[7] of probabilities (of recession or expansion), the variables that formed the clusters containing the probabilities of the cycles under study are thus used in the Dynamic Factorial Models with regime change. The clusters analysis is suitable in this case because it approximates probabilities by means of a distance measure, a procedure that can be used even for non-linearity conditions.

The results of the clusters containing government expenditures and revenues are presented in Table 10. For the analysis of expenditures, we used the probability of expansion for all series, since the univariate dating indicated that this is the longest-lasting phase, and we used the probability of recession in the case of tax collection for the same reason. The results show that the series with probabilities of expansion closest to the probability of expansion of government expenditures are the series related to economic activity, which indicates that the increase in industrial activity (PIM METALURGIA, CNI RENDIMENTO IND TRANSFORMA- CAO), trade and services activity (PMC and PMS), energy consumption (EPEs) and investments (FBCF) are associated with increases in the probability of expansion of expenditures. Unemployment, in turn, has an expansion probability associated with the expansion of government expenditures. The transmission channel of these probabilities alone would be a research of its own.

Table 10 – Probability Clusters for Tax Cycles

Cluster de Probabilidade de Recessão - Arrecadação Governamental	Cluster de Probabilidade de Expansão - Despesas Governamentais
PIM	PIM METALURGIA
PIM TRANSF	CNI RENDIMENTO IND TRANSFORMACAO
PIM MINERAIS NMET	PMC VAREJO AMPLIADO
PIM MAQ EQUIP	PMS
FIESP HTP	DESEMPREGO IBGE
CNI HS TRAB PROD BORR PLAST	EPE NORTE OUTROS
CNI NUCI PROD BORR PLAST	EPE SUDESTRE TOTAL
CNI EMPREGO BORR PLAST	EPE CO TOTAL
CNI MASSA SAL METALURGIA	EPE CO COMERCIAL
BACEN RESULT PRI ESTATAIS SCAM	EPE BRAIL TOTAL

[6] Considering the high-dimensional context it would not be possible to present the results of these estimations without breaking the allowed page limit, however they are available and can be requested from the author.

[7] The results of the estimated clusters are in the Appendix in Tables 20 and 21. We note that the total number of time series of estimated probabilities was less than the total number of series of the study (2571), this is due to the fact that some series presented degenerate probabilities and, therefore, were not considered ne step of cluster analysis.

SAL ADMISSAO CAGED CNAE ATIV IMOB	CONSUMO GAS NATURAL ANP
SAL DEMISSAO CAGED CNAE IND TRANSF	IBGE - FORMAÇÃO BRUTA DE CAPITAL FIXO
SAL DEMISSAO CAGED CNAE TRANSP CORREIO	
EPE NORTE RESIDENCIAL	
EPE SUL TOTAL	
EPE SUL INDUSTRIAL	
COMMODITIES SPOT GASOLINA	
CNC ICF RENDA ATUAL	
EUA EXP BENS	
CONSUMO DERIVADOS PETRO ANP	
IBGE - SERVIÇOS - Comércio	
IBGE - SERVIÇOS - Total	
IBGE - VA	
IBGE - PIB	
IBGE - FORMAÇÃO BRUTA DE CAPITAL FIXO	

* The clusters were obtained using the K-means algorithm, establishing: a minimum number of 5 series per cluster, a maximum number of clusters of 100, a convergence criterion of 0.05. In all cases there was convergence.

The results for tax collection show that the probabilities of recession are also strongly associated with economic activity, the set of series is wider with more series of industrial activity (PIM, CNI) some salary variables (admission of real estate activities, and dismissal of transformation industry activities), energy consumption (EPEs), activity and investment variables (The GDP itself, service GDP and GFCF). The only variable not directly linked to economic activity is the primary result of state-owned enterprises without exchange rate adjustment calculated by the Central Bank (BACEN RESULT PRI ESTATEIS SCAM). From these variables we implement the Dynamic Factorial models with regime change.

4.13 Identifying the Lag Order of the Series in the MS-DFM Model

The next step after selecting the variables to be used in the MS-DFM model is to determine the lag order that will be used to estimate the common factor dynamics in the model. For this purpose, we use cross-correlation analysis and we are in two different ways its validity: we test individually each cross-correlation for each lag considered; we estimate and test the statistical significance of the DCCA, which is a measure of cross-correlation that identifies the degree of long-range cross-correlation in time series.

The results presented in Table 11 for the MS-DFM model of expenditures indicate that we should consider the first lag, as individually we have statistically significant cross-correlation with government expenditures, in addition to high DCCA for most of the selected series, particularly for the trade (PMC), service (PMS), and Unemployment series.

Table 11 – Cross Correlation Analysis regarding Government Spending

Variáveis	1° defasagem	2° defasagem	3° defasagem	DCCA	P-valor DCCA
PIM METALURGIA	0,01	0,09	0,15	0,81	0,01
CNI RENDIMENTO IND TRANSFORMACAO	0,02	0,08	0,16	0,61	0,01
PMC VAREJO AMPLIADO	0,01	0,09	0,17	0,89	0,00
PMS	0,01	0,09	0,17	0,87	0,01
DESEMPREGO IBGE	0,00	0,06	0,17	0,91	0,00
EPE NORTE OUTROS	0,05	0,10	0,19	0,49	0,01
EPE SUDESTE TOTAL	0,01	0,12	0,15	0,61	0,01
EPE CO TOTAL	0,06	0,13	0,19	0,31	0,01
EPE CO COMERCIAL	0,06	0,12	0,16	0,63	0,02
EPE BRASIL TOTAL	0,00	0,06	0,11	0,78	0,00
CONSUMO GAS NATURAL ANP	0,07	0,15	0,18	0,51	0,02
IBGE - FORMAÇÃO BRUTA DE CAPITAL FIXO	0,06	0,11	0,16	0,71	0,01

*DCCA: Detrended Cross Correlation Analysis and significance test for DCCA proposed by **?** P-values of Ljung-Box Test for lag in relation to Government Expenditure.

