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Impacts of Central Bank Credibility on Public Debt Management

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Abstract

This paper seeks to analyze, through a partial solution of an analytical model, how the credibility of the Central Bank - that is, whether agents effectively believe that ex-post inflation will be equal to ex--ante inflation - impacts the decision of private agents to formulate a portfolio of government bonds. The historical and empirical analysis of the Brazilian case seeks to validate the model's results.

Keywords: Sovereign Debt, Debt Management, Government Bonds, Expectations, Macroeconomics

JEL Classification: B22, E44, HG3

SUMÁRIO

Introduction

Governments constantly face a range of social and political demands, which they seek to meet in a feasible way.[1] These demands are sometimes tied to the government's unique authority in areas where the private sector has neither the authority nor the legitimacy to act, for example in intergenerational transfers, such as social security systems and taxes. To do this, however, the government must first absorb some of the resources from the economy. There are three basic ways to do this: taxing, issuing debt, and creating money.

As is well known, in case it decides to go into debt, the government must go to the market to seek resources to finance its expenses. Thus, it is natural to assume that the public debt manager acts as a rational agent seeking to minimize the cost of financing the government. Since taxes are distortive, the agent tends to borrow in order to soften the possible impacts of an increase in the tax burden, which may become necessary if excessive indebtedness occurs over time.

The need for government financing inspires questions about the possibilities of executing it. According to some simplifying assumptions, the type of security under which the debt is issued makes no difference. However, a basic analysis of the composition of sovereign debts around the world goes in the opposite direction. As Fischer (1982) aptly summarizes: "Governments in inflationary difficulties issue indexed bonds and those that can avoid it, do not."

The apparent paradox goes back to the theory of bond pricing. As inflation reduces the real payoff of bonds, agents will charge more to hold them in their portfolios, thereby increasing their cost of issuance to the government. However, how much more agents will be charged considers how much of the nominal yield on bonds will be eroded by inflation.

Thus, inflation-indexed bonds seem to be a feasible and cheap alternative for governments that are lax with their monetary policy. Note that estimates of real bond yields first require agents' formulation of inflation expectations. Among the various factors to be considered in the construction of expectations, the main one is the credibility of the Central Bank.

Confidence is one of the most important tools for monetary policy managers. As presented by Lucas and Sargent, a credible disinflation promise lowers the cost of disinflation. At the same time, so-called open mouth operations can smooth moves by directing agents' expectations[2]. There are several ways to define central bank credibility, but one can summarize that a credible central bank is one that convinces other agents that it will deliver a given level of inflation.[3]

It is worth noting that this process does not occur in a vacuum. Central bank and government coexist, each trying to advance its objectives. As Acemoglu et al. (2008) argue:

1 Fischer (1982)

2 See Taylor (2001)

3 See Blinder (2000)

"Most instances of high inflation are directly related to the inability of governments to fund their (often politically-motivated) expenditures through taxation and borrowing."

They argue that the effectiveness of institutional reforms, specifically the de jure construction of central bank independence, depends on the a priori institutional framework in place. Their results indicate that central bank independence has a greater impact on inflation in countries with a median level of institutional robustness. Nevertheless, they also find an increase in government spending resulting from the reduction of inflation, and, consequently, of inflationary revenue.

The Brazilian case seems to be an example of the above. Until 2021, the Brazilian Central Bank did not have de jure independence[4], relying only on de facto independence. This balance was extremely precarious and dependent on the good will of the installed heads of the Executive Branch. As shown in Section 2, the Brazilian institutional arrangement was not capable of sustaining immaculately the main purpose of a Central Bank. Roberto Campos' famous anecdote with Gen. Figueiredo summarizes well the fragile situation. Concerned with government interference in the Central Bank, Campos warns the president that the Central Bank must preserve its role as guardian of the currency. To which, Figueiredo replies, "I am the guardian of the currency!"[5]

Due to the chronic problem with high inflation rates, Brazil became a laboratory for the creation of the most diverse assets that sought to maintain both their protected real value and replicate the liquidity of paper money. According to Carneiro (1999), such creativity allowed not only the relatively normal functioning of the economy in the face of a highly disruptive and perverse inflation scenario, but also the growth of a financial system that was, at the limit of possibilities, robust and complex, and, surprisingly, not dollarized.

Brazilian particularities go beyond this. The successive shocks experienced by the Brazilian economy, together with the difficulties in managing economic policy, led to several crises in the payment of the foreign debt. In this way, starting in the 1980s, an environment ended up being created in Brazil where most of the holders of the public debt are domestic.

Garcia and Bevilaqua (1999) analyze the good performance of the Brazilian economy in the face of the devaluation experienced in 1998 in accordance with the literature on exchange rate and confidence crises. They find that the fact that there was a strong domestic bias, even if difficult to measure[6], reduced the volatility of demand for bonds in the country.

4 COMPLEMENTARY LAW No. 179, OF FEBRUARY 24, 2021 - DOU - National Press (in.gov.br)

5 The Stern Lantern

6 There are several indirect means by which foreign investors can hold Brazilian government bonds. However, the authors argue that regulatory changes in the period allow the assessment that the foreign participation in the demand for government bonds was small.

At the same time, characteristics already mentioned above of the development of the domestic financial market also contributed to the stability of the government bond market during the turbulent period of the 1990s. According to the authors, banks and the Central Bank were partners in rolling over domestic debt, the former receiving a high interest remuneration and incurring a small market risk. The counterpart came from the banks' offer of an almost perfect substitute for paper money: demand deposits that carried interest.

Such characteristics are not lost in decades alone. Even with the restoration of the country's credibility over time, the bulk of the public debt is still made up of domestic bonds. Alfaro and Kancuzk (2018) analyze the increased participation of foreign investors in the domestic bond market in light of the government's choice to issue domestic bonds and accumulate reserves.

Thus, it is important to analyze recent events concerning government debt decisions in light of these characteristics. Externalizing possible causal relations between the Central Bank's credibility and public debt management is fundamental. The paper follows with a brief historical analysis of this relationship in Brazil in the next section; section three points to the literature review of both fields of study; section four deals with an initial analysis of the data; section five defines characteristics of the model to be used; section six points to the results; finally, section seven concludes.

Historical Perspective

In Brazil, unlike most countries that experienced high inflation, inflation persistence did not prevent formal government financing. This was due to the creation of price readjustment instruments indexed to inflation. Initially aimed only at government bonds, indexation would, over the course of two decades, infiltrate all sectors of the Brazilian economy.

The success of the instrument is illustrated by Garcia et. al (1999), according to whom indexed government bonds, ORTNs, were seen by local investors as an effective protection against inflation and that allowed them to keep government bonds in their portfolio. The success was such that, during the 1960s, demand for the securities grew beyond the government's need for financing.

In the 1980s, with the increasing volatility of inflation, securities denominated in nominal terms proved increasingly risky. Consequently, Central Bank interventions to stabilize institutions positioned in such securities became increasingly frequent.[7]

In order to correct the problem, new securities were created: the LBC (Central Bank Bill) and the LFT (Treasury Financial Bill). As was to be expected, these securities, besides being post-fixed, had very short maturities (the shortest being that of the LFT, 48 days). Throughout the eighties, the average maturity of public bonds fell from 20 months to 11 months.[8]

7 Garcia et. alli (1999)

8 Ibidem

This process of indexation and inflationary spiral was accompanied, as Roberto Campos feared, pari passu by the deterioration of the Central Bank's institutional capacity to fulfill its mission as guardian of the currency. As Franco (2017) states, the deviation of the BCB from its core function was due to four primary factors: "the assumption of agricultural fostering functions by the BCB; the movement account; the monetary budget; and destruction of original governance"[9]

The first point was basically due to a political necessity. As Franco describes Roberto Campos' interpretation of the issue:

"`Bulhões felt that the establishment of the BCB was already in itself a difficult task; it should not be soured by a dispute with the rural works in Congress. Thus, among the objectives of the Central Bank was established the promotion of agriculture through subsidized credit.[10]

The Movement Account was adopted as a provisional measure to allow the Central Bank's operations to run smoothly even though its bureaucratic structure was not completely finished. It would therefore follow in a similar fashion to the usual behavior of the account when it linked the Superintendence of Money and Credit (Sumoc) to the Banco do Brasil - Sumoc, despite being the regulator of the national financial system was not able to receive deposits, which thus ended up being deposited at the Banco do Brasil. However, the structure, which was supposed to be temporary, became permanent:[11]

"With the passage of time, however, the perception grew that the movement account established in practice an extremely convenient arrangement for those who imagined a system of thematic public banks receiving automatic supplies from a central bank with no ability to limit the credit operations it was obliged to finance."

