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Measurement of Fiscal Risk based on the Financial Weakness Hypothesis Applied to the Public Sector

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Abstract

This paper proposes a model for measuring fiscal risk as required in the Fiscal Risk Annex (ARF) set forth in the Budget Guidelines Law (LDO). To this end, theoretical approaches are used, such as the Financial Weakness Hypothesis Applied to the Public Sector, risk and uncertainty, and risk management. To measure fiscal risk, the financial positions of the public entity are calculated, descriptive statistical procedures are used, and scenarios are estimated following the assumptions of the Portfolio Theory. The model proved to be pertinent to use, especially for small municipalities due to its simplicity and efficiency of its applicability. This application minimizes degrees of subjectivity in the elaboration of the ARF, is based on an economic theory and on descriptive statistical procedures of dispersion and variability. Additionally, it contributes with an empirical approach to research on this topic.

Keywords: Financial Fragility Hypothesis; Minsky; LDO; Fiscal Risk.

JEL Classification: D81; E12; E62.

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1 INTRODUCTION

The Budget Guidelines Law (LDO) has among its attributions the presentation of goals and priorities of the public administration, guide the preparation of the Annual Budget Law (LOA), provide on changes in tax legislation, among others (Brazil, 1988). Additionally, with the approval of the Fiscal Responsibility Law, the LDO has added technical provisions aimed at fiscal control, such as assisting in the balance between revenue and expenditure, criteria for limiting commitment, cost control, Fiscal Risks Annex (Brazil, 2000).

These fiscal control measures are somehow linked to the search for conditions of efficiency in the management of public resources. They demand not only control, but also measurement, since the indexes of a certain expense or even the condition of allocation, indebtedness, finances, education, have the advantage of bringing an almost instantaneous informational content for the inference of effectiveness and efficiency in the conduct of public management.

However, in the budget execution process, government failures, added to macroeconomic imbalances, can lead or contribute to situations of fragility, specifically financial fragility in government management. To better understand how such restrictions can affect the public sector, theoretical contributions such as those of Hyman Minsky help explain the behavior of economic agents and their consequences.

Minsky developed the Financial Fragility Hypothesis in which he relates economic instabilities to the leverage process of firms, that is, the use of third-party capital to finance their assets. According to Minsky, depending on the way firms leverage and manage their cash, three types of financial positions are evidenced: Hedge, Speculative, and Ponzi (Minsky, 1977, 1986, 1992). These positions denote qualities of security, restriction, and fragility.

Minsky's theoretical assumptions were applied to the public sector, providing the emergence of a stream of studies that analyzes the Financial Fragility Hypothesis Applied to the Public Sector. Among works developed by this current, Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 2020) can be mentioned. These authors developed the Financial Fragility Index Applied to the Public Sector.

The Hypothesis of Financial Fragility Applied to the Public Sector (HFFSP) can help in understanding economic policies implemented by governments, as well as highlighting financial positions of security or fragility. Such attributes can also contribute to the analysis of the causes of financial fragility of governments, as well as to the estimation of fiscal risks.

The management of fiscal risks provides a greater and better understanding of what kind of risks the organization or even the government is exposed to. In addition, it creates conditions for measuring risk, estimating the degree of tolerance of public accounts, strategies for coping, mitigation measures, and monitoring (STN, 2018).

The STN (2019) in its Fiscal Statements Manual, presents a ready-made template of the Fiscal Risk Annex (ARF). However, the detailed procedures for estimating fiscal risk are not presented. Regarding studies on the estimation of fiscal risk, Azevedo et al. (2019), identified few studies on the subject and that in their analysis, they concluded that in certain cases the availability of the ARF as an integral part of the LDO, is done more in a ceremonial way, i.e., to comply with legal determinations, than as a tool in helping and preventing risk situations.

Because of this gap, this study proposed to present a method to assess and estimate fiscal risks, precisely those related to the budget and the LDO. Such intent is justified due to the deficiencies presented above, both in the practical aspect for the fulfillment of this obligation, and to contribute to empirical studies related to this topic. It also brings implications as to suggest a method based on theoretical literature about risks, applied to the budget and public finances. Such attributes may provide the public finance professional with a pragmatic, yet efficient alternative for measuring and estimating fiscal risk.

The contribution of this paper is in integrating the economic theory developed by Minsky regarding financial instabilities and their positions, added to the HFFSP developed by Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 2020) regarding the bases for risk measurement. Such an approach is innovative and no other works are known to have performed such a procedure.

This study is divided into 5 parts. It begins with an introduction contextualizing the theme, motivations for the study, and evidenced gaps. The theoretical framework presents the theoretical approaches on the hypothesis of financial fragility, risk and uncertainty, risk management and its application in the public sector, the Portfolio Theory as a basis for the measurement of Fiscal Risk, the LRF, the LDO and the ARF, ending with considerations on HFFSP applications for the estimation of fiscal risks. In the methodology part, the procedures proposed for the estimation of fiscal risks based on the indexes of financial fragility applied to the public sector were described, plus statistical procedures and assumptions of the Theory of Portfolios. In the Results of the Proposed Model part, the procedures for measuring fiscal risk are presented in detail, using real data from a municipality. It ends with the final considerations, in which the final comments, synthesis of the findings and defense of the model's relevance are presented.

2 THEORETICAL FRAMEWORK

2.1 Hyman Minsky's Financial Fragility Hypothesis - an overview

Economist Hyman Minsky was responsible for complementing the Keynesian approaches by relating firms' cash flow expectations to periods of instability and economic crises (Keen, 1995). In his analyses, Minsky (1977, 1992) links the process of leveraging investments with agents' expecta-

tions, rising interest rates, ability to pay off debt, changing drivers of economic viability evidences, financial positions, and their meanings.

Minsky (2013) when analyzing the dynamics of capitalist economies, considers that the greatest of its defects is to be unstable. This instability, for the author, is inherent to the economy. The process of economic instability is a consequence of a financial system and its complexity in constant evolution, seeking at every moment new forms of financial operations that offer greater gains. Even though Minsky's work was written between the 1970s and the early 1990s, his considerations are becoming more and more current and show us a horizon of considerations and actions to be taken. Considerations regarding the recognition of economic cycles, the role of institutions, the dynamism of the regulatory sector necessary and agent in countercyclical actions. (Minsky, 1992; 2013).

Resulting from these contexts, Minsky (1977, 1986, 1992) proposed the Financial Fragility Hypothesis, in which he states that instabilities and crises arise from positions of financial fragility incurred by firms, resulting from the process of leveraging resources for investment. This leverage process, that is, the use of third-party capital in the capital structure of companies, is determined based on the expectations of growth felt by these economic agents who take on debt to increase their production and thereby generate higher profits.

This process of formation of these financial liabilities of companies occurs from periods of economic growth. However, before these periods of optimism, economic agents, in general terms in the economy, present solid balance sheets, having the price of assets established in a conservative manner and the debts do not exceed the value of their assets (Bahry and Gabriel, 2010). In this phase of the financial cycle, the economy is doing well, with profit rates higher than interest rates, and economic agents are conservative in their indebtedness. This conservatism is due to a certain memory of these agents to recent periods of financial difficulty, low availability of credit, high interest rates, and banks with credit restrictions. However, as the economy grows, asset prices rise and banks, as well as investors, become less conservative in lending and borrowing due to the prospect of increased future cash flows (Keen, 1995).

However, after this period, growth comes to a halt and indebted companies must honor their financial and operational obligations. The cash flow of companies, this in a generalized way in the economy, not being realized in the expected manner to honor their obligations, can lead to moments of panic in financial management, in addition to deflation of assets, if the economy stalls in a period of recession, and may lead to depression. Thus, from the confrontation between cash outflows and inflows in the realization of these expected flows, also called quasi-rents, three types of positions are presented: Hedge, Speculative and Ponzi (Minsky, 1985, 1986, 1992, 2013).

