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A study on the dynamics of sustainability of Minas Gerais' State Public Debt between 2003 and 2020: an econometric approach

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Abstract

In the 1980s and 1990s, a specialized literature was developed in the investigation of public debt sustainability around the world. This literature was based on unit root tests, VECM modeling and public debt forecasts. These tests were intended to verify whether governments are incurring an unsustainable debt trajectory, and whether they are implementing a Ponzi scheme. The models aimed to reduce uncertainty about the Public Debt, becoming an important ally to public finance managers. This work aimed to verify evidence of the sustainability of public debt in Minas Gerais between 2003 and 2020 through the econometric approach of testing the presence of unit-roots and the cointegration between Revenues, Expenditures and other relevant variables. The conclusion was that the Public Debt of Minas Gerais shows signs of weak sustainability, lacking actions to re-establish a path of sustainability between the variables Primary Revenues and Primary Expenditures.

Keywords: Public Debt, Cointegration, Unit Roots, Sustainability, VECMs.

JEL Code: C32, C53, H74

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LIST OF ABBREVIATIONS AND ACRONYMS

ADF - Augmented Dickey-Fuller

FJP - João Pinheiro Foundation

IBCR - Regional Economic Activity Index - Minas Gerais

ICMS - Value-Added Tax on Sales and Services

IO - Innovational Outlier

KPSS - Kwatikowski-Phillips-Schmidt-Shin

LC - Complementary Law

LN - Natural Logarithm

LRF - Fiscal Responsibility Law

MAE - Mean Absolute Error

MAPE - Mean Absolute Percentage Error

NPG - No Ponzi Games

OECD - Organization for Economic Co-operation and Development

GDP - Gross Domestic Product

PIM - MG - Minas Gerais Monthly Industrial Survey

PP - Phillips-Perron

RMSE - Root Mean Square Error

ROI - Intertemporal Budget Constraint

SEF-MG - Secretary of State for Finance of Minas Gerais

SEPLAG-MG - Secretary of Planning and Management of Minas Gerais

STN - National Treasury Secretariat

Theil - Theil Coefficient of Inequality

VAR - Vector Autoregression

VECM - Error Correction Model

SUMMARY

1 Introduction

Historically, the State has been concerned about the fiscal sustainability of public debt, through the creation of a framework of legislation that sought to balance the budget in Brazil. On the occasion of the growth of debt stocks around the world, especially in the USA and Brazil, a literature specialized in testing the sustainability of public debt emerged in the mid 1980's and 1990's, either by means of unit root tests, or by means of error correction vector models and public debt forecasts (Hamilton e Flavin 1985; Wilcox 1989; Trehan e Walsh 1988; Bohn 1991). Thus, in general terms, these tests intend to verify whether governments, in their fiscal policy incur in an unsustainable debt trajectory and whether they are implementing a Ponzi scheme, which means rolling over their debt indefinitely and incurring in default (Luporini 2006). With the development of such approaches, it has allowed the public administrator a reduction of uncertainty in the paths that will be taken by the debt, since such models have at their core the prediction of the debt as one of their gains.

In this sense, a great debate was observed mainly at the national level of Brazilian public finances (Pastore 1994; Rocha 1997; Issler e Lima 1997). However, regarding the federation units, this debate is not yet closed, since there are not many contributions in this field. Therefore, new contributions in the literature are necessary, especially in the realm of state public finances, since the states do not have the characteristic instruments of the national entity (issuing bonds, setting interest rates, seigniorage in general, etc.), the question of how to equate the real deficits of the states becomes difficult to solve. Given this lack of mechanisms, the subnational entities are left with strict fiscal and tax control to provide the sustainability of their debts, which should always avoid continued deficits, contrary to what has been observed in the recent history of several states of the federation.

In view of this behavior, this paper gains importance as it investigates the dynamics of the internal public debt of Minas Gerais between the years 2003 and 2020, and the existence of signs of sustainability of its public finances, according to the tests that the literature indicates. Moreover, this work focuses on forecasting the public debt for the year 2020.