The monetary budget, in turn, initially had - when Sumoc still existed - a merely statistical character of the means of payments. However, with the emergence of the Central Bank, and throughout the 1970s, the monetary budget would come to play a key role in the escalation of inflation. It would become the means of concentrating all credit resources available to the state and redistributing them in whatever way the government decided was best. The advantage was that such a budget, even with its expansionary fiscal character, bypassed the National Congress and was under the direct tutelage of the National Monetary Council - CMN.[12]

9 Franco (2017) pp. 422

10 Ibidem pp. 420

11 Ibidem pp. 424

12 Ibidem pp. 429-430

Finally, we have the reformulation of the original composition of the CMN, an institution hierarchically superior to the Central Bank. In the beginning, this composition was made in order to establish the main objectives of a Central Bank. The institutional fragility, however, was evident in the excessively personalistic structure. With the fall of the most "orthodox" team, represented by Roberto Campos and Otávio Gouvêa de Bulhões, the composition of the CMN was altered in order to serve not the primary mission of a central bank - monetary stability - but rather the most diffuse interests. The CMN, which initially had nine members, grew to 26 in 1986. However, with the reforms introduced along with the Real Plan, in 1994 its composition was reduced to three members.[13]

Therefore, it can be seen that the Real Plan propitiated not only the end of the Brazilian hyperinflationary process, but also opened space for a better structuring of public governance, among them, including the public debt. However, as Garcia and Salomão (2006) state:

"The conquest of hyperinflation was rightly seen as a sine qua non condition for achieving more 'civilized' fixed income markets.... Hyperinflation was defeated by the Real Plan more than eleven years ago. Nevertheless, progress in lengthening public fixed income markets has been well below expectations."

The next section revisits the arguments in the literature that respond to such an impasse.

Literature

The economic literature generally considers public debt management as an optimizing agent seeking to minimize the cost of financing the government. If the assumptions of complete markets and perfect information are satisfied, there is no benefit in using several categories of securities for debt financing.

However, it is easy to notice that such assumptions seem disconnected from reality. Thus, Goldfajn and De Paula (1999) enumerate the characteristics attached to the issuance of sovereign debt:

- **inflationary risk:** full indexation of the debt would only be optimal in the case where real income and expenses are constant
- **real exchange rate risk:** the cost of debts denominated in foreign currency is directly related to exchange rate volatility. Ideally, in a floating exchange rate regime, exposure to this category should therefore be reduced
- **correlation between spending and inflation:** if the correlation between government spending and inflation is positive, the optimal would be to increase the nominal debt exposure, since an increase in inflation would lead to a lower real cost of debt.

13 Ibidem pp. 435

Nevertheless, it is necessary to emphasize the risks associated with their issuance: roll-over risk and government credibility.

Following this reasoning, Missale, Giavazzi and Benigno (1997) study 62 episodes of fiscal adjustment in OECD countries between 1975 and 1995; fiscal adjustment is considered the period in which the primary surplus increased by at least 1 percent. In addition, by the authors' definition, long-term debt is characterized by pre-fixed nominal securities and loans denominated in domestic currency with a maturity of more than two years. Short-term debt, on the other hand, encompasses foreign currency debt and indexed securities. The credibility of the adjustment, in turn, is measured in an ex-post manner, using the spread between the rates of long-term securities in the country and German securities.

The result found indicates that more credible adjustments tend to lead to a greater maturity in the issuance of bonds. At the same time, a government that intends to signal its commitment to adjustment tends to issue shorter-term debt seeking, in the near future, to refinance it at a lower cost.

Garcia and Salomão (2006) emphasize the importance of systemic risk for the bond market. They point out that not even the various forms of issuing them - whether nominal, indexed, or denominated in foreign debt - are capable, on their own, of fully diversifying the systemic risk of the domestic market. This impossibility contributes to an atrophy of the national financial markets.

As presented in the introduction, the Central Bank also has an important role - even if indirect - in the management of public debt, as it is responsible for anchoring expectations around the behavior of inflation. Consequently, a review of the literature on central bank credibility and how to measure it becomes fundamental.

Soares (2018), based on a methodology presented by Gaglione and Issler (2015) and using mainly information from the Focus Expectations System, creates a micro-founded credibility index based on the expectation of inflation 12 months ahead. The result indicates that the Central Bank of Brazil proved credible in 65 percent of the sample under study, with the exception of the periods of the first months of 2007, and between 2013 and 2016.

Several other indices have been created specifically for Brazil; however, the vast majority focus on short-term inflation expectations. In this interval, however, the economy is subject to exogenous shocks that tend to generate biases in the credibility indices, even if these shocks are orthogonal to the Central Bank's behavior. The 2002 election, in which there was the risk of institutional rupture with threats of non-payment of the public debt, can be considered an example of such shocks.[14]

Gaglianone and Oliveira (2017) propose to create a Central Bank credibility index with a long-term perspective, that is, focusing on the Central Bank's ability to influence agents' long-term inflation expectations. The interpretation differs slightly from that most common in the literature,

14 For more, see the critique present in Gagliaone and Oliveira (2017)

following Blinder (2000), in which a credible Central Bank is one that is committed to a low inflation rate. For the authors, a credible Central Bank is one that is able to anchor long-run expectations.

To build the index, several metrics were used as signals for the model, which is subsequently smoothed through a Kalman filter. Thus, we used inflation expectations taken from the database of the Central Bank's Focus System, nominal government bonds (LTN), floating rate bonds (NTN-B) and, finally, information regarding the swap market - the so-called ``Future-DI".

We use the yield to maturity spread of LTN bonds versus NTN-B bonds as a metric for implicit inflation, following the methodology initially proposed by Svensson (1993). Although not a ``pure" metric of inflation expectations, the authors reaffirm its importance by arguing that the Federal Reserve, the US central bank, constantly refers to a similar metric as a possible indicator of inflation expectations.

Lowenkron and Garcia (2007), in turn, derive an asset pricing model following a Taylor Rule model aiming at quantifying the implicit inflation (as defined above). The intent, therefore, is to evaluate the credibility of the Central Bank, dividing such a measure into two components: the expected inflation, in fact, and the risk premium for inflation.

The result found points out that short-term shocks in inflation have an effect in the medium-term, here also used the metric of expected inflation over 12 months. The effect could be the result of two factors: inertia, due to the still high degree of indexation in the economy, or the lack of credibility of the Central Bank. The authors conclude that indexation cannot be the cause since it has no correlation with the inflation risk premium.

Stylized Suits

The data regarding inflation expectations from specialized analysts were taken from the Focus Expectations System of the Brazilian Central Bank (BCB).

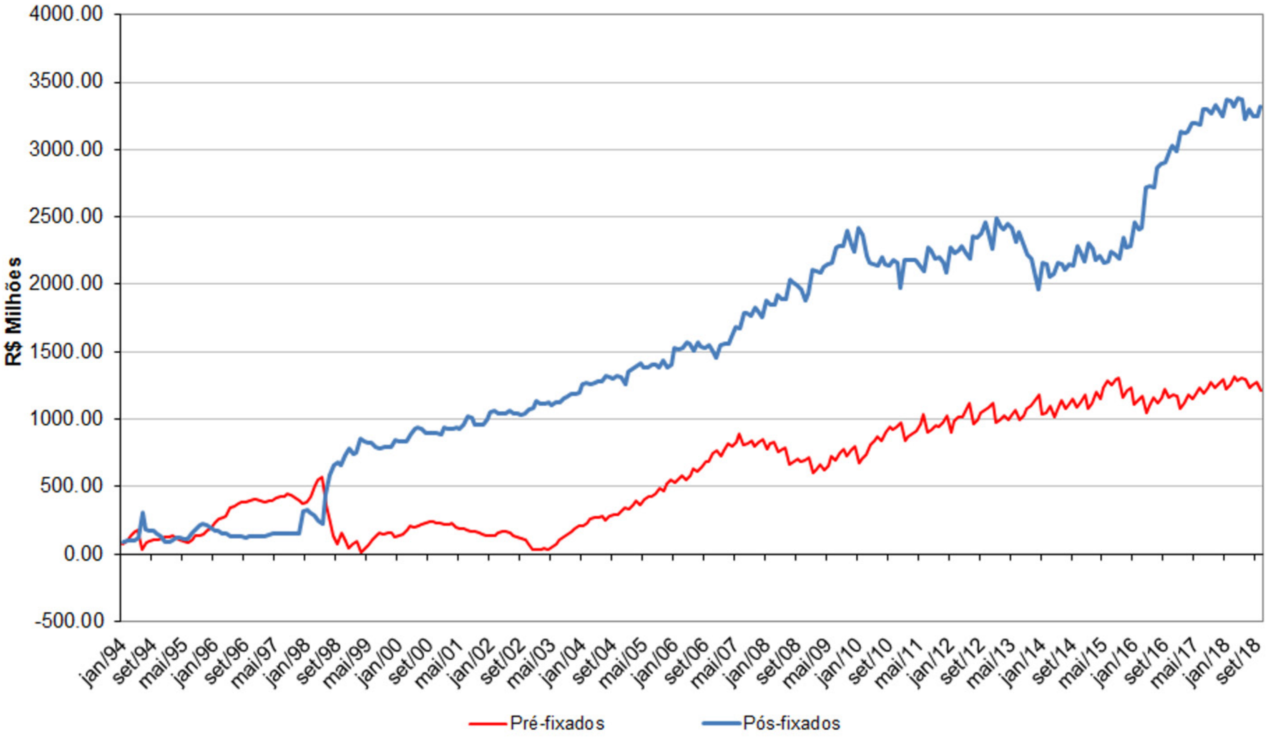
The data referring to the federal public debt were taken from several sources: National Treasury, Central Bank, and Ministry of Finance. Finally, prices of financial assets, as well as their other characteristics, were taken from the Brasil, Bolsa e Balcão (B3) system.