The distinction between the Hedge and Speculative position is that the former occurs when cash flows from the firms' operations are expected to be sufficient to cover the debt commitments made (Minsky, 1977). In this description, Minsky makes no distinction whether the debts are financial

and/or operational. The Speculative position would occur when these same cash flows from operations (in this case, the operational ones, that is, those arising from the economic activity of the firms), would not be sufficient to meet the payment obligations, and the economic agent would must refinance the payment of the principal, that is, of the amortization (Minsky, 1977).

If profits are analyzed, if they decrease, the Hedge units become Speculative units. If this decrease persists, speculative units become Ponzi units. These induced transformations of the financial structure, leads to falling capital asset prices which in turn leads to declining investments (Taylor and O'Connell, 1985).

Minsky (1992) in a later moment, describing the financial units, relates them to the payment capacity and the structure of liabilities for the acquisition of assets. Hedge financial units will be those that hold a higher proportion of net worth in relation to liabilities. A Speculative unit will be the one that to honor the payment of the principal (amortization) takes new loans, rolling its debt. The Ponzi units are those that the flows of the operations are not enough to honor the interest and the principal. However, in periods of prosperity, the units move from safer positions to riskier ones, that is, from Hedge to Speculative and Ponzi positions. This shift is motivated by the bet on obtaining higher future cash flows. This increase in risk, when widespread, can lead to unstable situations in the economy (Minsky, 1992).

Thus, financial fragility stems from the widespread practice of firms taking on debt to purchase capital assets to finance their production, one of the characteristics of capitalist economies with sophisticated capital asset financing operations (Minsky, 1977; 1992; Foley, 2003). Another explanation is that firms fail to earn rates of return higher than interest rates, making many investments unviable. Such situations lead enterprises that were Hedge to conditions of speculative, and speculative in Ponzi (Keen, 1995).

Thus, in the growth phase of the economy, asset prices rise, as does the share of the debt burden, as agents bet on the future increase of their cash flows. However, this practice leads firms to levels of debt that are unbalanced in relation to their assets, causing asset prices to fall because of the economic slowdown (Minsky, 1977, 1986, 1992).

In this context, the studies on financial fragility address aspects not only related to the positions arising from the indebtedness that form capital structures, but also address the relationship between the growth rate of the economy and the interest rate resulting from monetary policy decisions.

In periods of crisis, Ponzi positions are widespread and bring out panics in the market. Therefore, interest rates increase, investment intentions are reduced, and profit rates fall. In this position, agents to fulfill their obligations must sell their assets, however, as their price decreases, due to the imbalance of supply and demand, such actions lead to an increase in losses and a decrease in the net worth of the companies (Taylor and O'Connell, 1985).

In the considerations of Minsky (2013) on the process of change from Hedge to speculative

positions, is the overconfidence of entrepreneurs, in which there are no financial or credit problems in the long term, they are inclined to take on debt to invest in capital assets. Even, according to Minsky, using short term credit to finance long term assets. For Minsky, such an environment can bring risk even to entities in a hedge position. On the other hand, rises in interest rates, difficulty in revenue realizations, increased costs, as well as influences on government policy behavior, both in its monetary and fiscal aspects can lead a Speculative position to Ponzi (Taylor and O’Connell, 1985; Foley, 2003, Minsky, 2013; Gonzalez, 2017).

This consideration of realized cash flows for the determination of financial fragility positions are evidenced in works such as those by Foley (2003), Torres Filho, Martins and Miaguti (2019), Davis, De Souza and Hernandez (2019). For these authors the categorization of positions is determined by the relationship between cash flow from operations and debt service. Hedge positions are derived from cash flows sufficient to cover the firm’s operating moves plus interest and principal on financing. In speculative situations firms roll over the principal of the financings, i.e. the repayments and in Ponzi positions firms must refinance even the interest (Davis, De Souza and Hernandez, 2019). These positions are summarized in Table 1 - Definitions of financial positions.

Although Davis, De Souza and Hernandez (2019) use the term Sources of cash to define the financial position, along the lines of the Financial Weakness Hypothesis, this term denotes not only cash inflows, but also the deduction of expenses/costs. Thus, operating income should be considered as Sources of cash. After the deduction of the debt service consisting of interest and amortization, if there is a positive balance, it can be considered as a Hedge position. The Speculative position will be determined when the resources are enough to cover the operational result of the cash sources and the interests. In the Ponzi position, the flows do not cover even the interest.

Table 1 – Definitions of financial positions

Position	Position Definition
Hedge	$[\text{Cash sources} - \text{Interest} - \text{Amortization}] > 0$
Especulativa	$[\text{Cash Flows} - \text{Interest}] > 0$ and $[\text{Cash Fountains} - \text{Interest} - \text{Amortization}] < 0$
Ponzi	$[\text{Cash Flows} - \text{Interest}] < 0$

Source: Adapted from Davis, De Souza, and Hernandez (2019).

Another assessment of financial positions is based on the relationship of the variables Revenues (R), Loans (D), Investments (I) and Debt Service (V). This valuation is proposed by Foley (2003) when describing financial positions and claiming that Minsky’s analysis is based on cash flows structured through accounting categories.

In this evaluation the company's equity is equal to the difference between the value of its assets and the value of its debts. Thus, according to Foley (2003), in a Hedge position the value of net revenues can cover debt service and investment. In speculative positions, firms generate investment opportunities that exceed their ability to finance themselves. In a firm in a Ponzi position, there will be a need to go into debt to pay part of the debt service. In this flow, when evaluating other aspects, such as trust, if creditors do not feel confident about the firm's ability to generate enough revenue (cash flows) to honor the contract, they will not grant new loans to roll over the debts. This breakdown in cash flow can lead companies into fragile conditions and in more drastic situations, bankruptcy. Under these conditions, with low growth rates and low profits, the economy will engage in widespread Ponzi positions (FOLEY, 2003)

In the study by Torres Filho, Martins, and Miaguti (2019) also used accounting variables to highlight the financial positions of Brazilian electricity distribution companies. The variables EBITDA, Financial Liabilities, and Short-term Debt Stock were used. The variable EBITDA represents Earnings Before Interest, Taxes, Depreciation and Amortization. The acronym is in English. The Financial Obligations represent the current interest, and the Short-term Debt Stock represents the financing obligations due within 12 months after the date of the firms' Balance Sheet. From this model one can understand that the authors, to determine the positions, made use of realized income and expense flows, that is, recognized in the financial statements under the accrual basis of accounting, income recognized when realized and expenses when incurred.

It is possible to see, both in the concepts and relationships presented by Minsky, and in the other referenced studies, that there are two distinct cash flows for evidencing fragility positions. The first is the estimated cash flow, that is, when economic agents analyze the feasibility of investments or even bet on increases in future cash flows, they rely on these estimates as a source for the analysis of investment feasibility. However, they may be attracted to increase their risk appetite, even though they do not have secure financial structures.

The second flow refers to what is realized. According to works that have investigated financial fragility and even Minsky's considerations, to determine the positions, Hedge, Speculative and Ponzi, it is necessary to use the realized cash flow, that is, the incurred one. Not in the projections, but the effective one.

This interpretation regarding the use of realized cash flows, or even costs incurred and revenues realized, when the approach is accounting, is reinforced when studying the expansion of the possibilities of using this theory. Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 2020) extended applications of the Financial Weakness Hypothesis, justifying its pertinence for analysis and evidencing financial positions in the public sector. This aspect is what is presented below.