Thus, in this context, this research has as its main question: are there indications of sustainability of the internal public debt of Minas Gerais, in line with the econometric approach advocated by the literature?

As a general objective, this work seeks to verify whether there is evidence of sustainability of the public debt of Minas Gerais between 2003 and 2020 according to the econometric approach of testing for the presence of unit roots and cointegration between revenue and expenditure.

Thus, the specific objectives of this work for the analysis of the sustainability of the internal public debt of Minas Gerais are the following: To conduct a literature review on the public debt sustainability literature; to evaluate the results of the econometric tests of unit roots of the variables that will compose the equations, in line with the specialized literature on public debt sustainability; to estimate Error Correction Vector Models - VECM for the internal public debt of Minas Gerais verifying its possible cointegrations; To carry out specific exogeneity/homogeneity tests for the Primary Revenue/GDP variable to verify its possible degree of exogeneity to the models developed; to verify the response of the public debt, Primary Revenue and Primary Expenditure when they suffer an exogenous shock from other variables (unconventional impulse-response analysis) and the projection of results for the variables in the model.

This work is composed of four sections, including this introduction. In the second chapter, a bibliographical review is made of the sustainability of the public debt, to present an overview of the state of the art in the discussion of this subject, both internationally and nationally.

The Third chapter will discuss the methodology used for the estimation of the proposed model, in addition to the description of the data and the discussion of the empirical results. Finally, the conclusion will revisit the main results and conclude on the effectiveness of the models.

2 Public Debt Sustainability

This section will address and conceptualize the issue of public debt sustainability considering the literature. To this end, a literature review will be conducted on the same, bringing to light the discussion held both internationally and in the Brazilian case.

2.1 Review of the literature on public debt sustainability

The interest in public debt sustainability as a branch of economic study arises because of its growth in the 1980s and 1990s, first in the United States and later in Brazil. The 1990s for Brazil was a decade of great turbulence for public finances and the stabilization of the currency, both at the federal, state and municipal levels. It was at this time that the first studies on public debt sustainability emerged, which will be treated in this chapter (Hamilton e Flavin 1985; Wilcox 1989; Kremers 1989).

First, in fact, public indebtedness is an instrument widely used by governments for the optimal intertemporal distribution of public policies. That is, the State, to provide basic public services and its public policies, uses debt (Costa 2009). In view of this, the concept of debt sustainability is something urgent for the State to continue providing public services to the population. According to (Luporini 2006)According to the author, sustainability as a concept would be the capacity of the State to indebt itself without heading towards an excessive degree of debt accumulation, which may generate the inability of the State to honor its commitments to creditors, that is, incur in default. For (Blanchard et al. 1990)the policy of public debt sustainability can be defined as the convergence of the debt-output ratio to a constant value or to its initial level.

The literature has developed over the years several types of approaches to determining public sector solvency. The sustainability indicator in which a ratio of Debt stock to Gross Domestic Product is made is possibly one of the most widely used concepts in the literature. This concept determines the importance of comparing the real after tax interest rate paid to holders of government bonds to the growth rate of the economy's real product (Domar 1944; Harrod 1949; Hamilton e Flavin 1985).

(Domar 1944) e (Harrod 1949) were pioneers in addressing the sustainability condition for government debt. For these authors, fiscal sustainability is maintained when the nominal growth rate of the economy is greater than the growth rate of the nominal stock of government bonds. In other words, the condition for sustainability to occur would be that the ratio between government bonds and output is not divergent over time. This implies that government debt will not be sustainable if the primary result remains equal to zero and the stock of debt is growing.