Chart 1 shows the evolution of the public debt composition in the period from 1994 to October 2018. It includes, therefore, the period prior to the Inflation Targeting Regime - established in 1999 by the then president of the Central Bank Armínio Fraga. The classification category used was pre-fixed securities, usually referred to as nominal in the literature, and post-fixed, or indexed, securities. Those denominated in foreign currency were excluded as they represent a small portion of the total debt when compared to the main categories: securities linked to the Selic rate and those linked to price indexes.

A first observation shows an inversion in the evolution of the type of debt right at the beginning of the sample. Between 1995 and September 1998, pre-fixed securities grow steadily, while indexed securities remain constant. This fact is due in part to the end of the Brazilian hyperinflationary process, thus allowing greater medium-term predictability for nominal rates. This behavior can also be attributed to the fixed exchange rate regime adopted during this period.

As for pre-fixed securities, it can be seen that they grow rapidly between 2004 and 2008, but have remained practically stable, or with a slight increase, since then.

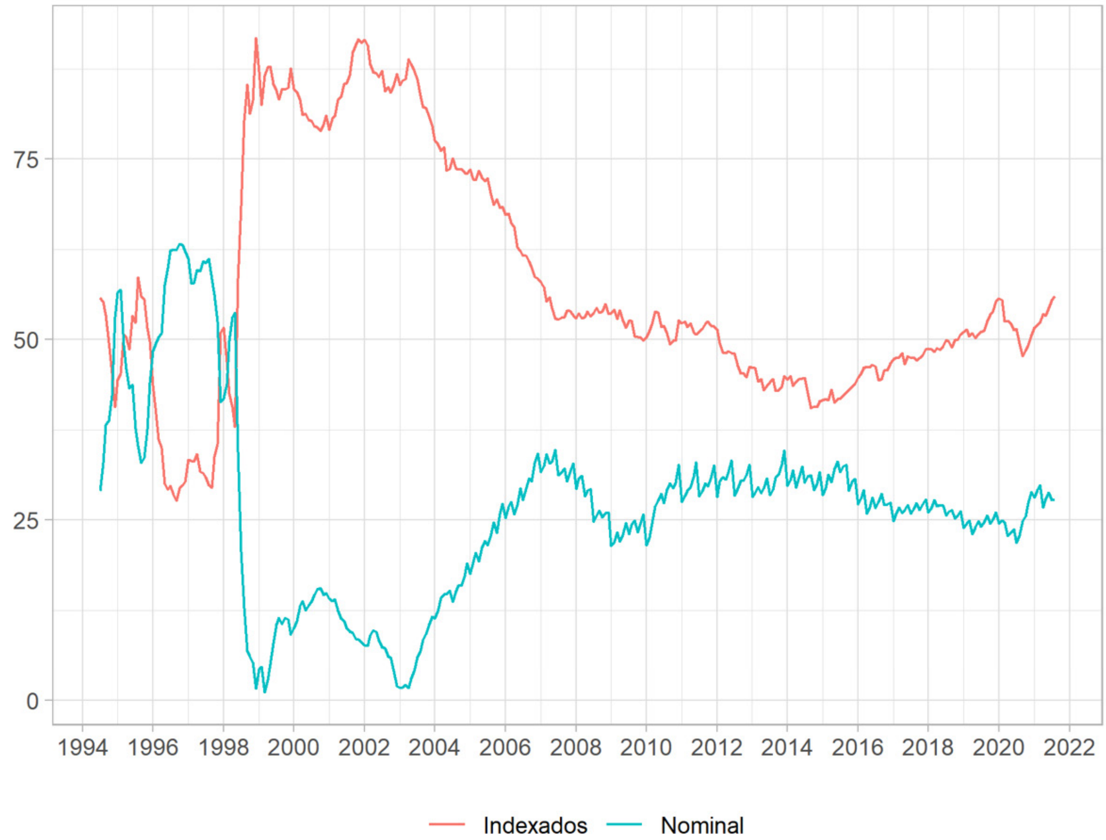
Chart 1 - Evolution of Public Debt by Category - Real Values



PRE-FIXED | POST-FIXED

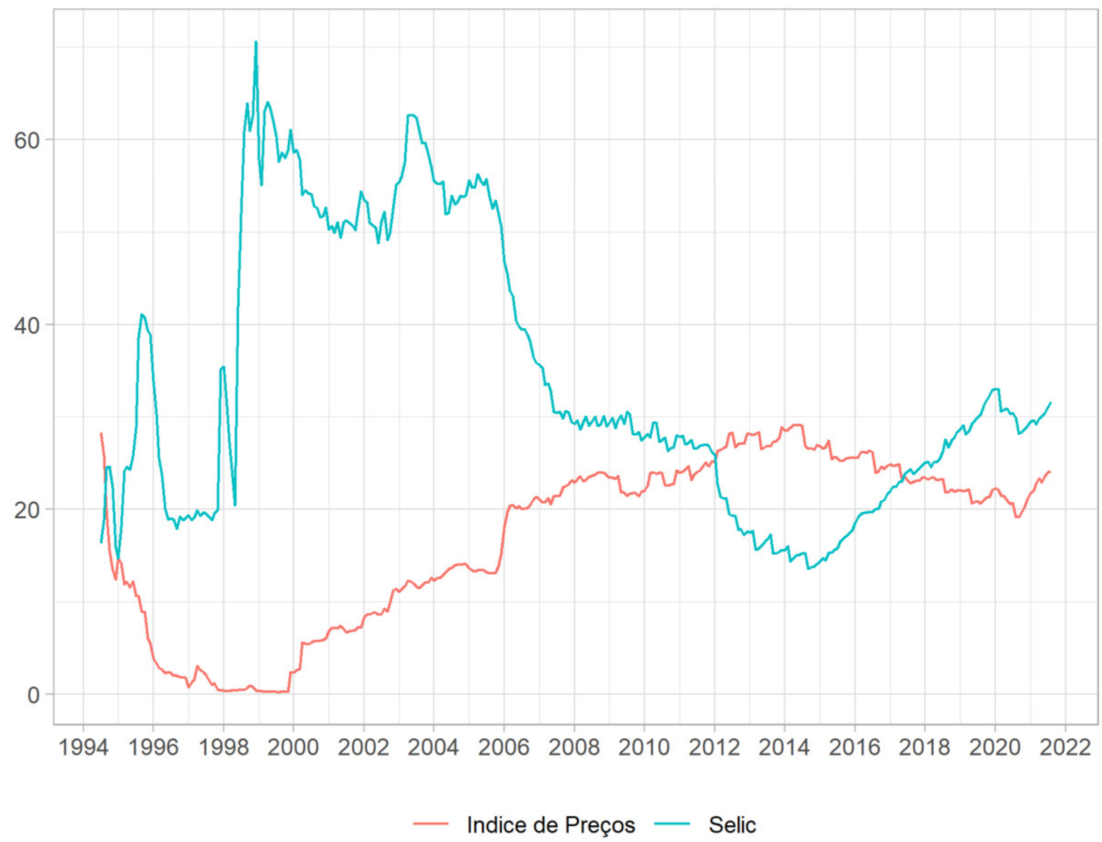
Chart 2 illustrates the composition of the category presented as "floating-rate securities". One can see that both categories converge to 50% each, in a process that lasts about 14 years. However, in 2016, this pattern is broken and the difference tends to increase, benefiting Selic-indexed securities.

Chart 2 - Evolution of the Title Composition (%)



INDEXED | NOMINAL

Chart 3 - Evolution of Title Composition (%)



PRICE INDEX | SELIC

The critique by Goldfajn and De Paula (1999) becomes fundamental in the Brazilian context. Since, due to the large amount of indexed and rigid spending tied to the 1988 Constitution, the Brazilian government has a high correlation between spending and inflation (Graph 3). Therefore, in bad states - in which the government should take advantage of its inflationary impact - one notices precisely the reduction in the percentage of nominal bonds in the composition of the debt

Another point pointed out by the authors is the credibility of the government. Since the Central Bank does not have consolidated de jure independence, agents may try to anticipate an inflationary drift, as already exposed.

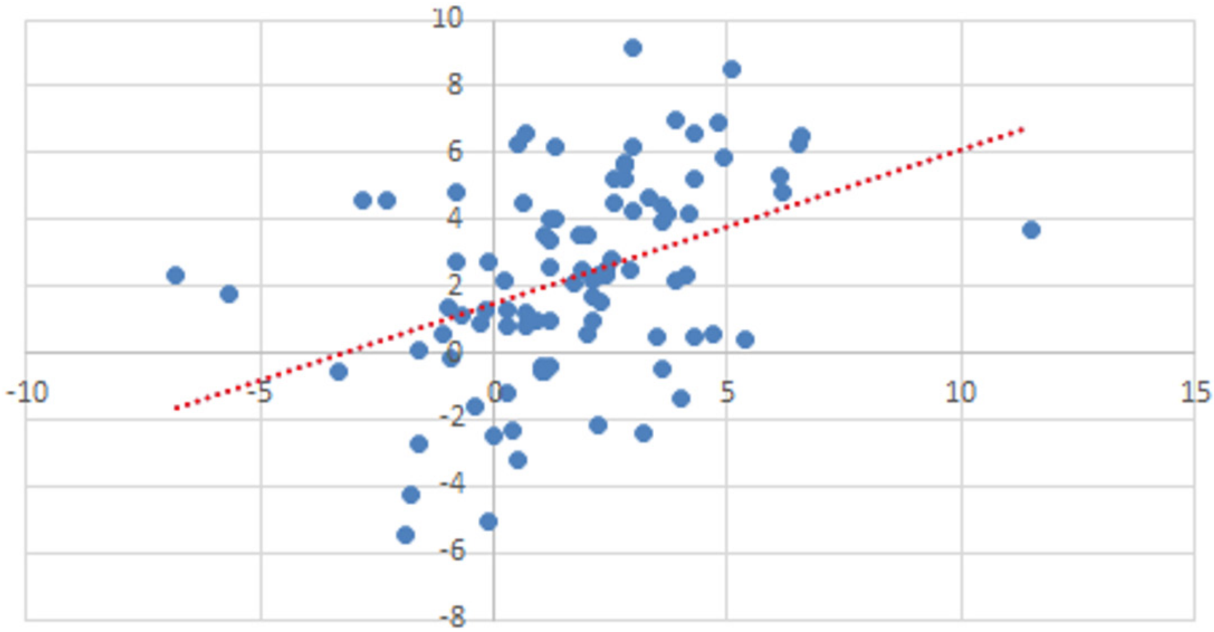
Charts 4 to 5 seek to illustrate the intuition. They describe the relationship between the percentage of indexed debt and the level of credibility of inflation policy, as measured by the deviation of inflation expectations from their targets, for each term of office of the Central Bank president since the introduction of the Inflation Targeting Regime. Finally, Chart 6 suggests that even after the legislation making the central bank independent, significant deviations from the target cause agents to react.