2.2 The Financial Weakness Hypothesis Applied to the Public Sector

These concepts and inferences of this theory as to the financial instabilities and their causes, as well as the evidencing and ascertainment form, can be used for understanding and measuring situations of security or difficulties in the public finance management. In addition, Terra and Ferrari Filho (2020) point out that this theoretical basis can be used to verify cyclical or counter-cyclical policies adopted in the Public Sector. For the public sector, this theoretical current is named as Financial Fra-

gility Hypothesis Applied to the Public Sector (HFFSP).

It should be noted that according to Minsky (1977, 1986, 1992, 2013) the conditions of how firms' debt is financed, their estimated and realized cash flows, and their respective financial positions, help explain financial crises. However, in the public sector the motivations for indebtedness are different from the private sector.

In the public sector, recourse to the debt procedure may be a result of strategies, such as choosing between increasing taxes or financing expenditures through public debt (Musgrave, 1973; Barro, 1979). The constitution of public debt may have the function of providing an intergenerational balance in its financing. This affirmation is due to Intergenerational Equity - given that the enjoyment of public assets acquired or built will take place in a certain period, generally in the long term. Thus, if different generations use these assets, there would be fiscal justice in dividing the cost of this financing over this time that will span more than one generation (Musgrave, 1973).

With due caveats, in the budget execution, the expenses with public debt service may have a relevant impact in determining its fragility condition, or not, as well as the evolution of public expenses in a disproportionate manner to revenues. Decreases in tax collection, resulting from changes in the economic cycle or uncompensated increases in expenditures, can lead to instability in the Public Sector.

The Financial Fragility Hypothesis proposed by Minsky, can be used for the analysis of an economy and governments. The financial fragility of governments will be ascertained depending on the Hedge, speculative or Ponzi positions that are ascertained (Taylor and O'Connell, 1985; Minsky, 1992; De Paula and Alves Jr, 2000; Foley, 2003; Gonzalez, 2017). However, despite the relevance and importance of this topic, few studies on financial fragility applied to the Public Sector are evidenced, in the light of Minskyan theory.

Argitis and Nikolaidi (2014, 2016) conducted studies on the financial fragility of the Greek government ex ante and ex post to the crisis in that country. There were two studies. The first in 2009 and the second in 2016. They evidenced indices that corroborated the situation of governmental financial fragility. In addition, they evaluated the permanence of the government in fragile situations, even after fiscal adjustments. The conclusion pointed out by the authors was that despite the Greek government implementing fiscal austerity policies, it did not produce relevant results for the change in financial fragility positions. On the contrary, it caused such an adverse financial position that they named it ultra-ponzi.

In Brazil, studies have advanced by incorporating the theory of financial fragility in the public sector. Picolotto (2016) proposed an index that evaluated government investments and their relationship with the public budget. He titled it the Minskyan Index of Government (IMG). Other studies, such as those by Terra and Ferrari Filho (2011, 2020) evaluated situations of financial fragility of the Brazilian federal government in the period from 2000 to 2016. These studies point to the relevance, or

at least, the possibility of using the Financial Fragility Hypothesis, with the proper adaptations, applied to the public sector.

In this effort and intension of using this theory to analyze governments' financial positions, Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 2020) developed the Financial Fragility Index Applied to the Public Sector. The characteristics, possibilities, advantages and applications of this index is what is presented in the next item.

2.3 The Financial Position Applied to the Public Sector

Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 2020) developed the Financial Fragility Index Applied to the Public Sector (IFFSP). Through this index it is possible to ascertain and analyze the financial fragility of governments, in any of the federal, state and municipal spheres. To construct this index, Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 2020) used the accounts of Revenue and Current and Capital Expenditures, presented in the Budget Balance Sheet.

The construction of the indices and their respective formulas is based on the accounts presented in the Budget Balance Sheet according to the Brazilian tax legislation, especially according to the structure of Law 4,320/1964. The revenues, for this model, are determined by the sum of current revenues and financial revenues (Terra and Ferrari Filho, 2020).

$$R_{fg} = R_{cfg} + R_{ffg} \quad (1)$$

Where,

R_{fg} represents the sum of current plus financial revenues;

R_{cfg} represents the current revenues; and R_{ffg} represents the financial revenues

As clarified by Terra and Ferrari-Filho (2020), the letters fg refer to the Federal Government. This is because their analysis was based on the Federal Budget Balance.

Another point that requires attention and clarification is that according to the structure of the Budget Balance, Current Revenues represent a synthetic account that includes, among other revenues, the financial ones. In the case of expenses, as will be seen below, Current Expenses include, among other accounts, the Interest on Public Debt, which has the function of showing the financial expenses considering the financial fragility theory. Thus, in order not to have replication both in the calculation of revenues and expenses, in the current revenues, for this model, financial revenues were excluded. Likewise, in current expenses, interest expenses were excluded.

Another important point in this method is that Capital Revenues and Expenses are not used, except for amortizations. This is because the calculation of the financial positions and the respective financial fragility index show the capacity of the public entity to generate enough revenue to honor the commitments of maintaining government activities (current expenses), the interest on financing

and the expenses with debt amortization will generate to leverage the investments of the governments, besides being an instrument of anti-cyclical policies.

In addition, the incorporation of Capital Revenues for the calculation of the financial positions would harm the evidence of insufficient funds. An example would be the situation of an entity that rolls over its debts. If there were the recognition of Capital Revenue, the need for financing due to negative cash would not be captured. It is worth remembering that in situations of balanced public finances, Current Revenue should be sufficient to honor government spending for maintaining state activities and debt service (BROWN-COLLIER and COLLIER, 1995; Lima and Diniz, 2016).

Then current and financial expenses are calculated (Ferrari Filho, Terra and Conceição, 2010; Terra and Ferrari filho, 2020):

$$E_{fg} = E_{cfg} + E_{ffg} \quad (2)$$

Where,

E_{fg} represents the sum of current plus financial expenses; E_{cfg} current expenses;

E_{ffg} , are the financial expenses, and these expenses are made up of the sum of amortization expenses plus interest. Thus,

$$E_{ffg} = A_{fg} + i_{fg} \quad (3)$$

Where,

A_{fg} represents the amortization expenses and

i_{fg} represents the interest expenses.

The balanced budget will be verified if the sum of current and financial revenues equals the sum of current expenses, interest and amortization. It should be noted that financial revenues and interest on debt were excluded from Current Revenues and Current Expenses, as informed above. Thus, according to Terra and Ferrari-Filho (2020) the following equation will form the equilibrium of the Budget Balance:

$$R_{cfg} + R_{ffg} = E_{cfg} + E_{ffg} \quad (4)$$

According to the accounts and formulas presented, the Public Sector Financial Fragility Index is calculated, denoting the three types of positions:

Table 1 - Indexes of the financial positions applied to the public sector

Índice apurado	Posição
$\frac{(R_{cfg} + R_{ffg}) - E_{cfg}}{A_{fg} + i_{fg}} > 1$	Posição de financiamento <i>hedge</i>
$\frac{(R_{cfg} + R_{ffg}) - E_{cfg}}{A_{fg} + i_{fg}} > 0 < 1$	Posição de financiamento especulativa
$\frac{(R_{cfg} + R_{ffg}) - E_{cfg}}{A_{fg} + i_{fg}} < 0$	Posição de financiamento <i>ponzi</i>

Source: Adapted from Terra and Ferrari-Filho (2020).

Source: Adapted from Terra and Ferrari-Filho (2020).

According to Table 1 - Indices of the financial positions applied to the public sector - depending on the result the positions are presented. Indexes greater than or equal to 1, a Hedged financial position is determined. In the public sector, the Hedge position is related to the conditions in which revenues cover the expenses of maintaining the government activity, that is, Current Expenses, plus the public debt service. The speculative position denotes a situation in which resources cover current expenses and interest. However, the repayments need to be refinanced. The Ponzi position represents conditions in which the resources are insufficient to cover even part of the Current Expenses. There is no capacity to honor the interest and amortization.