In line with the first approach brought to light, there is the so-called Intertemporal Government Budget Constraint (ROI). This starts in nominal terms as expressed in equation (1):

$$B_t = (1 + i_{t-1}) * B_{t-1} + G_t - T_t (1)$$

Where B_t is the nominal value of the stock of public debt in the market in period t; it is the nominal interest rate on public debt in the previous period; and T_t e G_t are the government revenues and expenditures in t. Subtracting from both sides B_{t-1} you have that:

$$\Delta B_t = i_t * B_{t-1} + G_t - T_t (2)$$

Dividing the two sides by the Gross Domestic Product (GDP), according to the economic literature in the area, one has that:

$$\Delta b_t = r_t * b_{t-1} + g_t - t_t \ (3)$$

Where b_t is the ratio of the debt stock to GDP; rt is the real interest rate of the economy g_t and t_t are the primary government expenditures and revenues relative to GDP. The budget constraint of a government for a given period should be extended in time for the concept of debt sustainability to be complete. Thus, we have that the expansion of equation (3) to infinity:

$$b_t = \rho^n (\sum_{n=1}^{\infty} t_{t+n} - \sum_{n=1}^{\infty} g_{t+n}) + \rho^n b_{t+n} \ (4)$$

Where $\rho^n = [(1+h)/(1+r)]^n$ is the discount factor, and h represents the real growth rate of the economy. To prevent the government from perpetually financing itself by rolling over its debt and servicing its debt by issuing new bonds, a No Ponzi Game (NPG) restriction is imposed, meeting the following transversality condition:

$$\lim \rho^n b_{t+n} = 0 \ (5)$$

Finally, applying recursively, condition (5) must be such that the stock of debt at t will equal the sum of future primary surpluses in present value terms, as shown below in (6):

$$b_t = \rho^n(\sum_{n=1}^{\infty} t_{t+n} - \sum_{n=1}^{\infty} g_{t+n}) \quad (6)$$

In sum, equation (6) shows that the debt will be sustainable when the growth rate of real GDP is greater than the growth rate of interest, ceteris paribus. Another condition for sustainability is when the primary surpluses, in terms of present values, offset the stock of debt. That is, the government's intertemporal budget constraint establishes that public debt in current values is equal to the sum of future flows, discounting the government's primary results.

However, as the literature points out, this approach has some problems in its formulation (Costa 2009; Bertussi 2013). First, the government cannot guarantee the promise of sustainability by controlling the nominal output growth rate, second, this approach does not consider the uncertainty of debt stock payments or the existence of primary surpluses or deficits that may occur in the real world. Thus, uncertainty could entail several possible debt trajectories.

From this, several works are developed that add uncertainty in the model. The first analysis of the intertemporal budget constraint, developed by (Hamilton e Flavin 1985) is done by means of unit root tests and sought to test two hypotheses about the limitation of government borrowing, namely: (i) the non-impediment of the government to incur permanent budget deficits either by paying interest due to increased debt or simply by issuing new debt; (ii) or, furthermore, the non-disposition of creditors to buy new government debt securities unless governments made a commitment to balance their public accounts in terms of present values. Thus, the authors' interest is to test whether the government obeys this restriction.

The authors performed two types of tests to verify the sustainability of the US public debt in the period 1962-1984. The first would be to test the hypothesis of stationarity of both debt and deficits (excluding interest), through the Dickey-Fuller (DF) test. This test applied to both series and rejected the null hypothesis of unit root, which would validate the hypothesis of sustainability of the American debt for the analyzed period.

The second was to use the test of (Flood and Garber 1980) cited in (Hamilton and Flavin 1985), and was intended to test the NPG condition. This test showed that it could not reject the NPG condition hypothesis, thus providing support for the debt sustainability hypothesis. Thus, (Hamilton e Flavin 1985) conclude, from the tests previously explained, that the recurrent deficits that occurred between 1960-1981 of the U.S. debt could not shake the sustainability of the U.S. public debt.

The introduction of stationarity tests opens a new path in the list of tests for intertemporal sustainability of public debt. After Hamilton and Flavin's analysis (1985) several others followed, in which cointegration tests were added between the variables studied. (Trehan e Walsh 1988)Hamilton and Flavin (1985), for example, using a larger data base than (Hamilton and Flavin 1985), comprising an analysis period from 1890 to 1986, explaining that, the real interest rate of the economy being constant, the test of debt sustainability could be made in two ways: (i) by testing the stationarity of the first difference of the debt (is the nominal deficit stationary or not?); (ii) or further by testing the cointegration hypothesis between government revenues and expenditures, i.e., interest payments are included in this calculation.