Having noted in the previous section the problems in constructing credibility indexes, the index used is based only on inflation expectations from the Focus System. We used an average of inflation expectations for the current year and the two subsequent years with their respective targets.

Charts 5, 7 and 8 seem consistent with the argument. In the latter, a horizontal displacement of expectations seems to have occurred after the first six months in office - which could be interpreted as a credibility shock.

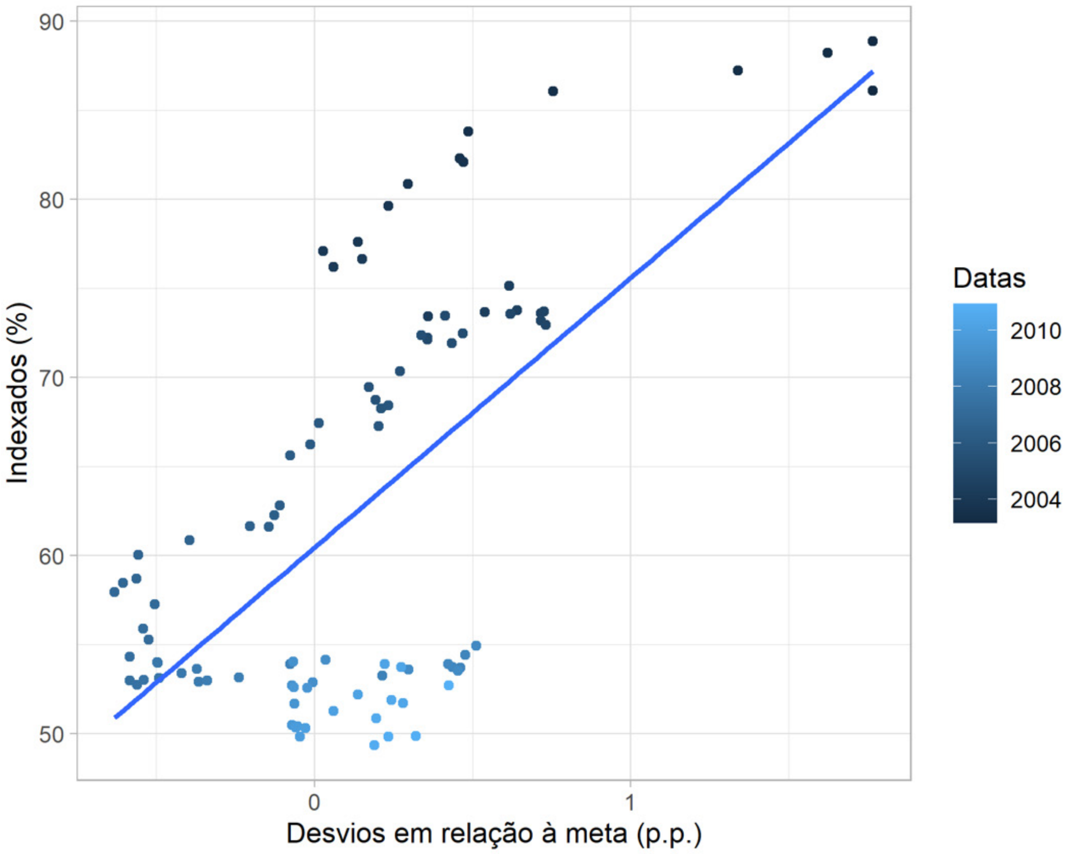
However, chart 6 points to evidence to the contrary. For in this period the relation was negative. This means a positive correlation between the percentage of debt pegged to nominal securities and deviations of expectations in relation to the inflation target.

Chart 4 - PIB x Government Consumption



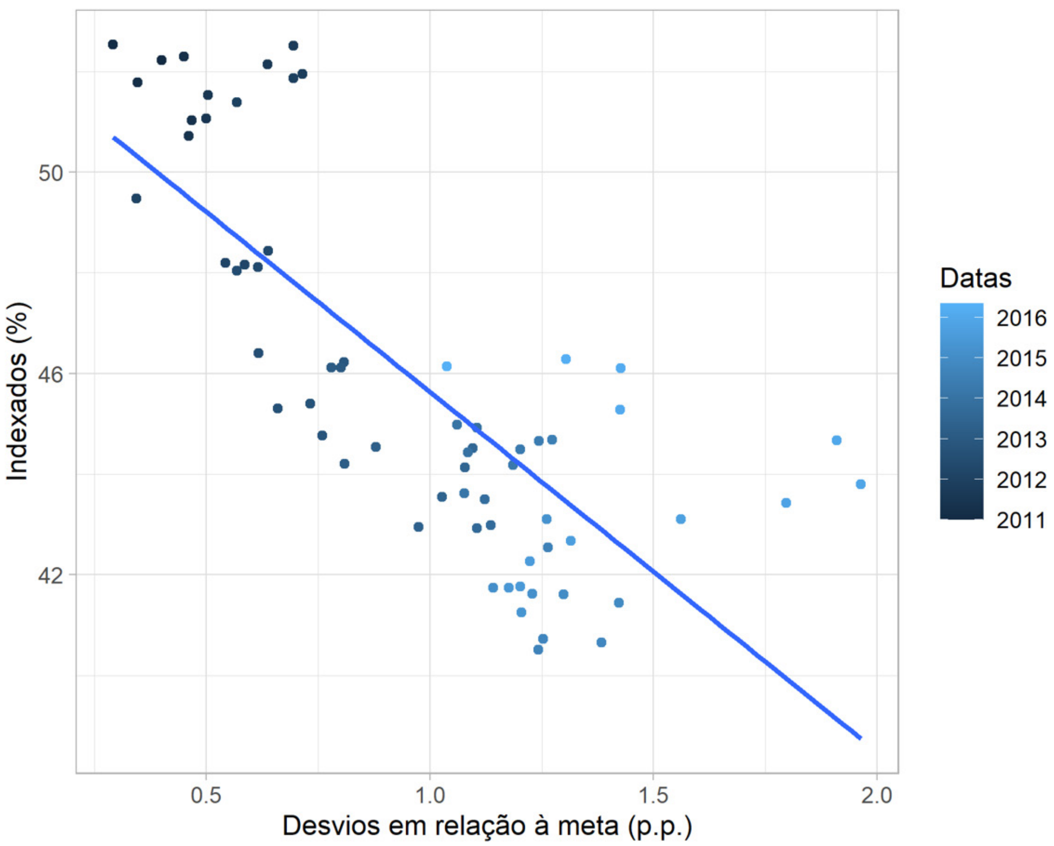
Source: IBGE Note: Quarter Against Quarter of the Previous Year