Besides the informational capacity of these indexes to classify the financial positions of public entities, Terra and Ferrari Filho (2020) point out that through them one can evaluate how fiscal policies are being conducted, including whether pro- or counter-cyclical, according to Keynesian theory and Minsky's observations.

Another aspect in which the disclosure of financial positions can collaborate is with respect to signaling to investors the government's ability to honor its commitments. If the government raises funds from third parties, the evidence of Hedge positions generates confidence in the market. Furthermore, when economic agents have the signal that the government can honor its commitments, they accept to incur lower interest rates, given the risk of default that is not remote. On the other hand, usual evidence of Ponzi positions leads investors or creditor financial institutions to embed higher risks in their risk and return analyses, thus demanding higher remuneration rates as a form of compensation for the increased risk (Terra and Ferrari Filho, 2020). These are even institutional aspects that cannot be disregarded.

Other points that value the use of these indexes deserve to be highlighted. Terra and Ferrari Filho (2020) listed the following points: a) they indicate conditions of solidity or fragility of the governments' accounts; b) these indexes focus on confrontation flows between revenues, expenses and debt service and not on analyses about debt sustainability; c) the use of the Budget Balance accounts for the calculation of financial positions, goes beyond the calculation of the Primary and Nominal Result; 4) it is possible to make comparisons and degrees of financial security or fragility; 5°) they show the type of fiscal policy adopted by the government in the period under analysis.

As for the indexes of the financial position applied to the public sector going beyond the calculation of the Primary and Nominal Result, the arguments arise from the fact that they can verify the fiscal solvency of the public entity. This is because their formulas aggregate the debt service, i.e., both interest and amortizations. According to Albuquerque, Medeiros, and Feijó (2008, p.72), the Primary Result evaluates "the sustainability of fiscal policy, that is, the capacity of governments to generate enough revenue to pay their usual bills [...] without compromising the capacity to manage the existing debt. The Nominal Result, by the below-the-line method, measures the Public Sector Financing Requirement (PSFN). However, according to the above-the-line method, used by the National Treasury Secretariat (Secretaria do Tesouro Nacional - STN), the Nominal Result is calculated by the Primary Result minus the interest of the public debt (GIAMBIAGI e ALÉM, 2008; ALBUQUERQUE, MEDEIROS E FEIJÓ, 2008).

Although these results, the Primary and the Nominal, depending on the method, show the debt sustainability before and after adding the interests, they do not consider the deduction of the amortization variable. For showing the fiscal solvency capacity of public entities, debt service presents a relevant participation in the composition of the public budget. An entity may present a positive Primary Result, but when the interest on the debt is deducted, it may have a nominal deficit. Adding the amortization expense, which is a budget expense, the fiscal solvency capacity can be even more compromised. On the other hand, governments that present a Hedge financial position can honor the government's expenses for maintaining state activity, plus interest and amortizations, denoting balance and fiscal security.

An analysis of the evolution of the Brazilian public debt, the ascertainment of financial positions and the evidencing of security conditions or fiscal insolvency, can be an example. Terra and Ferrari Filho (2020) evidenced that from 2000 to 2013, the Federal Government presented a speculative financial position, despite presenting a surplus Primary Result. In 2014 and 2015, the final period of the analysis, it presented Ponzi financial position and primary deficit. However, despite presenting a surplus Primary Surplus Result from 2000 to 2013, the national public debt grew due to successive nominal deficit results resulting from the recognition of interest expenses. The increase in public debt was to finance this interest. Besides these deficiencies in the information, with respect to budget solvency, although the calculation of the Nominal Result recognizes the effect of interest, it does not add the impact of amortizations that influence the cash and payment capacity of public entities.

This informational constraint can be exemplified by the steady growth of current expenditures and in the evolution of Brazil's public debt in the period from 2000 to 2015, being aggravated from 2014 onwards.

2.4 The concept of budget solvency as a complement to the study of financial positions

The condition of an insolvent entity derives from its inability to pay its obligations when due (Gitman, 1997). It is linked, concomitantly, to the loss of profitability and the insufficiency of cash flows to honor the commitments assumed by the organization (Wruck, 1990; Castro Júnior, 2003).

In the public sector, the solvency issue derives from the ability of governments to generate primary surpluses that support present obligations (ARTIS and MARCELINO, 2000). Its verification can also be established by comparing the discounted present value of current and future primary expenses with the discounted present value of current and future revenues, net of any debt (Goldfajn and Guardia, 2003).

In this context, the budget solvency represents the budget balance, manifested by the abilities to produce financial reserves to meet emergencies and present sufficient liquidity to cover current expenses (Lima and Diniz, 2016). The balanced budget is linked to the budget solvency that carries within itself the ability of the government entity to generate sufficient revenues to cover budget expenses, without incurring in deficits (Groves and Valente, 2003). Thus, the budget surplus will be an indication of the governments' ability to generate sufficient revenues to meet public spending (Lima and Diniz, 2016).

Thus, the choice of budget solvency/insolvency as a concept and complement for understanding the pertinence of using financial positions applied to the public sector in the analysis of government accounts, is made due to the following points:

1º) the model proposed by Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 2020) uses variables obtained from the Budget Balance Sheet;

2º) Although the model proposed by Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 20120) uses the term "financial position", as well as Minsky's works use the concepts of estimated and realized cash flows, for the public sector adaptations were made. Besides obtaining the data from the Budgetary Balance Sheet, it is considered that the recognition of revenues and expenses only when they are received and paid may generate distortion of information. In the Budgetary Balance, mainly the expenses are recognized according to their commitment, as per article 35 of Law 4.320/1964. Even if the expense is not paid, the government entity's obligation to pay is recognized (BRASIL, 1964).

The information obtained from the Cash Flow Statement, or the Balance Sheet, derives from operations in which cash inflows and outflows were recognized. However, if the government entity recognizes an expense or has issued a commitment but not paid the expense, that expense will not be

part of the realized cash flows. If the government entity does not liquidate the interest expense and the amortization payment, posting them in Accrued Liabilities, they would not configure the calculation by the cash flow, showing a false situation of positive cash flows.

Because of these arguments, it is understood that the Budget Balance, or even the budget execution spreadsheets published by the STN, are adequate for the ascertainment of financial positions in the public sector because they show conditions of cash flow insufficiency, debt rollover, imbalance between current revenues and expenses, among other points.

2.5 The Public Sector Financial Position as a basis for measuring fiscal risk

The LRF institutes the Fiscal Risk Annex (ARF) to be presented when the Budget Guidelines Law (LDO) is prepared. Due to this obligation, it is considered that the Financial Weakness Hypothesis Applied to the Public Sector can be a tool to help prepare this annex.

For this purpose, an understanding of the relationships of risk and uncertainty, risk management, and the fundamentals of Portfolio Theory is necessary.

2.5.1 Risk, Uncertainty and Risk Management

The issue of risk is related to the probability of occurrence of an undesired effect or event, i.e., a possibility of an adverse situation on an expected outcome (Vaughan and Vaughan, 2014). Risk arises from the real world and takes place because of varied circumstances that may offer the likelihood of loss.

This adverse situation may arise from the prospect of a tax loss, losses in the production process, in investments, among other situations. According to Aven and Renn (2009) and Spikin (2013) these adversities are divided into two categories: the risk measured through probabilities and expected values and the risk estimated through uncertain events and consequences.

The risk estimated through probabilities and expected values is measured according to scenarios and their respective probabilities of occurrence. The risk estimated through uncertain events and consequences is guided by the uncertainty that is a condition that hinders both inferences and measurements of probabilities and adverse effects (Aven and Renn, 2009).