The studies of (Trehan e Walsh 1988) studies point to diverse conclusions about the sustainability of the American public debt. The divergence of results may be due, according to the authors, to the low power of the tests previously used or to the possibility of non-stationarity of the real interest rate. However, they have shown the need for the government's budget to be balanced. Therefore, for these authors, there would be the need for the cointegration of government expenditures, tax revenues and seigniorage. Thus, the condition of stationarity of the deficit is necessary, but not sufficient, for intertemporal budget equilibrium. Therefore, solvency would be tested by the stationarity of the total surplus (interest payments included), being equivalent to the cointegration test between the debt stock and the surplus.

(Wilcox 1989) in turn, dialogues with and extends the work of the pioneer studies of (Hamilton e Flavin 1985) and extends their work by assuming some assumptions, such as: (i) the admission of stochastic interest rates as opposed to the pioneer authors' assumption that interest rates would be constant; (ii) the assumption of the nonstationarity of primary surpluses (without interest), unlike Hamilton and Flavin; (iii) in addition to allowing for the possibility of stochastic violations in the ROI, while the two authors assumed that any such violation would be nonstationary.

The author in question modeled his hypotheses by means of an ARIMA model, estimating the discounted debt stock, and the sustainability of fiscal policy would be given as a function of the forecast of the debt trajectory. His main conclusion was that, contrary to the (Hamilton e Flavin 1985) there was strong evidence of unsustainability in the conduct of American fiscal policy, especially in the post-1974 period, and the ROI did not seem to be satisfied. Moreover, the 1960-1984 period had strong evidence of structural changes in fiscal policy and could not be treated.

(Kremers 1989) (Hamilton and Flavin 1985), in a seminal article, examined the conduct of American fiscal policy since 1920 and verified whether it was influenced by the increase in the stock of federal public debt. His main hypothesis was that the equilibrium between the debt/GDP ratio guaranteed by the existence of primary surpluses over time, would be sufficient to maintain the sustainability of the public debt. The author's analysis concluded that the long-term restrictions arising from the increase in the debt stock, influenced the annual conduct of fiscal policy since 1920 and the following periods before and after World War II had stabilizing effects on the debt-output ratio. We also found signs of changes in the conduct of fiscal policy after 1981, where increases in debt service and debt stock were generated, generating deficits not consistent with those of previous decades. In addition, the (Kremers 1989) found evidence suggesting that the pioneering article by (Hamilton e Flavin 1985) used a unit root test with low explanatory power.

In this same vein, (Hakkio e Rush 1991) test whether there is cointegration between tax revenues and primary and non-primary budget expenditures as a proportion of GDP, between 1950 and 1988, using the budget constraint in real terms, since the interest rate must be stationary.

However, unlike other authors, (Hakkio e Rush 1991) separate their analysis into time periods such as 1950:II through 1988:IV; 1964:I through 1988:IV; and 1976:III through 1988:IV. This break into sub-periods demonstrates the authors' concern over structural breaks in the historical series. The authors noted that U.S. government spending policy remained in violation of the intertemporal budget constraint due to high budget deficits. Therefore, government spending should be reduced and taxes raised.

With regard to the cointegration tests, the authors suggested that from 1950:II to 1988:IV there was cointegration between revenues and expenditures in real and per-capita terms. The periods 1964:I to 1988:IV and 1976:III to 1988:IV rejected cointegration between the revenue and expenditure series, indicating a change in the fiscal behavior of the government and a possible indication of a violation of the intertemporal budget constraint, that is, there is evidence of unsustainability of the American public debt between 1975 and 1988 (Hakkio e Rush 1991).