Chart 5 - BCB under Meirelles



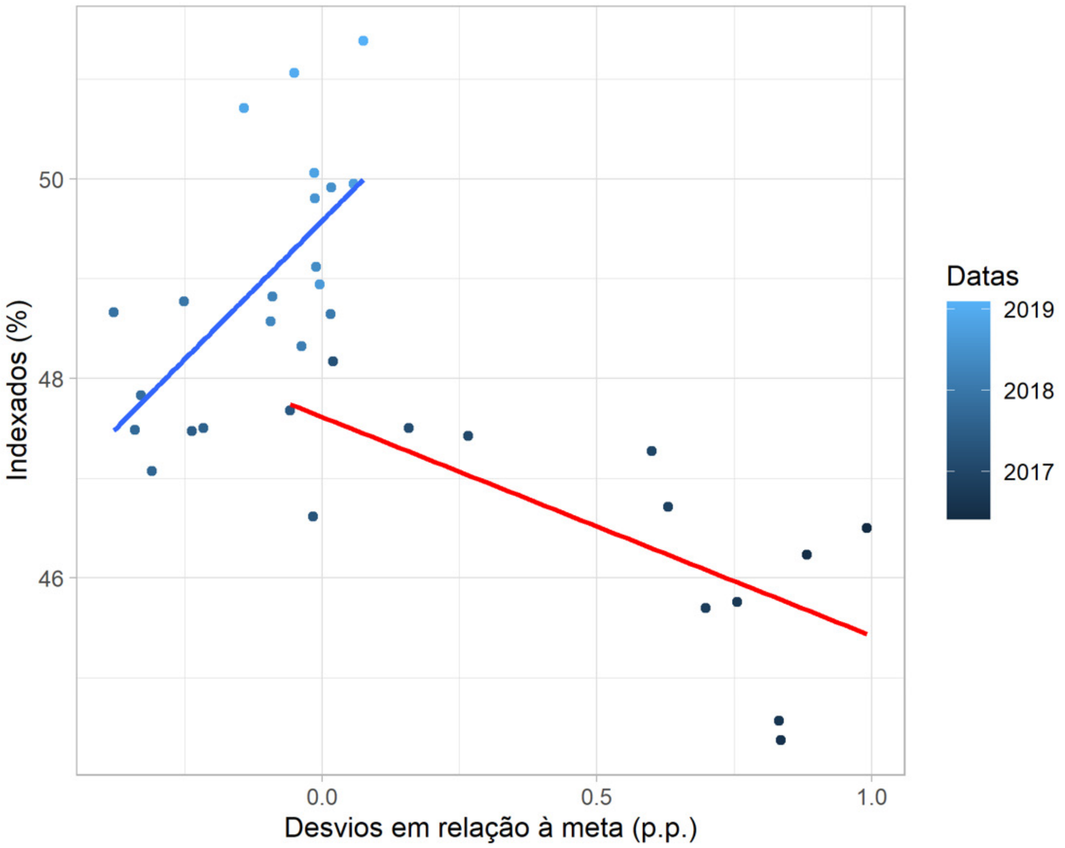
Indexed (%) | Dates | Deviations from target (p.p.)

Chart 6 - BCB under Tombini



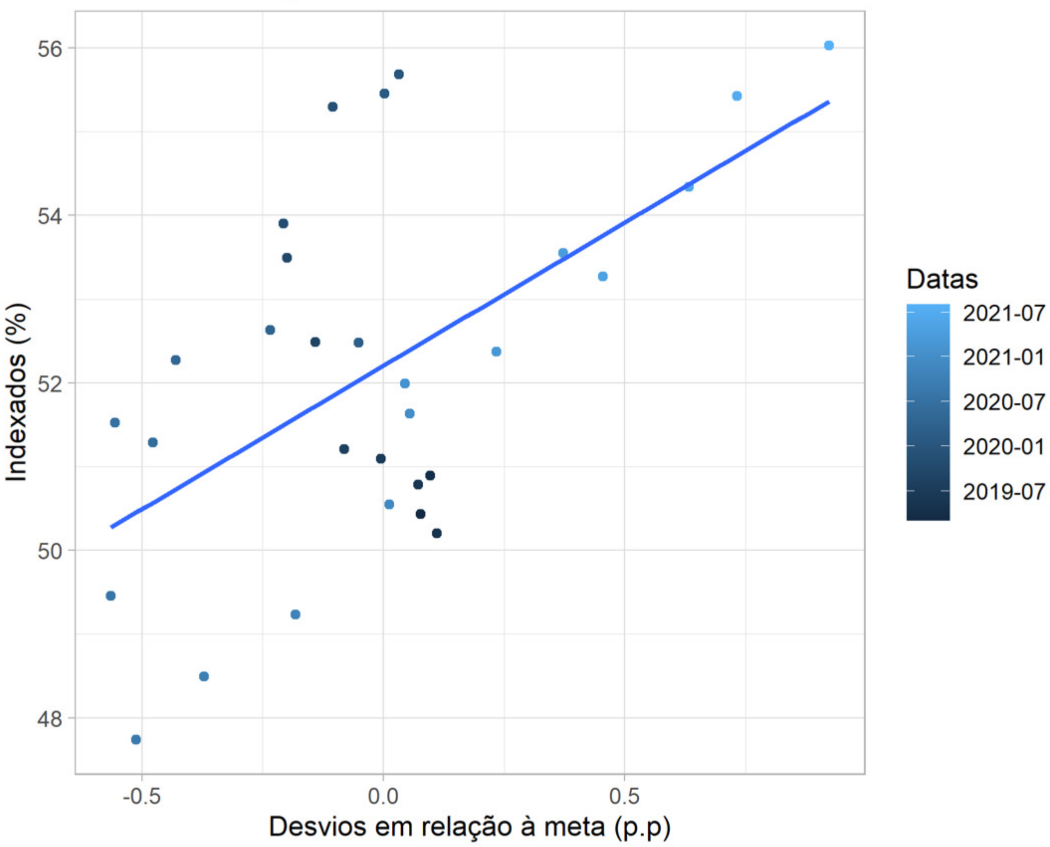
Indexed (%) | Dates | Deviations from target (p.p.)

Chart 7 - BCB under Goldfajn



Indexed (%) | Dates | Deviations from target (p.p.)

Chart 8 - BCB under Goldfajn



Indexed (%) | Dates | Deviations from target (p.p.)

The Model

Such a move is believed to be caused by the government's lack of credibility in keeping its inflationary policy in check. The argument is similar to the model developed in Du, Pflueger, and Schreger (2016). The authors formulate a model that is compatible with the empirical evidence found; these point out that precisely the countries that would benefit most from this consumption smoothing between states of nature are those with a lower percentage of nominal bonds.

The channel through which the equilibrium mechanism occurs is the risk premium. This increases the cost of issuing nominal bonds, which prevents the government from financing itself exclusively through this category. Thus, the government is forced to seek other means of financing.

The model developed by the authors deals with diversification between securities denominated in local currency and securities denominated in foreign currency. Due to the historical context presented in Section 2, Brazil has an extremely low percentage of debt denominated in foreign currency, and, in addition, the vast majority of holders of its securities are domestic.

Model-Base

Thus, a simplified version of the model used in Du, Pflueger, and Schreger (2016) was first developed to explain the Brazilian reality. The model follows with two periods, government and investors. The latter, however, will be domestic, and will have, among the investment options, nominal bonds (with uncertain real payoffs) and floating-rate bonds (with constant real returns regardless of the states of nature).

The objective of the baseline model is to illustrate the mechanism by which it is optimal for the government to deviate from its inflation policy, given its objective function. The government therefore seeks to maximize the expected utility of consumption of its services (or transfers) in period 2.

Note that, as corroborated by the literature, it is natural to assume that there are two different types of agents: those who receive government services (transfers), and those who hold government securities.

To do this, it needs to issue an amount of debt in period $\bar{D}$ of debt in period 2 that will be delivered as goods and services to the population in t = 2.

$$\text{U}(x_2) = (1 + \bar{D})x_2 - D_2 - b\pi^2$$

Equation 1

Where x_2 represents the domestic product of the economy in period 2 and has normal distribution N $(\bar{X}, \sigma^2)$. D_2 is the amount of debt to be paid in period 2. Finally, the parameter b represents inflation costs.

The government, therefore, in trying to maximize the utility of the population, actually seeks to minimize the cost of its debt. To finance it, there are only two means: nominal bonds (n) and inflation-indexed bonds (ind). Therefore, one can think of the government's problem as follows:

$$F_G(\pi) = \max\{(1+\bar{B})x_2 - D_2 - b\pi^2\}$$

Equation 2

Subject to:

$$(1-s)Q_{ind} + sQ_n = \bar{D}$$

Equation 3

Where Q_i is the price of security i.

Note that because the indexed bond presents a constant pay-off in all states of nature, it represents something as close as possible to a riskless asset in this economy. Thus, the prices of such bonds in equilibrium should be:

$$Q_{ind} = \frac{1}{1+R}$$

$$Q_n = \frac{1-\ \pi^e}{1+R}$$

Now suppose that the government sells these bonds to a rational, risk-averse representative agent who seeks to maximize his utility as follows:

$$F_A(s) = max_s \{ E(R_p) - \ \gamma V(R_p) \}$$

Equation 4

Where R_p is the return on the portfolio of the representative agent.

Suppose that the agent receives, exogenously, an endowment in period 1 (dx_1), which will be fully saved and used to purchase the bonds. Note that since we are in a closed economy: $dx_1 = \bar{D}$. Suppose also that, for agents, the Central Bank is perfectly credible.

The investors' portfolio is given by:

$$R_P = s\left[\frac{1-\pi_i}{Q_n} - 1\right] + (1-s)R_{ind}$$

Equation 5

You can replace Q_n in (5):

$$E[R_p] = s\left(\frac{1+R}{1-\pi^e}\right)E(1\cdot\pi) + (1-s)R_i$$

Let be $(1+R)/(1-\pi^e) = \beta$, we have:

$$E[R_p] = s\beta E[1-\pi] + R_i - sR_i$$

Equation 6

Analogously, the variance of the portfolio is given by:

$$V(R_p) = \left[\frac{1+R}{1-\pi^e}\right]^2 s^2 V(\pi)$$

Equation 7

Substituting in (4):

$$F_A(s) = \max_s(\beta s E[1-\pi] + R_{ind} - sR_{ind} - \gamma p^2 s^2 \sigma_\pi^\alpha)$$

Equation 8

$$\frac{\partial F_A}{\partial s} = \beta E[1-\pi] - 2s(\gamma p^2 \sigma_\pi^2) - R_{ind}$$

At the optimal point, we have:

$$s = \frac{E[1-\pi]}{(1-\pi^e)}(1+R)\frac{1}{2\gamma\sigma_\pi^2} - R_{ind}$$

Equation 9

Note that one can rewrite the above equation as:

$$s = E\left[\frac{1-\pi}{1-\pi^c}\right](1+R)\frac{1}{2\gamma\sigma_\pi^2} - R_{ind}$$

Equation 10

Taking a Taylor approximation:

$$E\left[\frac{1-\pi}{1-\pi^e}\right] = \frac{E[1-\pi]}{E[1-\pi^e]} \approx 1$$

Therefore,

$$S = \frac{1+R}{2\gamma\sigma_\pi^2} - R_{ind}$$

Equation 11

Note that $\partial s/\partial \gamma < 0$, $\partial s/(\partial \sigma_\pi^2) < 0$ and $\partial s/(\partial R_{ind}) < 0$

In other words, the more risk-averse, the greater the variability of inflation or the greater the return on indexed securities, the lower the proportion of the portfolio allocated to nominal securities.