The conditions of uncertainty are different from those of risk. In uncertainty, the scenarios and probabilities of occurrence of events cannot be estimated with any degree of confidence (Skipin, 2013). On the other hand, in risk conditions, despite the lack of certainty of occurrence in the future, it can be estimated by probabilities. In uncertainty conditions, one does not even know the possible scenarios and probabilities (Knight, 1921 Apud Hermans; Fox and Van Asselt, 2012). Thus, uncertainty arises from the lack of knowledge about what may or may not happen in the future, inhibiting even the estimation of probabilities (Vaughan and Vaughan, 2014).

While in environments of uncertainty the estimation of scenarios and probabilities of occurrence of a given event are impaired, in risk conditions, this can be managed. This management will depend on the determination of certain conditions and information (Spikin, 2013).

The first studies that sought to evidence and estimate risks were undertaken by Knight and based on positivist approaches. This approach, considered classical, is formed by two bases. The first is the identification and evaluation of risk. The second is concerned with control measures, i.e., risk management (Hermans; Fox and Van Asselt, 2012).

Risk management is made up of a set of processes that monitor both positive and negative factors and how they may affect the organization in certain activities or even compromise its continuity (Van Staveren, 2009; Spikin, 2013).

Because of the need and relevance of monitoring risks and the consequences they may cause to organizations, several entities that study, support, disseminate, advise, among other actions related to risk management were created. These include the Association of Insurance and Risk Managers (AIRMIC), the National Forum of Risk Management in Public Sector and the Institute of Risk Management (IRM) (SPIKIN, 2013). Related to the public sector specifically, the United States Government Accountability Office (GAO), publishes several documents on risk management, addressing natural, population health, fiscal, and even terrorist risks (GAO, 2005).

In the public sector, risk management arises from different adverse situations, such as diseases, pandemics, natural disasters, terrorism, among others. Likewise, there is risk in budget management, i.e., fiscal risk (GAO, 2005). The pertinence or even necessity of measuring and managing fiscal risk is corroborated by the World Bank, which has published works on this topic. This institution recognizes the benefits of managing fiscal risks and that these are related to adverse conditions in the budget execution, hidden contingent liabilities and other unforeseen fiscal risks (Brix and Schick, 2002).

2.5.2 Portfolio Theory as a basis for measuring Fiscal Risk

The basis of the Portfolio Theory employed in financial risk management is the diversification of risk presented by Markowitz in his 1959 study Portfolio Selection. Markowitz incorporated statistical elements into finance to evaluate risk-related problems in investments.

The evidencing of risk and that this can be diversified through an investment portfolio are the fundamentals of the Portfolio Theory. Markowitz considered that assets carry their own risk, also called idiosyncratic risk, and a systematic risk that cannot be diversified through a portfolio. The measurement of risk is done through statistical measures of variability and association such as variance, standard deviation, coefficient of variation and covariance (Markowitz, 1991; Sweeney; Williams and Anderson, 2013).

The risk measures, in the model proposed by Markowitz, are the variance and the standard deviation. Such measures show the variability, the dispersion of possible expected prices. This variability,

variance and standard deviation, is what will denote the risk measures (Assaf Neto, 2006).

Likewise, fiscal risks can be estimated taking into account budget forecasts, as well as possible variability. As will be detailed in the methodological procedures and developed in the data analysis, the suggestion of a model to measure fiscal risk is based on the Portfolio Theory and the Financial Fragility Hypothesis Applied to the Public Sector. This model is based on the contributions of Markowitz (1991), Ripamonti et al. (2001), Ramaswami (2002), Assaf Neto (2006), Minsky (2013) and Terra and Ferrari Filho (2020).

2.5.3 The LRF, the LDO and the Fiscal Risks Annex

Brazil in the late 1980s and part of the 1990s presented a fiscal imbalance that led to the need to implement tighter budget constraints (Macedo and Corbari, 2009). In addition to these issues, there were influences of management practices advocated and disseminated by the New Public Management current, as well as regulations, aiming to effect greater control of public spending. This context contributed to the approval of the Fiscal Responsibility Law - LRF (Pereira and Biondini, 2014).

The LRF can be considered a watershed in the Brazilian public finances, having among the innovations brought, additions to the preparation of the LDO, as well as new instruments of control and disclosure of information. In its control instruments, the Summary Report of Budget Execution (RREO) and the Fiscal Management Report (RGF) stand out, in addition to determining sanctions to the public manager for noncompliance with the approved rules (Pereira and Biondini, 2014; Macedo and Corbari, 2009; Brazil, 2000).

It is worth mentioning that the LDO was created with the approval of the 1988 Constitution, because of the dismemberment of the budget process through 03 instruments, namely: The Multi-Year Plan (PPA), the Budget Guidelines Law (LDO) and the Annual Budget Law (LOA). Regarding the LDO, the Constitution of 1988 gave it the status of being a link between the PPA and the LOA, an instrument to enable government programs, ascertainment and evidencing of goals, risks and priorities, changes in tax legislation, authorization for granting advantages, creation of public positions, among others (Couto, 2018; Giacomoni, 2017; Albuquerque; Medeiros; Feijó, 2008).

The LRF gave the LDO attributes that determined criteria for limiting commitments, financial programming, establishment of targets and fiscal risks, among others (Giacomoni, 2017; Albuquerque; Medeiros; Feijó, 2008). It is worth remembering that until then, the preparation of procedures aimed at measuring and evidencing the fiscal targets was not institutionalized, as well as the fiscal risks that could change the execution of the annual budget forecast (Azevedo et al., 2019; Pereira and Biondini, 2014).

In this context and in the search for greater transparency, as well as reducing uncertainties in the management of the budget and public resources, the LRF establishes the Fiscal Risk Annex (ARF) as an integral part of the LDO. According to the STN, the management of fiscal risks is composed of

6 functions, which are the identification and exposure of risk, the measurement, estimation of the degree of tolerance of public accounts, strategic decisions to address the risk, implementation of mitigation measures, and finally the continuous monitoring. The annex of fiscal risks is regulated by the STN, being a model to be followed by government entities, whether municipal, state, and the Union (STN, 2018).

The ARF consists of two basic tables, in which contingent liabilities and other fiscal liabilities risks will be shown. Contingent liabilities express government commitments, legal or contractual, which depend on the occurrence of one or more future events to generate cash outflows. The other fiscal liability risks refer to budgetary risks, i.e., the possibility of fiscal obligations suffering negative impacts such as revenue frustration or under realization, assumption of expenses initially not set or under budgeted (Brazil, 2000; STN, 2018).

Despite the relevance and applicability of the ARF and its power to minimize risks and uncertainties in public budget management, according to Azevedo et al. (2019) there are few studies that discuss empirical evidence regarding the use of fiscal risk management data presented by this fiscal control instrument. Moreover, according to the authors, the informational content of fiscal risk reports is low and among the probable reasons for this is the little attention from external control (from the Courts of Accounts and Legislative) and the linking of the ARF to the dynamics of the annual budget cycle, which already faces discredit and high inaccuracy.

In addition to this constraint on the informational content of the ARF and its effectiveness, Ramaswami (2002) points out that it is not an easy task to measure or estimate fiscal risk for several reasons. First because it is a risk that is the result of both explicit and implicit commitments, hence the difficulty of finding a realistic measure. Second, because there are no organized insurance markets for transferring the risks of these types of processes. Added to this, and this is also a Brazilian reality, is the indifference to these types of budgetary risks, since in many cases, fiscal insolvencies of subnational entities are absorbed by the central government (Ramaswami, 2002). This belief in the help of higher governmental instances may facilitate favorable conditions for moral hazard findings.