When we consider the selected series in the tax collection cluster presented in Table 12, we have a similar result, statistically significant cross-correlation for the first lag and statistically significant DCCA for all series and particularly high for the GDP, PIM, Hours worked in São Paulo industry (FIESP HTP) series.

Table 12 – Cross Correlation Analysis regarding Government Income

Variáveis	1° defasagem	2° defasagem	3° defasagem	DCCA	P-valor DCCA
PIM	0,00	0,06	0,11	0,91	0,00
PIM TRANSF	0,05	0,09	0,15	0,81	0,01
PIM MINERAIS NMET	0,01	0,12	0,16	0,61	0,01
PIM MAQ EQUIP	0,06	0,13	0,17	0,89	0,00
FIESP HTP	0,06	0,12	0,17	0,91	0,01
CNI HS TRAB PROD BORR PLAST	0,00	0,06	0,17	0,61	0,00
CNI NUCI PROD BORR PLAST	0,07	0,10	0,19	0,49	0,01
CNI EMPREGO BORR PLAST	0,06	0,12	0,15	0,61	0,01
CNI MASSA SAL METALURGIA	0,06	0,13	0,19	0,31	0,01
BACEN RESULT PRI ESTATAIS SCAM	0,06	0,12	0,16	0,55	0,02
SAL ADMISSAO CAGED CNAE ATIV IMOB	0,00	0,06	0,11	0,78	0,00
SAL DEMISSAO CAGED CNAE IND TRANSF	0,07	0,15	0,18	0,51	0,02
SAL DEMISSAO CAGED CNAE TRANSP CORREIO	0,06	0,06	0,16	0,71	0,01
EPE NORTE RESIDENCIAL	0,01	0,09	0,16	0,71	0,01
EPE SUL TOTAL	0,00	0,09	0,15	0,81	0,01
EPE SUL INDUSTRIAL	0,05	0,08	0,16	0,61	0,01
COMMODITIES SPOT GASOLINA	0,01	0,09	0,17	0,89	0,00
CNC ICF RENDA ATUAL	0,06	0,09	0,17	0,87	0,01
EUA EXP BENS	0,06	0,06	0,17	0,51	0,00
CONSUMO DERIVADOS PETRO ANP	0,00	0,09	0,19	0,49	0,01
IBGE - SERVIÇOS - Comércio	0,00	0,08	0,15	0,79	0,01
IBGE - SERVIÇOS - Total	0,06	0,09	0,19	0,77	0,01
IBGE - VA	0,06	0,09	0,16	0,63	0,02
IBGE - PIB	0,00	0,06	0,11	0,91	0,00
IBGE - FORMAÇÃO BRUTA DE CAPITAL FIXO	0,07	0,10	0,18	0,51	0,02

* DCCA: Detrended Cross Correlation Analysis and significance test for DCCA proposed by **?** P-values of Ljung-Box Test for lag in relation to Government Expenditure.

4.14 Dating and Chronology of the Brazilian Expenditure, Economic and Political Cycle in High-Dimension

With the series selected and lags defined, we estimate the MS-DFM model[8]. From the probabilities estimated by this model, we again define a recession quarter:

$$D_{recession}^{quarter} = \begin{cases} 1, Pr_i > \overline{Pr_i} + 0.5\ x\ \sigma Pr_i \\ 0, \quad otherwise \end{cases}$$

Figure 14 – Expense Cycle, Economic Cycle and Brazilian Political Cycle

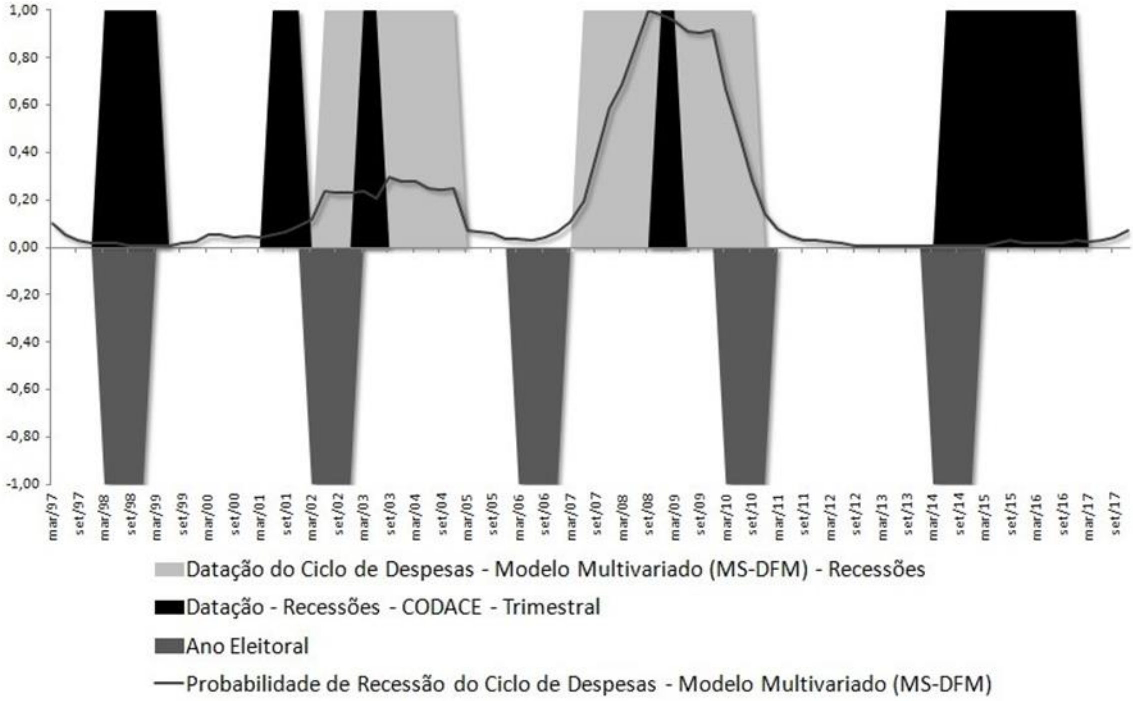


Figure 14 presents the dating results for dating the government spending cycle. In this model only two long recessions were dated (2002Q2 -2004Q4 and in 2007Q2 - 2010Q3), lasting respectively 11 and 14 quarters, with average growth of 1% and 2%, and turning points with drops of 6% 3% respectively for a single quarter as is presented in Table 13. The largest phase of expenditure expansion began in the last quarter of 2010 and lasted until the end of the time sample, lasting 29 quarters and with an accumulated increase of 21%, with a turning point of 9% growth in a single quarter. Even in the CODACE economic recession period, the probability of a government spending recession remained low. Government spending recessions always occur after CODACE economic