Once agents set the ratio between the types of securities, the government can choose to divert its inflationary policy so as to maximize the utility described in (2).

$$F_s(\pi) = \left[1 + \frac{1-s}{1+R} + \frac{s(1-\pi_e)}{1+B}\right] x_2 - b\pi^2 - [(1-s) + s(1-\pi)]$$

Equation 12

Consequently:

$$\frac{\partial F}{\partial \pi} = -2b\pi + s$$

At the optimum:

$$\pi(2b) = s$$

Equation 13

In an equivalent way:

$$\pi = \frac{s}{2b}$$

Equation 14

Note that ∂π/∂b > 0, 0The higher the proportion of nominal bonds, the higher the inflation generated to minimize the cost of debt. At the same time, ∂π/∂b < 0, the larger the parameter that represents inflation costs, the lower the optimal inflation will be.

Complete Model

Now, still following Du, Pflueger, and Schreger (2016), suppose the population has power utility and is financed by an accumulation of debt in t = 1 de $\bar{D}$.

$$\mathrm{U}(C_2) = \frac{C_2^{(1-\gamma)}}{1-\gamma}$$

Equation 15

The government can finance this deficit by issuing two types of bonds: nominal and indexed. D^n and indexed D^{ind}. It is worth remembering that the second type presents a real return of $R^{ind} = R$. So that:

$$Q^n D^n + Q^{ind} D^{ind} = \bar{D}$$

Equation 16

You can normalize, so that we have:

$$Q^n s + Q^i (1-s) = \bar{D}$$

Equation 17

Where s represents the total proportion of debt issued through nominal bonds and Q^i, is the price (cost) of the security of type *i*.

As in the original model, the population's income endowment is assumed to have a log-normal distribution:

$$X_2 = \bar{X} e^{\frac{x_2}{\bar{X}}}, \qquad x_2 \sim N(0,\sigma), \qquad \bar{X} = 1 + \bar{D}$$

Equation 18

The consumption of government spending in the second period must therefore be equivalent to its spending in the period minus the repayment of the debt incurred:

$$C_2(X_2) = X_2 - D_2(x_2)$$

Equation 19

where the payment is described by:

$$D_2(X_2) = D^n[e^{-\pi_2 X_2}] + D^i$$

Equation 20

Which can be rewritten as:

$$D_2(X_2) = s[e^{-\pi_2 X_2}] + (1 - s)$$

Finally, we introduce the investors. They are assumed to have time separable CRRA preferences with a risk aversion γ^* and discount rate δ^*:

$$\mathrm{U}(C_1, C_2) = E\left[\sum_{t=1}^{2} (\delta^*)^t \frac{(C_t^*)^{1-\gamma^*}}{1-\gamma^*}\right]$$

Investors receive an endowment in period 1, which is fully saved (i.e. used to buy the bonds). While in period 2, they receive $X_2^* = \exp(x_2)$. Their goal is therefore:

$$\max_{C_1, C_2} E\left[\sum_{t=1}^{2} (\delta^*)^t \frac{(C_t^*)^{1-\gamma^*}}{1-\gamma^*}\right]$$

Thus, we arrive at the following Euler Equation:

$$1 = E_t\left[\delta^* \left(\frac{C_{t+1}}{C_t}\right)^{-\delta^*} (1 + Ret_{p,t+1})\right]$$

Equation 21

Assuming a portfolio is composed only of floating-rate securities, which always pay a real rate R, therefore indicating the closest to a risk-free rate that exists in this economy. [15] We are left with:

$$1 = E_t\left[\delta^* \left(\frac{C_{t+1}}{C_t}\right)^{-\delta^*} (1 + R)\right]$$

15 See Fischer (1982)

And so, by replacing C_t e C_{t+1} we arrive at the Stochastic Discount Factor (M) of the economy:

$$\frac{1}{1+R} = E_t[\delta^* e^{(-\delta^* x_2)}]$$

The Stochastic Discount Factor (M) is the term within the expectancy.

$$\frac{1}{1+R} = E_t[\delta^* \exp(-\gamma^* x_2^*)]$$

Once the Stochastic Discount Factor is derived, at equilibrium, the price of the bonds should reflect their actual return discounted at that rate:

$$Q^n = E[Me^{-\pi_2 X_2}]$$

Equation 22

$$Q^{ind} = E[M] = \frac{1}{1+R}$$

First, it should be noted that M has a log-normal distribution, therefore:

$$E[M] = -\delta^* \exp\left(E[-\gamma^* x_2^*] + \frac{1}{2}\gamma^{*2}\sigma_2^2\right)$$

Then we have that:

$$\frac{1}{1+R} = \delta^* \exp\left(\frac{1}{2}\gamma^{*2}{\sigma_2}^2\right)$$

Substituting M in (22), we have:

$$Q^n = E\left[\delta \exp(-\gamma^* x_2^*) \exp\left(-\pi_2(x_2)\right)\right]$$

$$Q^n = E\left[\delta^* \exp\left(-\gamma^* x_2^* - \pi_2(x_2)\right)\right]$$

Equation 23

We know that x_2 e π_2 are jointly normal, and therefore

$$E[\exp(z)] = exp\left(E(z) + \frac{1}{2}Var(z)\right)$$

Where $z = -\gamma^* x_2^* - \pi_2 (X_2)$. We can define the Variance as:

$$Var\big(-\gamma^* x_2^* - \pi_2(X_2)\big) = \gamma^{*2}\sigma_x^2 + Var\big(\pi_2(X_2)\big) + 2\gamma^* Cov(x_2, \pi_2)$$

Which leads us to the following equation:

$$Q^n = \delta \exp\left(\frac{1}{2}\gamma^{*2}\sigma^2\right) \exp\left(-E[\pi_2(x_2)] + Var\big(\pi_2(X_2)\big) + 2\gamma^* Cov(x_2, \pi_2)\right)$$

Finally, the price of the nominal bond is given by:

$$Q^n = \frac{1}{1+R} \exp\left(-E[\pi_2] + \frac{1}{2} V(\pi_2) + \gamma^* Cov(x_2, \pi_2)\right)$$

Equation 24

Here we see the first channel through which inflation, and therefore monetary policy, can affect this type of bond. The price of such a bond has a direct and negative relationship to expected inflation.

Nevertheless, there is also another channel through which agents' expectations can affect the issuance of such securities: through the risk premium.

You can define the risk premium of the nominal bond by its expected excess return:

$$PR^n = \ln\left(E\left[\frac{exp(-\pi_2)}{Q^n}\right]\right) - \ln(1+R)$$

Once again resorting to the properties of the log-normal distribution:

$$PR^n = \ln\left(\frac{1}{Q^n} exp\left(-E[\pi_2] - \frac{1}{2}Var(\pi_2)\right)\right) - r$$

Substituting Q^n in the equation we arrive at:

$$PR^n = \ln\left(\frac{1}{1+R}\right) - \ln\left(\gamma^* Cov(x_2, \pi_2)\right) - r$$

Thus, the risk premium is defined by:

$$PR^n = -\gamma^* Cov(x_2, \pi_2)$$

Equation 25

Thus, it can be seen that the risk premium is inversely proportional to investors' risk aversion and the covariance between output and inflation.

This result implies that if a government tries to smooth consumption by inflating in periods of falling output, and consequent falling revenues, agents will charge a higher price to hold their nominal securities.

Empirical test

As explained by the previous model, it is natural that we notice two fundamental impacts of lack of inflation policy credibility: an increase in the cost of debt issuance and an increase in the percentage of indexed debt.

Using the methodology adopted in Lowenkron and Garcia (2007), we can adopt as a measure of the risk premium (PR^n) the difference between the implicit inflation (π_I) and the actual inflation expectation (π^e). Where:

$$\pi_I = \frac{1 + i^n}{1 + i^{ind}} - 1$$

The cost associated with issuing bonds can therefore be understood as the risk premium associated with inflation (inflation premium):

$$PR^n = \pi_I - \pi^e$$

We used as the nominal interest rate variable the prefixed interest rate of the term structure of the LTN referring to 12 months[16], while the real interest is given by the DIxIGP-M Swap[17]. The decision to choose the latter was made because securities referenced to the IPCA, the index on which the Central Bank's inflation target is based, were not very common until the mid-first decade of the 2000s

16 Available at: Ipeadata, "Prefixed Interest Rate - Term Structure - LTN - 12 months

17 Taken from the Refinitiv Eikon data system (code "DIIGPM1Y=BVMF")

The objective becomes to identify whether the risk premium on inflation leads the government to optimize its debt management by reducing its stock of nominal bonds.