Due to the little information and suggestions of methods for estimating fiscal risks, as described in this topic, a method will be proposed to fill this gap. Its description, as well as the presentation of its results, will be exposed in the methodological procedures and results analysis part.

2.5.4 HFFSP as a tool to help prepare the Fiscal Risk Annex

There are no restrictions for studies on risk management in the public sector. One example is the use of assumptions or even models derived from the Portfolio Theory, with the proper adjustments, to be employed in the public sector, specifically to measure budgetary or fiscal risk (RAMASWAMI, 2002).

The HFFSP, due to the informational content it carries, can serve as a tool for evidencing and even preventing fiscal crises. The calculated financial position, which is the basis for the Financial

Fragility Index Applied to the Public Sector, developed by Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 2020), can provide conditions and empirical bases for estimating fiscal risks.

Budget balance is linked to the Hedge position and budget solvency stems from the governments' ability to generate revenues that support budget expenditures, without incurring in deficits (Groves and Valente, 2003). Conversely, fiscal risk will be the condition of not supporting current and capital expenditures, as measured by the financial position. Thus, estimates of speculative and Ponzi positions denote the fiscal risk attributed to a given government, according to budgetary conditions and estimated scenarios.

Thus, considering the information that can be generated and inferred by the financial fragility indexes applied to the public sector, added to the risk measurement tools offered by the Portfolio Theory, can be elements to predict future crises or fragilities and collaborate to the elaboration of the ARF.

The logic of this combination of theories and empirical procedure is that, depending on the financial position ascertained in the last fiscal year, considering the scenarios for the following year, complemented by probabilities to be estimated, the public entity will present greater or lesser risk of more fragile financial positions. This estimate will enable the public manager to better plan for possible fiscal constraints and prepare the ARF based on statistics and economic theory.

3 METHODOLOGICAL PROCEDURES

To determine the tax risk, the approach will be quantitative, since descriptive statistical procedures will be used by means of variability and dispersion measures. As to its objectives, this research can be considered exploratory.

Exploratory studies, according to Kothari (2004), are researches that aim to gain greater familiarity with a given phenomenon, or even, gain greater insights into the subject. Beuren (2010) complements, clarifying that this type of research occurs when there is not much knowledge about the theme addressed, as well as when one seeks a deepening of concepts that are still preliminary in the phenomenon studied.

3.1 The Variables used

The variables used to estimate the fiscal risk were obtained from the budget income and expense statements. These statements are available on the STN website, through SICONFI, at: <https://siconfi.tesouro.gov.br/siconfi/index.jsf>. SICONFI provides accounting, financial and fiscal statistics information from the municipalities, states, Federal District and Union.

Specifically, the variables to be used to estimate fiscal risk are those that determine the Financial Fragility Index Applied to the Public Sector, developed by Ferrari Filho, Terra and Conceição (2010) and Terra and Ferrari Filho (2011, 2020). Table 2 presents the variables and the theoretical basis for their use.

Table 2- Variables for estimating fiscal risk

Variable	Forming Variables	Theoretical Foundation
Current Revenues	Tax Revenue - RT Equity Revenue - RP Current Transfers - TC Current Revenue - CR (difference of the total current revenues less the forming revenues presented above)	Taylor and O'Connell (1985); Minsky, (1992); De Paula and Alves Jr. (2000); Ferrari Filho, Terra and Conceição (2010); Terra and Ferrari Filho (2011, 2018); Picolotto (2016)
Current Expenses	Personnel Expenditure - Personnel Current Expenditure - DC (difference between total current expenditure minus personnel expenditure and interest and debt charges)	Taylor and O'Connell (1985); Minsky, (1992); De Paula and Alves Jr. (2000); Ferrari Filho, Terra and Conceição (2010); Terra and Ferrari Filho (2011, 2018); Picolotto (2016)
Interest	Interest	Taylor and O'Connell (1985); Minsky, (1992); De Paula and Alves Jr. (2000); Foley (2003); Ferrari Filho, Terra and Conceição (2010); Terra and Ferrari Filho (2011, 2018); Picolotto (2016)
Amortization	Amortization - Amortiz	Taylor and O'Connell (1985); Minsky, (1992); De Paula and Alves Jr. (2000); Foley (2003); Ferrari Filho, Terra and Conceição (2010); Terra and Ferrari Filho (2011, 2018); Picolotto (2016)
HFFSP	Financial position calculated according to the Financial Fragility Hypothesis - HFF - applied to the public sector	Taylor and O'Connell (1985); Minsky, (1992); De Paula and Alves Jr. (2000); Foley (2003); Ferrari Filho, Terra and Conceição (2010); Terra and Ferrari Filho (2011, 2018); Picolotto (2016)

Source: Prepared by the authors.

It must be emphasized that in order not to have replication in the sum of the values, the Interest and Expenses of the Debt were deducted from the Current Expenses, since they are part of this budget group.

3.2 Model for estimating Fiscal Risk

The STN (2019) in the Manual of Fiscal Statements presents a model of the ARF, containing a spreadsheet with the evidencing of fiscal risks and steps to be taken. However, it does not describe which procedures or method/model for estimating fiscal risk.

Pereira and Biondini (2014) commented on the use of probability tables of risk consequences as a suggestion of procedures to prepare the ARF. They also emphasize that once the fiscal risk has materialized, actions to minimize it should prioritize fiscal balance.

In a more applied approach, Ramaswami (2002) has proposed a technique to measure the fiscal risks of a budget in the public sector. This technique, or method, is part of a series of papers on government risks, contingent liabilities, and fiscal risks, published by the World Bank. According to the author, descriptive statistical tools are used to measure and balance risk factor effects. He called it a factor model, in which the randomness of investments and expected revenue/expenditure flows are related to variations in macroeconomic variables such as GDP, measuring individual or idiosyncratic risk by standard deviation.

In the model suggested by Ramaswami (2002), the probability distribution is related to random shocks, both in units and in different periods, with the standard deviation as the standard measure.

For this study, different scenarios will be considered, but derived from trends in financial positions in a single unit and one-year period, i.e., according to forecasts for the annual budget. These scenarios will be estimated based on the assumptions and risk measures described in the Portfolio Theory framework.

Based on the contributions of Markowitz (1991), Ripamonti et al. (2001), Ramaswami (2002), Assaf Neto (2006), Minsky (2013) and Terra and Ferrari Filho (2020), a model for measuring fiscal risk was developed, more precisely for budget risk. In accordance with the model suggested by Ramaswami (2002), the budget risk will be derived from historical series of the HFFSP, associated with the estimation of 05 distinct scenarios and their respective probabilities of occurrence. The basis for the estimates is the HFFSP calculated in the last fiscal year. For this model the GDP variation is used to correct revenues and the inflation variation for expenses. For inflation, the index will be the IPCA because it is the index provided for in the Constitutional Amendment 95, which limits the spending cap according to the fiscal regime. This amendment is applied to the federal budget, however, it is considered relevant to this study because of its institutional aspect.

After determining the scenarios and their probabilities, the variables for determining the financial position contained in the HFFSP were calculated. This calculation followed the same logic of a sensitivity analysis. To determine the budget risk, the standard deviation was used for weighting, playing the role of idiosyncratic risk. Idiosyncratic risk or specific risk is the exclusive risk of a single variable. In the finance literature it is exemplified as the risk of a single stock and is considered independent of market variations (Fu, 2009). It should be remembered that in the Capital Asset Pricing Model - CAPM the market or macroeconomic risk is represented by the Beta (β).

Specifically for this model, the idiosyncratic risk was calculated based on the variation of the estimated HFFSP in the 5 scenarios. This procedure is pertinent, since it presents the same logic as the CAPM model. In the CAPM, the systematic risk, represented by the beta, is calculated. The non-systematic risk, that is, the idiosyncratic risk, is calculated based on the variation of a portfolio of shares or of a single share (Sharpe, 1977; Fama and French, 2004).