[8] The detailed results of the estimated models are presented in the Appendix in Tables 18 and 19.

recessions. With the exception of the second major spending recession, there is always an expansion in pre-election years.

Table 13 – Quarterly Chronology of the Brazilian Government Expenditure Cycle - Duration and Amplitude

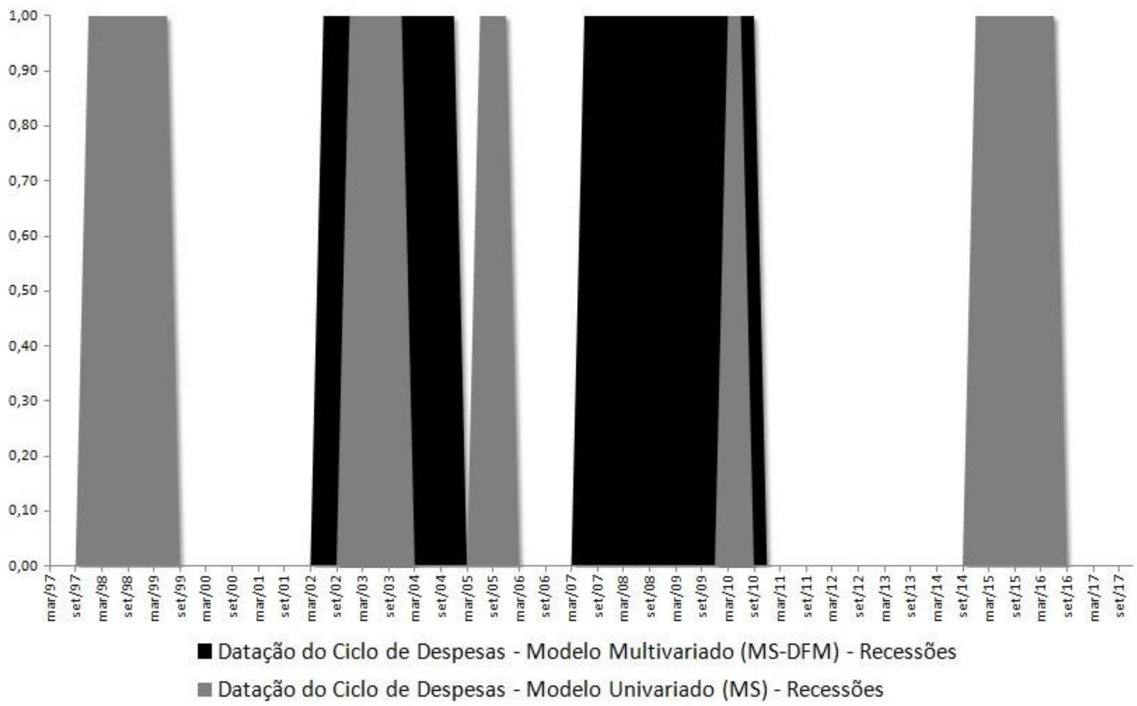
Multivariate Model

Período	**Recessões**			**Expansões**			**Pontos de Inflexão**	
	Duração Trimestral	Variação Acumulada na Fase	Variação média na Fase	Duração Trimestral	Variação Acumulada na Fase	Variação Média na Fase	Variação no Ponto de Inflexão no Vale	Variação no Ponto de Inflexão no Pico
1997T1 - 1998T2	-	-	-	21	0,32	0,02	-	0,11
2002T2 - 2004T4	11	0,09	0,01	-	-	-	-0,06	-
2005T1 - 2007T1	-	-	-	9	0,22	0,02	-	0,07
2007T2 - 2010T3	14	0,26	0,02	-	-	-	-0,03	-
2010T4 - 2017T4	-	-	-	29	0,21	0,01	-	0,09

*All changes were calculated using the seasonally adjusted quarterly series.
*Multivariate Model: Dynamic Factorial Markov Switching on the intercept and variance.
*The criterion adopted for determining the inflection point considers the smoothed probability of state 1 (recession) being equal to or greater than the unconditional mean plus 0.5 of the unconditional standard deviation. The highest or lowest values in the range of this condition, was considered the inflection point.

Comparatively we observe in Figure 15 the MS-DFM model captured the second and third recession of the Univariate model, and did not identify as contraction, considering the co-movement of the other series, the recession dated between 2014Q4/2016Q2 by the Univariate model.