The regression is given by:

$$\%nominal = PR^n + dummies_{presidentes} + dummies_{mês} + \varepsilon$$

Equation 26

The results obtained are illustrated in Table 1. There one can find a negative correlation between the percentage of nominal bonds and the inflation risk premium. It can also be seen that this measure is statistically significant at the 1% level. The dummy for the period of former president Tombini is positive and significant at 1%.

Regression (2) in Table 1 indicates that the percentage of nominal bonds is negatively correlated with the 12-month accumulated IPCA, and is relevant at 1%. In it, it tests whether the effect captured by the inflation risk premium and the accrued inflation is the same.

That is, it tests whether expectations are adaptive, so that by including past inflation, the inflation risk premium, forward looking metric, would become non-significant. This is not the case. Both regressors, plus the dummy for presidents, are significant at the 1% significance level. Thus, a 1% increase in the inflation risk premium leads to a 1% reduction in the percentage of nominal government bonds.

Finally, Table 2 repeats the experiment, but using the reference rate of the 360-day DIxPre Swap[18] as the nominal interest rate to calculate the inflation risk premium. The results are similar, corroborating the model presented above.

18 Taken from the Refinitiv Eikon data system (code "BRPRE1Y=BVMF")

Conclusion

The benefits of a Central Bank's credibility over its monetary policy objectives are well-structured throughout the literature. Notwithstanding the various credibility metrics and attempts to measure the impact of structural reforms aimed at central bank independence, little is said about the relationship between government and the central bank.

With the model elaborated, it is shown that inflation expectations influence the options of public debt managers, allowing them to change the type of debt issued according to the needs of the moment. Nevertheless, the opposite is also possible. A central bank may act in order to allow the government to finance its expenditures.

It can be seen, therefore, that in a troubled institutional scenario, the lack of independence, de jure or de facto, of the Central Bank encouraged agents to anticipate the Central Bank's movements always taking into consideration the need to finance government spending. With the consolidation of the Central Bank's independence in agents' expectations, the tendency is for this risk to dissipate.

As shown through the analysis, Brazil seems to have belonged, at different historical moments, to both cases. In the former, the Central Bank was totally distorted and placed at the service of various lobbies. The context, after the consolidation of the Real Plan and its reforms, has shown that the lack of de jure independence of the guardian of the currency has a cost. It can be perceived that, at all times, agents seek to anticipate a possible breach in de facto independence - this being extremely fragile and dependent upon the good will of the characters of the moment - leading to constant variations in the stock of government bonds. Nevertheless, in the long run it is possible to perceive the impressions that each president leaves on the public debt.

Tables

Table 1

	Dependent variable:	
	Nominal	
	(1)	(2)
inflation_premium	-1.424***	-1.008***
	(0.210)	(0.170)
IPCA_12meses		-1.213***
		(0.109)
Meirelles	0.901	2.035**
	(1.057)	(0.842)
Tombini	5.912***	8.623***
	(1.037)	(0.856)
Goldfajn	1.152	1.170
	(1.167)	(0.923)
Constant	23.767***	29.023***
	(1.106)	(0.994)
Observations	220	220
R^2	0.389	0.620
Adjusted R^2	0.347	0.592
Residual Std. Error	4.612 (df = 205)	3.647 (df = 204)
F Statistic	9.329*** (df = 14; 205)	22.185*** (df = 15; 204)
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 2

	Dependent variable:	
	Nominal	
	(1)	(2)
inflation_premium	-1.166^{***}	-0.836^{***}
	(0.211)	(0.168)
IPCA_12meses		-1.260^{***}
		(0.110)
Meirelles	0.218	1.644^{*}
	(1.075)	(0.851)
Tombini	5.673^{***}	8.582^{***}
	(1.070)	(0.875)
Goldfajn	1.052	1.105
	(1.205)	(0.943)
Constant	23.961^{***}	29.345^{***}
	(1.141)	(1.010)
Observations	220	220
R^2	0.349	0.603
Adjusted R^2	0.305	0.574
Residual Std. Error	4.760 (df = 205)	3.727 (df = 204)
F Statistic	7.856^{***} (df = 14; 205)	20.648^{***} (df = 15; 204)
Note:		*p<0.1; **p<0.05; ***p<0.01

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Appendix

Complete Model Derivation

This has power utility and is financed by an accumulation of debt in t = 0 de $\bar{D}$.

$$U(C_2) = \frac{C_2^{1-\gamma}}{1-\gamma}$$

Equation 27

The government can finance such a deficit by issuing two types of bonds: nominal and indexed D^{ind}. VIt is worth remembering that the second type presents a real return of $R^{ind} = R$. So that:

$$Q^n D^n + Q^{ind} D^{ind} = \bar{D}$$

Equation 28

You can normalize, so that we have:

$$Q^n s + Q^{ind}(1-s) = \bar{D}$$

Equation 29

Where s represents the total proportion of debt issued through nominal bonds and Q^i is the price (cost) of bond type i.

As in the original model, the government's income endowment is assumed to have a log-normal distribution:

$$X_2 = \bar{X} \exp\left(\frac{x_2}{\bar{X}}\right), \qquad x_2 \sim N(0, \sigma), \qquad \bar{X} = 1 + \bar{D}$$

Equation 30

The consumption of government spending in the second period must therefore be equivalent to its spending in the period subtracting the payment of the debt incurred:

$$C_2(X_2) = X_2 - D_2(X_2)$$

Equation 31

Where payment is described by:

$$D_2(X_2) = D^n \exp\big(-\pi_2(X_2)\big) + D^{ind}$$

Equation 32

Which can be rewritten as:

$$D_2(X_2) = s * \exp\big(-\pi_2(X_2)\big) + (1 - s)$$

Finally, we introduce the investors. They are assumed to have time-separable CRRA preferences with a risk aversion γ^* and discount rate δ^*:

$$U^*(C_1^*, C_2^*) = E\left[\sum (\delta^*)^t \frac{(C_t^*)^{1-\gamma^*}}{1-\gamma^*}\right]$$

Investors receive an endowment in period 1, which is fully saved (i.e. used to buy the bonds). While in period 2, they receive $X_2^* = \exp(x_2)$. Their goal is therefore:

$$\max E_t\left[\sum (\delta^*)^t \frac{(C_t^*)^{1-\gamma^*}}{1-\gamma^*}\right]$$

Your budget constraint in each period is given by:

$$W_{t+1} = \big(1 + Ret_{p,t+1}\big) * (W_t - C_t)$$

Such equations imply the following first-order condition:

$$U'(C_t) = E_t\big[\delta^* U'(C_{t+1})(1 + R_{p,t+1})\big]$$

Dividing the above equation by $U'(C_t)$, we have:

$$1 = E_t\big[\delta^*(1 + Ret_{p,t+1})\big]$$

You can also replace $U'(C_t)$ por $C_t^{-\gamma^*}$. Thus, we arrive at Euler's Equation for our problem:

$$1 = E_t\left[\delta^*\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma^*}(1 + Ret_{p,t+1})\right]$$

Equation 33

Assuming that the portfolio is composed only of floating-rate securities, which always pay a real rate R and therefore indicates the closest to a risk-free rate that exists in this economy, we are left with:

$$1 = E_t\left[\delta^*\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma^*}(1+R)\right]$$

And so, by replacing C_t e C_{t+1} we arrive at the Stochastic Discount Factor (M) of the economy:

$$\frac{1}{1+R} = E_t[\delta^*\exp\left(-\gamma^* x_2^*\right)]$$

$$\frac{1}{1+R} = E[M]$$

Once the Stochastic Discount Factor is derived, at equilibrium, the price of the bonds should reflect their actual return discounted at that rate:

$$Q^n = E[M * \exp\left(-\pi_2(X_2)\right)$$

Equation 34

$$Q^{ind} = E[M] * 1 = \frac{1}{1+R}$$

First, it should be noted that M has a log-normal distribution:

$$E[M] = \delta^*\exp\left(E[-\gamma^* x_2^*] + \frac{1}{2}\gamma^{*2}\sigma_2^2\right.$$

Then we have that:

$$\frac{1}{1+R} = \delta^*\exp\left(\frac{1}{2}\gamma^{*2}\sigma_2^2\right)$$

Substituting M into Q^n, we have:

$$Q^n = E[\delta^*\exp\left(-\gamma^* x_2^*\right)\exp\left(-\pi_2(X_2)\right)]$$

$$Q^n = \delta^* E[\exp\left(-\gamma^* x_2^* - \pi_2(X_2)\right)]$$

Equation 35

We know that x_2 e π_2 are jointly normal, and therefore

$$E[\exp(z)] = \exp\,(E(z) + \frac{1}{2}V(z))$$

Where $z = -\gamma^* x_2^* - \pi_2 (X_2)$. Substituting, therefore, we have:

$$E[\exp(z)] = \exp\,(E[-\gamma^* x_2^* - \pi_2(X_2)] + \frac{1}{2}Var(-\gamma^* x_2^* - \pi_2(X_2))$$

$$E[\exp(z)] = \exp\left(-E[\pi_2(X_2)] + \frac{1}{2}Var\left(\gamma^* x_2^* - \pi_2(X_2)\right)\right)$$