In the wake of the works that use the analysis of the presence of unit roots in time series, to determine their stationarity or not, as well as the verification of cointegration between revenues and expenditures, (Quintos 1995) in her article investigates the conditions of sustainability for the public debt besides searching for structural breaks in the American fiscal policy between 1947-1986. The author reached the conclusion that the American debt is sustainable, despite the presence of a structural break in the mid-1980s, indicating a change in the American fiscal policy for the analyzed period. The great innovation in her work was the development of two concepts of sustainability, strong and weak. The first, called strong, inspired by the works of (Hamilton e Flavin 1985; Trehan e Walsh 1988) where public debt must be stationary and public revenue and expenditure must be cointegrated, using for this purpose a cointegration vector (1,-1), that is, an increase in expenditure must be accompanied by an equal increase in revenue or an increase in revenue induces an increase in expenditure.

According to (Quintos 1995)the weak sustainability condition has as main assumption the stationarity in first difference of revenues and expenditures and their cointegration with a vector of order (1, -b), being $0< b \leq 1$. That is, in this condition, the cointegration between revenues and expenditures is sufficient for the sustainability of public debt.

Although the previous methodology of stationarity testing was established, (Bohn 1995) has made a number of criticisms of these models. The models test whether fiscal policy is consistent with ROI, however, as is shown by the literature, there are controversies among the results of public debt sustainability for the U.S., even if the same time series are analyzed (Kremers 1989; Trehan e Walsh

1988; Hamilton e Flavin 1985; Wilcox 1989). One of the aspects cited by the author is the deterministic framework that often ends up limiting the analysis, since this premise would imply that the economies analyzed are dynamically efficient, where the real risk-free interest rate of the economy is always greater than the real growth of GDP.

According to (Bohn 1995), unit root tests and cointegration analysis between revenues and expenditures would present a series of questionable results, since they are strictly based on asymptotic conditions. That is, the state incurring significant primary deficits does not guarantee that it is not complying with the intertemporal budget constraint. Since in the future, the same government can obtain considerable surpluses and recover the ROI condition on debt sustainability.

Bohn (1995; 1998) introduces a new approach to government fiscal sustainability, based on new econometric tests. His analysis focuses on the relationship between the primary result and changes in the debt/GDP ratio of the U.S. economy, the so-called fiscal reaction function (FRF). This function is strictly positive, linear and one of the conditions for public debt sustainability. It is a different approach from the traditional one because it does not consider the behavior of the interest rate. This can be described by the following regression:

$$rp_t = \beta b_t + \alpha X_t + \varepsilon \quad (7)$$

Where rp_t is the government primary result as a ratio of GDP, b_t is the ratio of the stock of debt to GDP over time, X_t is a vector of components in which are several variables responsible for the primary result and ε is the error term. (Bohn 1998) reaches the conclusion that if the series rp_t and b_t are stationary, it will be necessary to explicitly include other variables in the vector X_t . In case of not including this vector explicitly modeled, there may be a risk of bias due to omitted variables. However, when the variables are integrated of order 1 (ARMA model) and cointegrated, we do not need to explicitly model X_t . Thus, the cointegration vector will be restricted to (1, -β), indicating a reaction to the primary outcome on public debt.

The model of (Bohn 1998) model has a number of limitations. As (Sarvi 2011) the FRF provides only retrospective information on fiscal policies. However, the interest of this type of study is also to project outcomes to see what pattern the debt should follow to become sustainable. Furthermore, Bohn's model in question, unlike traditional approaches, does not provide a quantitative measure of sustainability, but is limited to accepting or rejecting the hypothesis of sustainability. Finally, the demographic transition that occurs in developing countries is a phenomenon that inspires caution for all scholars of the subject, since the political-institutional environment and even the situation of public finances is somewhat volatile. The presence of financial skeletons and contingent liabilities related to social security makes this kind of approach to be looked at with more caution.

2.2 Literature review for the Brazilian case

In the Brazilian case, the investigation of the sustainability of the public debt in an econometric way began with (Pastore 1994). This author, in his work, analyzed the period between 1974 and 1989, testing the first difference of the Brazilian public debt. His results indicated the sustainability of the Brazilian public debt for the period; however, this sustainability would be conditioned to obtaining revenues from seigniorage[1].