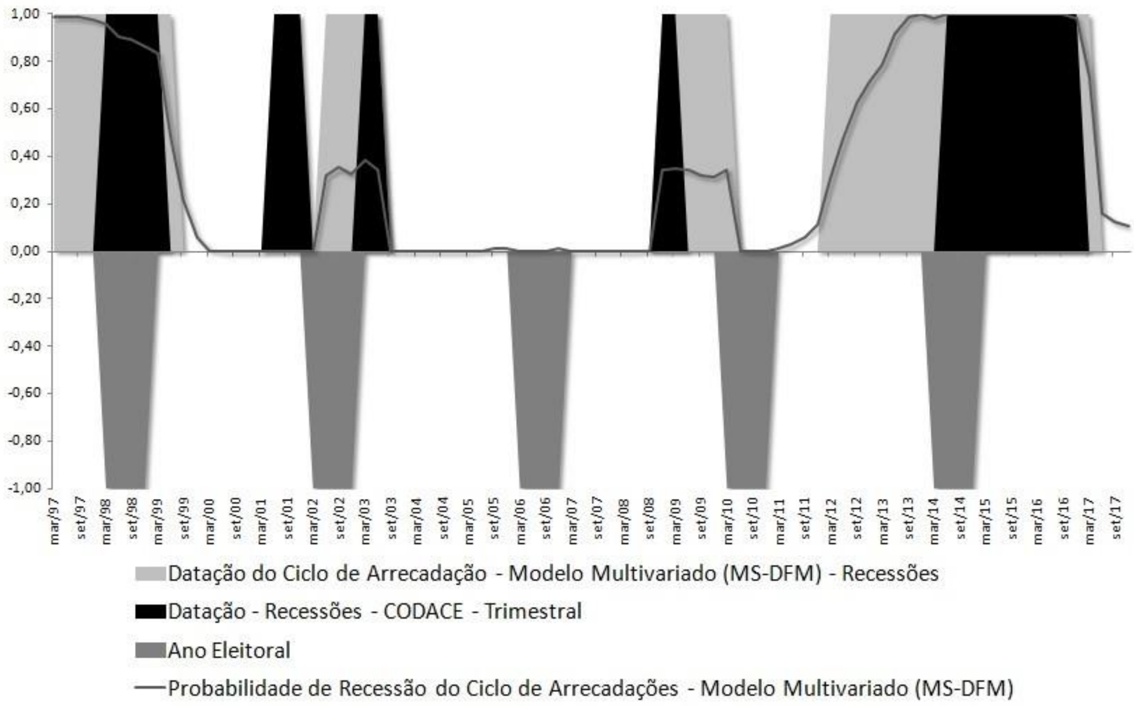
Figure 15 – Comparison of Dating for the Expenditure Cycle with Univariate and Multivariate Models



4.15 Dating and Chronology of the Brazilian High-Dimensional Economic and Political Cycle

The results of the MS-DFM model for dating the revenue cycle are graphically presented in Figure 16. We observe 4 recessions of the gross revenue cycle, with the last revenue recession crossing the entire CODACE-dated economic recession period. The recessions in the MS-DFM model, as in the univariate model, are closely related to the economic recessions, but they are not graphically related to the electoral cycles.

Figure 16 – Brazilian Revenue Cycle, Economic Cycle and Political Cycle



The longest recession dated by the MS-DFM model lasted 21 quarters (2012Q1-2017Q1) as we can see in Table 14, with a cumulative drop of 8% and an inflection point with a 5% drop. The longest expansion lasted 21 quarters (2003Q3- 2008Q3), with a cumulative expansion of 76% and inflection point with growth of 38% in a single quarter. Over time the duration of the expansion phases is decreasing, as are the variations in inflection points.

Table 14 – Quarterly Chronology of the Brazilian Government Revenue Cycle - Duration and Amplitude

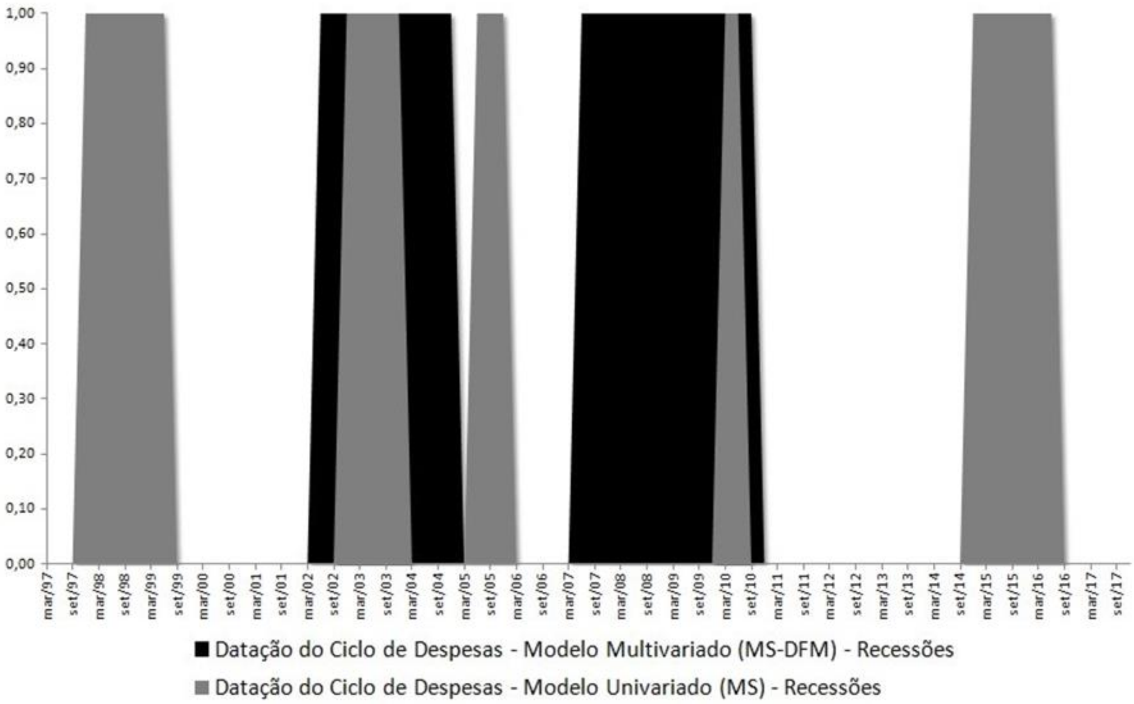
Multivariate Model

Período	**Recessões**			**Expansões**			**Pontos de Inflexão**	
	Duração Trimestral	Variação Acumulada na Fase	Variação média na Fase	Duração Trimestral	Variação Acumulada na Fase	Variação Média na Fase	Variação no Ponto de Inflexão no Vale	Variação no Ponto de Inflexão no Pico
1997T1 - 1999T2	10	0,33	0,03	-	-	-	-0,04	-
1999T3 - 2002T1	-	-	-	11	0,33	0,03	-	0,16
2002T2 - 2003T2	5	0,00	0,00	-	-	-	-0,04	-
2003T3 - 2008T3	-	-	-	21	0,76	0,04	-	0,38
2008T4 - 2010T1	6	0,03	0,01	-	-	-	-0,08	-
2010T2 - 2011T4	-	-	-	7	0,15	0,02	-	0,06
2012T1 - 2017T1	21	-0,08	0,00	-	-	-	-0,05	-
2017T2 - 2017T4	-	-	-	3	0,06	0,02	-	0,03