Equation 36

We can define the Variance as:

$$Var\left(-\gamma^* x_2^* - \pi_2(X_2)\right) = \gamma^{*2}\sigma_x^2 + Var\left(\pi_2(X_2)\right) + 2\gamma^* Cov(x_2, \pi_2)$$

Substituting in (36) we get:

$$\exp\,(-E[\pi_2(X_2)] + \frac{1}{2}(\gamma^{*2}\sigma_x^2 + Var\left(\pi_2(X_2)\right) + 2\gamma^* Cov(x_2, \pi_2)$$

Substituting the above equation into (35), we have:

$$Q^n = \delta^* \exp\,(-E[\pi_2(X_2) + \frac{1}{2}(\gamma^{*2}\sigma_x^2 + Var\left(\pi_2(X_2)\right) + 2\gamma^* Cov(x_2, \pi_2)])$$

Reorganizing, we have

$$Q^n = \frac{1}{1+r} * \exp\,(-E[\pi_2(X_2) + \frac{1}{2}Var\left(\pi_2(X_2)\right) + 2\gamma^* Cov(x_2, \pi_2)])$$

Finally, the price of the nominal bond is given by:

$$Q^n = \frac{1}{1+R}\,\exp\,(-E[\pi_2] + \frac{1}{2}Var(\pi_2) + \gamma^* Cov(x_2, \pi_2))$$

Equation 37

You can define the risk premium of the nominal bond by its expected excess return:

$$PR^n = \ln\left(E\left[\frac{-\pi_2}{Q^n}\right]\right) - \ln(1+R)$$

Note that ln (1+R) ≈r

We know that Q^n is constant, so

$$PR^n = \ln(\frac{1}{Q^n}E[\exp(-\pi_2)]) - r$$

Once again resorting to the properties of the log-normal distribution:

$$PR^n = \ln\left(\frac{1}{Q^n}exp\left(-E[\pi_2] - \frac{1}{2}Var(\pi_2)\right)\right) - r$$

Substituting Q^n in the equation:

$$PR^n = \ln\left(\frac{1}{\frac{1}{1+R}exp\left(-E[\pi_2] + \frac{1}{2}Var(\pi_2) + \gamma^* Cov(x_2, \pi_2)\right)}exp\left(-E[\pi_2] + \frac{1}{2}Var(\pi_2)\right)\right) - r$$

$$PR^n = \ln\left(\frac{1+R}{\exp(\gamma^* Cov(x_2, \pi_2))}\right) - r$$

$$PR^n = r - \ln(\exp(\gamma^* Cov(x_2, \pi_2)) - r$$

Thus, the risk premium is defined by:

$$PR^n = -\gamma^* Cov(x_2, \pi_2)$$

Equation 38

Now back to the definition of inflation. Previously, we defined π_2 as:

$$\pi_2 = \pi_2(X_2)$$

Substituting X_2, we have:

$$\pi_2 = \pi_2 \left(\bar{X} \exp \left(\frac{x_2}{\bar{X}} \right) \right)$$

We can assume that π_2 is defined by a log-linear function of X_2. Thus, we can rewrite π_2 as simply:

$$\pi_2 = bx_2$$

Equation 39

Note that:

$$\bar{X} = 1 + \bar{D}$$

Soon:

$$\bar{X} - \bar{D} = 1$$

At the same time:

$$C_2 = X_2 - D_2$$

Using the log-quadratic expansion
exp (z)-1 = 1/2 z^2 , we have that

$$C_2 = C_2 - (\bar{X} - \bar{D})$$

replacing C_2:

$$\left[\bar{X} \exp \left(\frac{x_2}{\bar{X}} \right) - D^n \exp(\pi_2) - D^{ind} \right] - (\bar{X} - \bar{D})$$

Reorganizing:

$$\left[\bar{X} \exp \left(\frac{x_2}{\bar{X}} \right) - \bar{X} \right] + [D^n \exp(\pi_2) - D^{ind} + \bar{D}]$$

Equation 40

Note that the second term of the equation can be written as:

$$D_2 - \bar{D}$$

Moreover, we know that D_2 equals $R_2 * \bar{D}$. Where R_2 is the return on the bond portfolio. To simplify the derivation, we will use the portfolio's excess return XR_2.

Note that the risk-free rate of the economy is precisely the rate paid by the indexed bond, i.e., the excess return of the bond portfolio is, of course, equal to the excess return of the pre-fixed bonds:

$$XR_2 = XR_2^n$$

$$xr_2^n = \ln\left(XR_2^n\right)$$

Therefore, we can rewrite (40) as:

$$\left[\bar{X}\exp\left(\frac{x_2}{\bar{X}}\right) - \bar{X}\right] + \left[\bar{D}\exp\left(xr_2^d\right) - \bar{D}\right]$$

Finally, we have (41):

$$\bar{X}\left[\exp\left(\frac{x_2}{\bar{X}}\right) - 1\right] + \bar{D}\left[\exp\left(xr_2^d\right) - 1\right]$$

Equation 41

Using the log-quadratic expansion in $x_2/\bar{X}$:

$$\bar{X}\left[\frac{x_2}{\bar{X}} + \frac{1}{2}\left(\frac{x_2}{\bar{X}^2}\right)^2\right] - \bar{D}\left[\exp\left(xr_2^d\right) - 1\right]$$

$$\exp\left(xr_2^d\right) = \frac{s * \exp\left(-\pi_2\right)}{Q^n}$$

$$= \frac{s * \exp\left(-\pi_2\right)}{\exp\left(-E[\pi_2] + \frac{1}{2}Var(\pi_2) + \gamma^* Cov(x_2, \pi_2)\right)}$$

$$= s * \exp\left(-(\pi_2 - E[\pi_2]) - \frac{1}{2}Var(\pi_2) - \gamma^* Cov(x_2, \pi_2)\right)$$

$$\approx s\left[-(\pi_2 - E[\pi_2]) + \frac{1}{2}(\pi_2 - E[\pi_2])^2 - \frac{1}{2}V(\pi_2) - \gamma^* Cov(x_2, \pi_2)\right]$$

Since we know that $E[\pi_2] = bE[x_2] = 0$, we have:

$$\approx s\left[-\pi_2 + \frac{1}{2}\pi_2^2 - \frac{1}{2}V(\pi_2) - \gamma^* Cov(x_2, \pi_2)\right]$$

Substituting in (41):

$$x_2 + \frac{1}{2\bar{X}}x_2^2 - \bar{D}\left[-s\pi_2 + \frac{s}{2}\pi_2^2 - \frac{s}{2}V(\pi_2)\right] - s\gamma^* Cov(x_2, \pi_2)$$

$$x_2 + \frac{1}{2\bar{X}}x_2^2 - \bar{D}s\pi_2 - \frac{s\bar{D}}{2}\pi_2^2 + \frac{s\bar{D}}{2}V(\pi_2) + s\gamma^* Cov(x_2, \pi_2)$$

Substituting $\pi_2 = bx_2$ e $V(\pi_2) = b^2 \sigma_x^2$, we have:

$$C_2 - 1 \approx x_2(1 + \bar{D}sb) + \left(\frac{1}{2\bar{X}} - \frac{s\bar{D}b^2}{2}\right)x_2^2 + \frac{s\bar{D}b^2}{2}\sigma_x^2 + sb\gamma^*\sigma_x^2$$

Note that:

$$V(C_2 - 1) = V(C_2)$$

and therefore

$$V(C_2) \approx V\left(x_2(1 + \bar{D}sb) + \left(\frac{1}{2\bar{X}} - \frac{s\bar{D}b^2}{2}\right)x_2^2 + \frac{s\bar{D}b^2}{2}\sigma_x^2 + sb\gamma^*\sigma_x^2\right)$$

We can make a first-order approximation of this expression such that:

$$V(C_2) \approx V(x_2(1 + \bar{D}sb))$$

$$V(C_2) \approx (1 + \bar{D}sb)^2 V(x_2)$$

Consequently, the standard deviation will be:

$$\sigma_C \approx (1 + \bar{D}sb)\sigma_x$$

Similarly, the expectation of C_2 will be:

$$E[C_2] \approx E[x_2(1+\bar{D}sb)+\left(\frac{1}{2X}-\frac{s\bar{D}b^2}{2}\right)x_2^2+\frac{s\bar{D}b^2}{2}\sigma_x^2+sb\gamma^*\sigma_x^2]$$

Note that this is an expression of the form:

$$E[ax+bx^2+c]=aE[x]+b(V[x]-E[x])+c$$

And so we have:

$$E[C_2]\approx 0+\left(\frac{1}{2X}-\frac{s\bar{D}b^2}{2}\right)\sigma_x^2+\frac{s\bar{D}b^2}{2}\sigma_x^2+sb\gamma^*\sigma_x^2$$

Which brings us to:

$$E[C_2] \approx 0+\frac{1}{2X}\sigma_x^2+sb\gamma^*\sigma_x^2$$

Equation 42

Discarding the variables that are not of interest, we have:

$$E[C_2] \approx sb\gamma^*\sigma_x^2$$

Equation 43

In this way, the government's problem becomes:

$$W_n=\max_b\left\{\frac{s\bar{D}}{2}\gamma b\sigma_x^2-\frac{\gamma}{2}(1+\bar{D}sb)^2\sigma_x^2\right\}$$

Equation 44