The risk measures, in this model, are the variance and the standard deviation. These measures show the variability, the dispersion of the possible expected financial positions. Thus, this variability, variance and standard deviation, will denote the risk measures (Assaf Neto, 2006).

The average to be calculated is not the arithmetic average, but an average weighted by the probabilities of the scenarios, according to the following formula:

$$\mu_x = E[X] = \sum_{j=1}^n X_j \times P(X_j)$$

X_j represents the Financial Position determined according to HFFSP for each scenario. $P(X_j)$ represents the probability determined for each scenario as estimated by the probability distribution.

The standard deviation will be calculated according to the following formula:

$$\sigma = \sqrt{\sum_{j=1}^n (X_j - \mu_x)^2 \times P(X_j)}$$

μ_x represents the calculated average. The other variables are the financial position and the estimated probability for each scenario, according to the average formula. The standard deviation will fit the estimates into a certain confidence interval, which will show the amount of estimated budget risk.

3.3 Period and scope

To estimate the fiscal risk, specifically the budgetary risks, the ARF and its estimate for the municipality of Belo Horizonte/MG, for the year 2020, was used as a basis.

The choice of the year 2020, to estimate the fiscal risk, is made because the budget statements of revenues and expenses for the year 2019 are available and show the realized revenue and executed expenses. On the other hand, on the date these research procedures were carried out, the budgetary statements of revenues and expenses for the year 2020, probably due to the annual closing and reconciliation process of the accounts, were not yet available on the STN website. Thus, the base is the year 2019 and the period to be estimated the fiscal risk is the year 2020.

As for the choice of the municipality of Belo Horizonte, it was made by convenience of the researchers, however it is noteworthy that the ARF of this municipality presents the necessary data for its comparison and estimation performed in this study. Thus, the basis for estimating fiscal risks in 2020 is based on the 2019 budget statements of revenues and expenses.

3.4 Organization of the variables, estimation of the scenarios and of the growth and drop rates

The first procedure was to organize the variables, as described in Table 2, to determine the financial position contained in the HFFSP.

The next step was to estimate the scenarios and the growth rates and/or drop in revenues and expenses that can be estimated based on an analysis of projections from official agencies. An example is the Focus Report from the Central Bank of Brazil (BACEN), which summarizes statistics calculated considering market expectations, or yet another report from some other agency or organization that publishes analyses and estimates for future scenarios. Scenarios can also be estimated based on the reality of a given region that historically and due to its characteristics and resources, presents higher rates than the national ones.

It is worth mentioning that for a more realistic estimate, the indexes for updating expenses cannot be at the same percentage as those for revenue. One option may be to apply the IPCA, based on Constitutional Amendment 95. However, it can still cause some significant discrepancy with the revenue indexes. One way to minimize this problem is to weight the estimate by evaluating the historical average of the percentage of current expenses in relation to revenues and weighting it with the scenarios. This procedure demands some subjectivity from the analyst, but also represents common sense so that the projections are not plastered. Such adjustments are also justified because the LDO is prepared before the presentation of the budget bill for the following year.

4 RESULTS FROM THE PROPOSED MODEL

The organization of the variables, according to Table 3, for the ascertainment of the financial position contained in the HFFSP is presented by Table 2 - Budget revenues and expenses 2019. The values are in thousands of R\$.

Table 2 - 2019 budget revenues and expenses

RC	RT	RP	TC	total CR	
713.864	4.226.663	323.836	5.596.955	10.861.317	
Staff	Interest	DC	Total DC	Amortiz	HFFSP
4.865.434	267.596	5.293.350	10.426.380	296.634,51	138.302,93

Source: STN- Siconfi.

The revenues are: CR- Current Revenue; RT- Tax Revenue; PR- Property Revenue; TC- Current Transfers and Total CR- Total Current Revenue. Expenses are: Personnel - Personnel Expenses; Interest - Interest and charges on public debt; DC - Current Expenses discounted the amounts of personnel and interest expenses; DC Total - Total Current Expenses; Amortiz - Expenses with amortization and HFSSP represents the financial position ascertained in 2019.

The estimation of the scenarios is presented as Table 3 - Probability Distribution. For this proposed model, the Deep Recession scenario was estimated with a probability of occurrence of 5%. Respectively, Moderate Recession by 20%, Stability 50%, Moderate Growth 20% and Accelerated Growth 5%.

Table 3 - Probability Distribution

Scenarios	Probability
Deep Recession	0,05
Moderate Recession	0,2
Stability	0,5
Moderate Growth	0,2
Accelerated Growth	0,05
Sum	1

Source: Prepared by the authors, adapted from Ripamonti et al. (2001).

Next, the rates to be applied to estimate revenues were estimated. For this model the rates were estimated in a deep and moderate recession scenario, at drops of -5% and -3% respectively. In a stability scenario a 1% increase was estimated, and for moderate growth 3% and accelerated growth 5%.

Table 4 - Variations in revenues

SCENARIOS	ESTIMATED RATES
Recession Prof.	- 0,05
Recession Mod.	- 0,03
Stability	0,01
Grow. Mod	0,03
Accelerated Growth Accel	0,05

Source: Prepared by the authors, adapted from Ripamonti et al. (2001).

After estimating the indexes for correcting the revenues, these are applied and the revenues for the following year are estimated. The calculated values are expressed in Table 5 Estimated Revenues.

Table 5 - Estimated Revenues

SCENARIOS	ESTIMATED RATES	RC	RT	PR	TC	Total CR
Recession Prof.	-0,05	678.170	4.015.330	307.644	5.317.107	10.318.251
Recession Mod.	-0,03	692.448	4.099.863	314.120	5.429.047	10.535.478
Stability	0,01	721.002	4.268.930	327.074	5.652.925	10.969.930
Grow. Mod	0,03	735.280	4.353.463	333.551	5.764.864	11.187.157
Accelerated Growth Accel	0,05	749.557	4.437.996	340.027	5.876.803	11.404.383

Source: Prepared by the authors.

The next procedure was to estimate the current expenses, plus interest and charges and amortization, according to the model for ascertaining the financial position under the HFFSP.

In the monetary updating of expenses, by the IPCA, a relevant discrepancy occurred in the calculation of the financial position. This was because the updated expenses were lower than the estimated revenues. Because of these considerations, current expenses were estimated at 98% of current

revenues for the recessionary scenarios; 97% of current revenues for the stability scenario, and 96% of current revenues for the growth scenarios. It is reiterated that since this is an estimation process, other criteria can be used if it is understood that they can bring more realistic estimates to the conditions of the government entity. Personnel expenses, because they originate from commitments linked to contracts and stability, do not vary in the scenarios, and were readjusted at 3.77%, i.e., the IPCA variation for the period.

Interest and amortization were estimated at another percentage, because they result from contracts that have their respective indexes, both for the adjustment of the outstanding balance and the agreed interest. For this model, both interest and amortization were estimated with an increase of 6.4% in relation to the amount committed in the previous year.

Table 6 - Estimated Expenses and HFFSP

SCENARIOS	Staff	Interest	DC	Total DC	Amortiz	HFFSP
Recession Prof.	5.048.861	284.722	4.778.304	10.111.886	315.619	-109.254
Recession Mod.	5.048.861	284.722	4.991.186	10.324.768	315.619	-104.910
Stability	5.048.861	284.722	5.307.250	10.640.833	315.619	13.479
Grow. Mod	5.048.861	284.722	5.406.088	10.739.671	315.619	131.867
Accelerated Growth Accel	5.048.861	284.722	5.614.625	10.948.208	315.619	140.556

Source: Prepared by the authors

Table 6 - Estimated Expenses and HFFSP presents the estimated expenses for the next period with the appropriate adjustments and procedures described.