**All changes were calculated using the seasonally adjusted quarterly series.
*Multivariate Model: Dynamic Factorial Markov Switching on the intercept and variance.
*The criterion adopted for determining the inflection point considers the smoothed probability of state 1 (recession) being equal to or greater than the unconditional mean plus 0.5 of the unconditional standard deviation. The highest or lowest values in the range of this condition, was considered the inflection point.

When we compare the dating by the two different models (MS-DFM and univariate), we observe that for the case of the revenue cycle there is a convergence between the dated recessions. The MS-DFM model joins the last two dated recessions in the univariate model into just one long recession from 2012Q1 to 2017Q1. The remaining recessions are almost coincident which ensures convergence of the conclusions about the revenue cycle.

Figure 17 - Comparison of Dating for the Revenue Cycle with Univariate and Multivariate Models



4.16 Synchronization Analysis - Multivariate Model

To fully understand the dynamics between fiscal cycles and economic and electoral cycles, following the example of what was used in the univariate models, we implemented the synchronisation analysis considering the multivariate models. The results, presented in Table 15, only confirm, with less intensity of the CI indexes and estimated synchronization, the results of the univariate models. There is an intense synchronization between the expenditure cycle and the CODACE economic cycle, as well as the election cycle. In these cases, the synchronization showed values of the CI index and estimated synchronization greater than the synchronism between expenditures and collections. This result indicates that government expenditures are more intensely related to the level of economic activity and electoral plan than to tax collection.

Table 15 – Synchronization Dynamics between the Fiscal, Economic and Political Cycles

Dinâmica	CI	Sincronização Estimada	P-valor
Despesas - Arrecadação	0,35	0,380952	0,0003
Despesas - CODACE	0,61	0,547619	<.0001
Despesas - Eleições	0,51	0,333333	0,0017
Arrecadação - CODACE	0,55	0,404762	0,0001
Arrecadação - Eleições	0,11	0,142857	0,1895
CODACE - Eleições	0,38	0,309524	0,0037

*Note: Results used Newey-West standard error

5. FINAL CONSIDERATIONS

The results of this study become a reference for studies of Brazilian fiscal cycles. The seasonality studied shows the month of December as a critical month from the fiscal point of view, because there is an average 38% increase in expenditures while there is a 16% decrease in collections on average. The quadratic trend analysis showed that during the study period expenses grew at an increasing rate while collections grew at decreasing rates. The univariate dating pointed to 5 expenditure recessions with an average duration of 5 quarters, while we have an average of 10 quarters of expansion, however the duration of the expenditure expansions has increased over the years. The revenue cycle according to the univariate model, showed 5 recessions with an average duration of 6 quarters, and expansions with an average duration of 9 quarters, however this duration of the revenue cycle expansions is decreasing over time.

The multivariate model identified a smaller set of expenditure recessions, and long periods of expenditure expansion, with an average duration of 20 quarters, regardless of the business cycle period. The revenue cycle showed more balance between the duration of its phases, 11 quarters on average for the contraction phase and 10 for the contraction phase, but the duration of the expansion phase has been decreasing over time. The longest expansion phase in the univariate model was 17 quarters (2010Q3 - 2014Q3), but in the multivariate model it was 29 quarters (beginning in the last quarter of 2010). In the univariate model for the revenue cycle, the longest recession lasted 7 quarters, while in the multivariate model it lasted 21 quarters (2012Q1 - 2017Q1), spanning the entire period of the longest economic recession dated by CODACE.

The synchronization results indicate that expenditures have synchronized movements with both the economic cycle and the electoral cycle, while the collection cycles presented synchronization with the CODACE economic cycle, but with no evidence of a statistically significant synchronization with the electoral period. The synchronization of expenditures with the economic cycle and with the electoral cycle is greater than the synchronization with the revenue cycle. The results indicate that the revenue cycle has presented more severe contractions and lesser recoveries. This behavior, associated with the increase in the duration of expenditure expansions, can generate a fiscal commitment in the medium and long term in an even more critical manner.

Thus, the study concludes the relevant chronology and dating of the Brazilian fiscal cycles that can be used in different economic studies: projection of fiscal variables: linear and non-linear forecasts; estimation of fiscal cycles in the frequency domain; use of dating to construct counterfactual variables to capture fiscal cycle effects; study of antecedent, coincident, and lagged variables of the fiscal cycle, among many other possible applications of the results of these studies.

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APPENDIX

APPENDIX

Table 16 – Dating Methods using the Markov-Switching Model

Despesas Governamentais - Modelo Markov Switching - Intercepto e variância com mudança de regime

```
***** Numerical Optimization Converged *****

Final log Likelihood: -169.1954
Number of estimated parameters: 6
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters


Switching Parameters (Distribution Parameters)

State 1
Model's Variance: 0.000373
Std Error (p. value): 0.0001 (0.00)
State 2
Model's Variance: 0.001946
Std Error (p. value): 0.0006 (0.00)

Switching Parameters (Regressors)

Switching Parameters for Equation #1 - Indep column 1

State 1
Value: 0.0164
Std Error (p. value): 0.0033 (0.00)
State 2
Value: 0.0993
 Std Error (p. value): 0.0079 (0.01)

Transition Probabilities Matrix (p-value)

0.93 (0.00) 0.11 (0.02)
0.01 ( 0.00) 0.98 ( 0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 13.76 time periods
Expected duration of Regime #2: 45.13 time periods
```

Despesas Governamentais - Modelo Markov Switching - Intercepto com mudança de regime

```
***** Numerical Optimization Converged *****

Final log Likelihood: -189.6867
Number of estimated parameters: 5
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters

There was no Non Switching Parameters for Indep matrix of Equation #1. Skipping this result

Non Switching Variance of model
Value: 0.74522
Std Error (p. value): 0.0231 (0.00)

Switching Parameters (Regressors)

Switching Parameters for Equation #1 - Indep column 1

State 1
Value: 1.8194
Std Error (p. value): 0.0964 (0.00)
State 2
Value: 0.1564
Std Error (p. value): 0.0773 (0.00)

Transition Probabilities Matrix (p-value)

 0.98 (0.00) 0.07 (1.00)
0.01 ( 0.00) 0.99 ( 0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 13.99 time periods
Expected duration of Regime #2: 2951605012.01 time periods
```

Despesas Governamentais - Modelo Markov Switching - Variância com mudança de regime

```
***** Numerical Optimization Converged *****

Final log Likelihood: -195.9207
Number of estimated parameters: 5
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters

Non Switching Parameter for Equation #1, Indep column 1
Value: 0.1762
Std Error (p. value): 0.0681 (0.00)

Switching Parameters (Distribution Parameters)

State 1
Model's Variance: 0.594477
Std Error (p. value): 0.1472 (0.00)
State 2
Model's Variance: 2.135401
Std Error (p. value): 0.9542 (0.03)

Switching Parameters (Regressors)

There was no switching parameters for the regressors in Equation #1. Skipping this result

Transition Probabilities Matrix (p-value)

 0.02 (0.00) 0.93 (0.01)
0.01 ( 0.00) 0.92 ( 0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 22.21 time periods
Expected duration of Regime #2: 9.75 time periods
```

Note: Within-sample estimation (from 1997Q1 to 2017Q4) of various univariate Markov-Switching models.

Table 17 – Dating Method using the Markov-Switching Model

Arrecadação Governamental - Modelo Markov Switching - Intercepto e variância com mudança de regime

***** Numerical Optimization Converged *****

Final log Likelihood: -137.0408
Number of estimated parameters: 6
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters

Switching Parameters (Distribution Parameters)

State 1
Model's Variance: 0.000983
Std Error (p. value): 0.1061 (0.00)
State 2
Model's Variance: 0.010113
Std Error (p. value): 0.0008 (0.01)

Switching Parameters (Regressors)

Switching Parameters for Equation #1 - Indep column 1

State 1
Value: -0.000983
Std Error (p. value): 0.0033 (0.00)
State 2
Value: 0.010113
Std Error (p. value): 0.0179 (0.01)

Transition Probabilities Matrix (p-value)

0.96 (0.00) 0.13 (0.02)
0.01 (0.00) 0.826 (0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 5.78 time periods
Expected duration of Regime #2: 7.03 time periods

Arrecadação Governamental - Modelo Markov Switching - Intercepto com mudança de regime

***** Numerical Optimization Converged *****

Final log Likelihood: -191.6851
Number of estimated parameters: 5
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters

There was no Non Switching Parameters for Indep matrix of Equation #1. Skipping this result

Non Switching Variance of model
Value: 0.958611
Std Error (p. value): 0.0011 (0.00)

Switching Parameters (Regressors)

Switching Parameters for Equation #1 - Indep column 1

State 1
Value: 0.8591
Std Error (p. value): 0.0951 (0.00)
State 2
Value: 0.1461
Std Error (p. value): 0.0083 (0.00)

Transition Probabilities Matrix (p-value)

0.83 (0.00) 0.16 (1.00)
0.01 (0.00) 1.00 (0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 13.81 time periods
Expected duration of Regime #2: 19947863605755.21 time periods

Arrecadação Governamental - Modelo Markov Switching - Variância com mudança de regime

***** Numerical Optimization Converged *****

Final log Likelihood: -199.9103
Number of estimated parameters: 5
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters

Non Switching Parameter for Equation #1, Indep column 1
Value: 0.6168
Std Error (p. value): 0.0578 (0.00)

Switching Parameters (Distribution Parameters)

State 1
Model's Variance: 0.524378
Std Error (p. value): 0.1071 (0.00)
State 2
Model's Variance: 0.165402
Std Error (p. value): 0.9333 (0.03)

Switching Parameters (Regressors)

There was no switching parameters for the regressors in Equation #1. Skipping this result

Transition Probabilities Matrix (p-value)

0.17 (0.00) 0.82 (0.01)
0.06 (0.00) 0.82 (0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 11.21 time periods
Expected duration of Regime #2: 4.31 time periods

Note: Within-sample estimation (from 1997Q1 to 2017Q4) of various univariate Markov-Switching models.