Following the model, adapted from Ripamonti et al (2001), the mean and standard deviation were calculated. The calculation of the mean, according to the formula presented, is:

$$\mu = (-109.254 \times 0,05) + (-104.910 \times 0,20) + (13.479 \times 0,50) + (131.867 \times 0,20) + (140.556 \times 0,05)$$

The standard deviation will be calculated according to the following operation:

$$\sigma_x = \sqrt{[(-109.254 - 13.696)^2 \times 0,05] + [(-104.910 - 13.696)^2 \times 0,20] + [(13.479 - 13.696)^2 \times 0,50] + [(131.867 - 13.696)^2 \times 0,20] + [(140.556 - 13.696)^2 \times 0,05]}$$

Table 7 presents the values organized by scenarios and the calculation of the mean and standard deviation.

Table 7 - Average and standard deviation

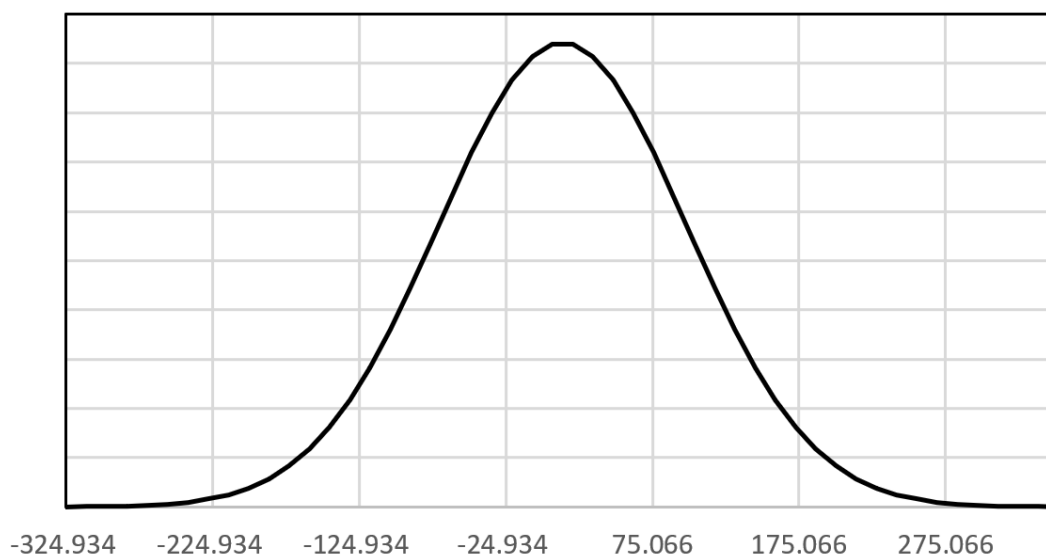
HFFSP	PROBABILITY.	AVERAGE HFFSP	DP
-109.254	0,05	-5.463	
-104.910	0,20	-20.982	
13.479	0,50	6.739	
131.867	0,20	26.373	
140.556	0,05	7.028	
	1	13.696	84.657

Source: Prepared by the authors.

As shown in Table 7, the average resulted in R\$ 13,696, in thousands of Reais, and the standard deviation was R\$ 84,657. The probability distribution, according to a normal distribution, recognizing the tax risk at two standard deviations down, was between R\$ -70,961 and R\$ -155,619.

Figure 1 - Distribution of Fiscal Risk

Distribuição do Risco Fiscal



Source: Prepared by the authors.

Figure 1 illustrates the probability distribution of fiscal risk. The dispersion involves the computed mean and 4 standard deviations up and down. The suggested model makes use of two standard deviations. In this case presented as an example and using real values, the budget fiscal risk is estimated at up to R\$ -155,619, in thousands of reais. The 95.44% confidence interval in this model ranges from R\$ -155,619 to R\$ 183,011. However, the values of these risks are the negative ones, as they represent the deficiency of resources, estimated in this probability distribution. That is, the fiscal risk is related to the probability of not having sufficient resources to meet the fiscal budget. The municipality of Belo Horizonte, in the LDO for 2020, estimated the budget risk at R\$ 118,109 and the total fiscal risk at R\$ 183,716.

Another point to be commented on is the high dispersion of the data, substantiated in the disparity of the standard deviation in relation to the average. As can be observed, the scenarios go from one extreme to the other, that is, from a situation of deep recession to accelerated growth. This in a single probability distribution. Thus, considering the probabilities for each scenario it is understandable that such variation occurs.

The advantage of this suggested model is that it is based on theoretical literature on risk, budgeting, and finance. Moreover, it is elaborated based on descriptive, dispersion, and variability statistical fundamentals. Such attributes have the ability to bring to the procedure a greater support in the estimates, minimizing projections with criteria that could be impregnated with greater subjectivity and randomness in their elaboration.

Nevertheless, observations that in this proposed model, there are doses of subjectivity of the preparer, as in any statistical model, are not rejected. However, it is necessary to verify the lack of works or even literature more applied to this procedure, as pointed out by Azevedo et al. (2019) regarding the sparse amount of studies related to the use of data in fiscal risk management.

And that is the contribution of this paper, that is, to bring empirical applications supported by Portfolio Theory and HFFSP. There is no evidence of works that have explored both descriptive statistics and risk measurement fundamentals for the budgeting process. Additionally, it brings new applications to HFFSP, demonstrating its ability to contribute to this field.

Despite the simplicity of the suggested model, formatted by basic descriptive statistical procedures, its relevance and applicability is precisely due to its ease of operation and application. This model can be useful in the elaboration of the ARF, especially for small municipalities that have modest resources or even restrictions of a more technical nature. It should be added that this model brings similar procedures to those undertaken by Ramaswami (2002) in the measurement of fiscal risks of a budget in the public sector, being referenced the relevance of his work, since it was edited by the World Bank.

In summary, the HFFSP may be a new option for measuring and estimating fiscal risk as advocated in this paper. Weak fiscal conditions denote fiscal risks that have been realized. The financial

positions calculated according to the HFFSP model can consider current revenues and expenditures, in addition to interest and amortization, accounts that impact government cash when realized. This construct goes beyond the Nominal Result, since it adds the portion of the amortization of public debt, which besides impacting the available resources, when not fulfilled its obligations, increase public indebtedness and bring the distrust of the lack of sustainability of the debt and even fiscal.

Thus, the applicability of the HFFSP added to the risk and return assumptions expressed by the Portfolio Theory is supported as one more tool for the elaboration of the ARF.

5 FINAL CONSIDERATIONS

This exploratory study presented a method for evaluating and estimating fiscal risks, precisely those related to the budget and the LDO. The motivation was the need for more studies that discuss empirical evidence in fiscal risk management, as well as models for their measurement and/or estimation.

Fiscal risk represents a problem to be faced in the management of government finances, aggravated by the informational content of fiscal risk reports being low, as also loaded with discredit and inaccuracy, as pointed out by Azevedo et al. (2019).

Thus, this work could contribute in bringing procedures based on theoretical literature on risk, budget and finance, providing the professional public finance technician with an uncomplicated alternative, however, efficient in measuring and estimating fiscal risk. Such defended attributes are capable of conferring to the estimation performed, a condition of less subjectivity and grounding in a scientific method in its elaboration.

The limitations of this study include the small number of studies focused on fiscal risk estimation and the lack of historical data of the model for follow-up and adjustments, since it is an alternative that is being presented at this time. As a suggestion for future research, a survey with finance and budget officials from small municipalities can provide a better understanding of their difficulties in preparing the ARF and the pertinence and acceptability of this type of model to determine fiscal risk.

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