Table 18 – Dating Method using the Markov-Switching Dynamic Factorial Model

Despesas Governamentais - Modelo Markov Switching Dinâmico Fatorial - Intercepto e variância com mudança de regime

```
***** Numerical Optimization Converged *****

Final log Likelihood: -29.4803
Number of estimated parameters: 6
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters


Switching Parameters (Distribution Parameters)

State 1
Model's Variance: 0.087235
Std Error (p. value): 0.1969 (0.00)
State 2
Model's Variance: 0.352555
Std Error (p. value): 0.1935 (0.02)

Switching Parameters (Regressors)

Switching Parameters for Equation #1 - Indep column 1

State 1
Value: -0.0231
Std Error (p. value): 0.0386 (0.00)
State 2
Value: 0.0382
 Std Error (p. value): 0.1978 (0.01)

Transition Probabilities Matrix (p-value)

0.91(0.00) 0.11 (0.02)
 0.01 ( 0.00) 0.96 ( 0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 7.07 time periods
Expected duration of Regime #2: 40.05 time periods
```

Despesas Governamentais - Modelo Markov Switching Dinâmico Fatorial - Intercepto com mudança de regime

```
***** Numerical Optimization Converged *****

Final log Likelihood: -389.0012
Number of estimated parameters: 5
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters

There was no Non Switching Parameters for Indep matrix of Equation #1. Skipping this result

Non Switching Variance of model
Value: 0.818621
Std Error (p. value): 0.0174 (0.00)

Switching Parameters (Regressors)

Switching Parameters for Equation #1 - Indep column 1

State 1
Value: 1.8194
Std Error (p. value): 0.0964 (0.00)
State 2
Value: 0.1564
Std Error (p. value): 0.0183 (0.00)

Transition Probabilities Matrix (p-value)

 0.98 (0.00) 0.14 (1.00)
0.01 ( 0.00) 1.00 ( 0.00)

Expected Duration of Regimes

 Expected duration of Regime #1: 93.91 time periods
 Expected duration of Regime #2: 15759.99 time periods
```

Despesas Governamentais - Modelo Markov Switching Dinâmico Fatorial - Variância com mudança de regime

```
***** Numerical Optimization Converged *****

Final log Likelihood: -45.3245
Number of estimated parameters: 5
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters

Non Switching Parameter for Equation #1, Indep column 1
Value: 0.7761
Std Error (p. value): 0.1988 (0.00)

Switching Parameters (Distribution Parameters)

State 1
Model's Variance: 0.694111
Std Error (p. value): 0.1071 (0.00)
State 2
Model's Variance: 2.135401
Std Error (p. value): 0.9542 (0.03)

Switching Parameters (Regressors)

There was no switching parameters for the regressors in Equation #1. Skipping this result

Transition Probabilities Matrix (p-value)

 0.02 (0.00) 0.86 (0.01)
0.16 ( 0.00) 0.72 ( 0.00)

Expected Duration of Regimes

 Expected duration of Regime #1: 10.98 time periods
 Expected duration of Regime #2: 4.68 time periods
```

Note: Within-sample estimation (from 1997Q1 to 2017Q4) of various Multivariate Markov-Switching Dynamic Factorial models.

Table 19 – Dating Method using the Markov-Switching Dynamic Factorial Model

Arrecadação Governamental - Modelo Markov Switching Dinâmico Fatorial - Intercepto e variância com mudança de regime

```
***** Numerical Optimization Converged *****

Final log Likelihood: -34.8332
Number of estimated parameters: 6
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters


Switching Parameters (Distribution Parameters)

State 1
Model's Variance: 0.104871
Std Error (p. value): 0.1268 (0.00)
State 2
Model's Variance: 0.001946
Std Error (p. value): 0.0006 (0.03)

Switching Parameters (Regressors)

Switching Parameters for Equation #1 - Indep column 1

State 1
Value: -0.4154
Std Error (p. value): 0.0515 (0.00)
State 2
Value: 0.1193
 Std Error (p. value): 0.0079 (0.01)

Transition Probabilities Matrix (p-value)

0.97 (0.00) 0.14 (0.02)
0.03 ( 0.00) 0.86 ( 0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 9.73 time periods
Expected duration of Regime #2: 6.83 time periods
```

Arrecadação Governamental - Modelo Markov Switching Dinâmico Fatorial - Intercepto com mudança de regime

```
***** Numerical Optimization Converged *****

Final log Likelihood: -59.6112
Number of estimated parameters: 5
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters

There was no Non Switching Parameters for Indep matrix of Equation #1. Skipping this result

Non Switching Variance of model
Value: 0.758622
Std Error (p. value): 0.0206 (0.00)

Switching Parameters (Regressors)

Switching Parameters for Equation #1 - Indep column 1

State 1
Value: 1.8194
Std Error (p. value): 0.0964 (0.00)
State 2
Value: 0.1564
Std Error (p. value): 0.0581 (0.00)

Transition Probabilities Matrix (p-value)

 0.85 (0.00) 0.17 (1.00)
0.01 ( 0.00) 1.00 ( 0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 83.81 time periods
Expected duration of Regime #2: 11945887605757.28 time periods
```

Arrecadação Governamental - Modelo Markov Switching Dinâmico Fatorial - Variância com mudança de regime

```
***** Numerical Optimization Converged *****

Final log Likelihood: -61.2356
Number of estimated parameters: 5
Number of Observations: 84
Number of Equations: 1
Optimizer: fminsearch
Type of Switching Model: Univariate
Distribution Assumption : Normal
Method SE calculation : 1

***** Final Parameters for Equation #1 *****

Non Switching Parameters

Non Switching Parameter for Equation #1, Indep column 1
Value: 1.6768
Std Error (p. value): 0.0988 (0.00)

Switching Parameters (Distribution Parameters)

State 1
Model's Variance: 0.594477
Std Error (p. value): 0.1472 (0.00)
State 2
Model's Variance: 2.135401
Std Error (p. value): 0.9542 (0.03)

Switching Parameters (Regressors)

There was no switching parameters for the regressors in Equation #1. Skipping this result

Transition Probabilities Matrix (p-value)

 0.15 (0.00) 0.83 (0.01)
0.06 ( 0.00) 0.82 ( 0.00)

Expected Duration of Regimes

Expected duration of Regime #1: 17.28 time periods
Expected duration of Regime #2: 5.61 time periods
```

Note: Within-sample estimation (from 1997Q1 to 2017Q4) of various Multivariate Markov-Switching Dynamic Factorial models.

Table 20 – Expansion Probability Clusters - Government Spending

Cluster	Frequency	RMS Std Deviation	"Maximum Distance from Seed to Observation"	Nearest Cluster	"Distance Between Cluster Centroids"
1	18	0,219	2,7522	24	2,2105
2	11	0,2945	3,1595	7	2,3551
3	80	0,0883	2,8508	29	1,9977
4	23	0,2059	2,8766	16	2,2185
5	13	0,2128	2,8498	38	2,8459
6	13	0,2565	2,9153	24	2,1551
7	18	0,2659	3,2201	2	2,3551
8	22	0,2872	3,518	26	3,0219
9	26	0,1197	3,1286	28	2,1167
10	20	0,2594	3,4748	21	1,8708
11	9	0,2611	3,1131	16	3,4716
12*	21	0,3161	3,815	43	2,9426
13	24	0,2439	3,3112	22	2,0456
14	13	0,2906	3,5801	44	2,3637
15	79	0,131	3,7956	47	1,706
16	36	0,1878	3,2172	23	2,1238
17	19	0,2928	3,9695	4	2,6926
18	11	0,2998	3,0262	37	2,3589
19	19	0,1545	2,857	27	1,7865
20	39	0,1118	2,5136	27	1,983
21	24	0,2224	3,3042	10	1,8708
22	28	0,1659	2,6656	13	2,0456
23	14	0,2129	2,5497	38	1,6775
24	30	0,1906	3,2493	6	2,1551
25	11	0,2604	3,279	28	2,5436
26	51	0,2044	2,9313	43	1,8983
27	39	0,1539	2,9448	19	1,7865
28	19	0,1982	2,9311	9	2,1167
29	42	0,1064	3,5163	3	1,9977
30	20	0,1761	2,204	22	2,2838
31	15	0,2595	2,911	32	2,2303
32	160	0,1411	2,9274	31	2,2303
33	16	0,2652	3,4815	38	2,0826
34	11	0,2674	3,0129	2	2,4734
35	7	0,2455	2,6614	15	2,917
36	14	0,3157	3,5223	26	2,7433
37	21	0,2098	3,1833	18	2,3589
38	79	0,1789	2,7112	23	1,6775
39	47	0,2032	3,5627	13	2,5324
40	20	0,28	3,7465	49	2,7269
41	64	0,1488	3,9146	50	1,5635
42	33	0,2356	3,4021	6	2,6384
43	14	0,2133	2,7052	26	1,8983
44	7	0,2575	2,8336	14	2,3637
45	18	0,2727	3,8575	46	2,6055
46	13	0,3023	3,4849	45	2,6055
47	155	0,0913	3,3249	50	1,5441
48	115	0,1101	2,398	20	2,0847
49	6	0,2071	2,026	40	2,7269
50	153	0,0899 2,939	47	1,5441	

Pseudo F Statistic = 199.6

Observed Over-All R-Squared = 0.85118

Approximate Expected Over-All R-Squared = 0.13972

Cubic Clustering Criterion = 3832.34

**cluster* containing Government Expenditures.

Table 21 – Probability Clusters Recession - Government Revenues

Cluster	Frequency	RMS Std Deviation	"Maximum Distance from Seed to Observation"	Nearest Cluster	"Distance Between Cluster Centroids"
1	21	0,2085	3,0495	23	2,4184
2*	49	0,1803	2,8378	33	2,29
3	20	0,2187	2,7395	41	2,2218
4	22	0,1988	2,7577	17	2,2785
5	42	0,1064	3,5163	34	2,0101
6	10	0,247	2,9478	47	2,9885
7	54	0,1647	2,4824	22	1,7735
8	12	0,211	2,887	7	2,8351
9	11	0,2435	2,6778	7	2,214
10	15	0,2702	3,0678	49	2,2706
11	12	0,3001	3,1966	15	2,6902
12	26	0,1197	3,1286	27	2,1077
13	34	0,2422	3,4134	28	2,2637
14	20	0,3128	3,7642	39	2,9744
15	12	0,2966	3,8541	46	2,3566
16	78	0,1283	3,7955	50	1,7077
17	36	0,1817	3,2372	4	2,2785
18	16	0,2618	2,9058	4	2,7343
19	11	0,2962	3,4845	48	2,7216
20	44	0,1413	2,8936	26	1,9812
21	28	0,1917	2,674	43	2,3258
22	11	0,2318	3,2276	7	1,7735
23	11	0,304	3,1298	1	2,4184
24	11	0,2604	3,279	27	2,541
25	25	0,2061	3,3368	2	2,503
26	36	0,1444	2,7866	33	1,6926
27	18	0,1895	2,9085	12	2,1077
28	9	0,2927	3,3497	13	2,2637
29	25	0,2173	3,5306	43	2,3774
30	11	0,23	2,7585	31	2,4412
31	165	0,149	3,0367	16	2,401
32	9	0,2699	3,1964	41	2,5579
33	16	0,1464	2,6359	26	1,6926
34	83	0,1054	3,3579	5	2,0101
35	12	0,2698	3,0914	45	2,427
36	9	0,2587	3,3831	40	2,4382
37	7	0,264	2,912	20	3,1096
38	9	0,2611	3,1131	32	3,4773
39	11	0,3185	3,3569	14	2,9744
40	23	0,1541	2,7341	41	2,0315
41	48	0,1729	2,773	7	1,9547
42	49	0,2033	3,0037	53	2,6499
43	61	0,1542	3,2007	53	1,8214
44	26	0,2471	3,2469	41	2,8038
45	12	0,2923	3,1459	10	2,3044
46	7	0,2575	2,8336	15	2,3566
47	11	0,2777	3,9294	52	2,7067
48	17	0,256	2,9895	19	2,7216
49	7	0,2904	2,9074	10	2,2706
50	155	0,0896	3,3244	53	1,5871
51	116	0,1121	2,9641	20	2,1555
52	9	0,2288	2,824	47	2,7067
53	168	0,1013	4,0018	50	1,5871

Pseudo F Statistic = 195,79

Observed Over-All R-Squared = 0,85641

Approximate Expected Over-All R-Squared = 0,14411

Cubic Clustering Criterion = 3871,587

**cluster* containing Government Revenue.