(Rocha 1997)(Rocha 1997), in turn, investigates the sustainability of the federal public debt between 1980 and 1993, using the methodology of (Trehan e Walsh 1988) e (Hakkio e Rush 1991), testing the stationarity of the first difference of the public debt and cointegration of tax revenues and expenditures, respectively. The results of his study indicated seigniorage as the main variable for intertemporal budget equilibrium.

(Issler e Lima 1997) in turn tested whether the Brazilian public debt in relation to GDP was sustainable between 1947 and 1992, and how the government balanced the budget aftershocks to revenues or expenditures. The authors used unit root tests, cointegration tests between variables, and exogeneity tests to verify the behavior of the variables considered, checking whether there was any behavior in them that did not allow cointegration in the VECM model. The tests made pointed to a sustainability of the debt if seigniorage revenue is included in government revenues. The Granger exogeneity test points to a weak exogeneity of government expenditures. Another conclusion comes from the unconventional impulse-response test, which shows that regardless of the origin of the initial fiscal imbalance (expenditure or revenue shocks), it is eliminated through a future change in taxes.

Dialoguing with the situation of the Plano Real at the time, the authors (Issler e Lima 1997) observe the decrease in seigniorage revenues and the increase in public expenditures, generating persistent deficits that could lead the Brazilian debt to an unsustainable path. Thus, these authors report that such a procedure would lead to either an increase in taxes, excluding seigniorage, or an increase in seigniorage revenues. These two measures would have harmful effects for the Brazilian population. In the first case, Brazilians would be the most heavily taxed citizens in Latin America, with few public services in return, and in the second, inflation would rise again and cause Brazilians to lose their purchasing power.

The literature has also investigated the sustainability of subnational governments (Fontenele et al. 2015; Moura 2017). In (Fontenele et al. 2015), the public debt of the 26 states of the federation and the Federal District in the period from 2000 to 2010 after the implementation of the LRF is analyzed. For this sustainability analysis the lm unit root test of (Pesaran and Shin 2003 cited by Fontenele et. al 2015), for panel data, was applied. This test considers that the autoregressive coefficients can vary freely for each analyzed unit, aiming to find out if there is stationarity in the debt/GDP ratio. The main

1 According to (Giambiagi e Além 2011) seigniorage is "the flow of nominal creation of the monetary base."

results found were that except for states in the Midwest Region, all the others indicated a tendency of unsustainability of the public debt, both in aggregate and disaggregated form. Finally, the authors conclude that the Federal States will not be able to honor their financial commitments without recourse to abrupt changes in economic and fiscal policy.

In (Moura 2017), it is analyzed whether there is sustainability of the indebtedness of Brazilian states in the period between 1995 and 2012. In this way, the authors disaggregate the fiscal balances into three, with the objective of verifying the impact of government transfers on the conduct of state fiscal policies. To do this, the author in question used the methodology of (Hakkio e Rush 1991; Quintos 1995) for a panel of 26 states and the Federal District, between 1995 and 2012. Another aspect noted by the said author is the existence of correlations between the cross-section units. The main conclusions of the author in his study were, the existence of evidence of sustainability of the fiscal policy of Brazilian states in the period, since the series of revenues and expenses (in % of GDP) are cointegrated, the finding of dependence on government transfers and the low reaction of tax revenues to state non-financial expenses.

3 Discussion of the results

In the following subchapters, the methodology of this work, the description of the historical series in the econometric modeling and the model itself will be discussed. First, the methodology used for estimating the model will be outlined, establishing the important variables, the tests required to guarantee the assumptions and how the forecasts will be made. Then, the characteristics of the data will be described and discussed, as explained in the methodology, and finally, the proposed domestic debt model and its forecasts will be analyzed.

3.1 Methodology

The present work, as already explained in previous chapters, aims at investigating the existence of sustainability of the internal public debt in Minas Gerais. Therefore, in the first place, a review of the literature on the subject of public debt sustainability was carried out, encompassing from ROI to the different econometric tests already explained in the last chapter, be they unit root, cointegration and modeling of ARIMA, VECM and FRF equations. (Domar 1944; Harrod 1949; Hamilton e Flavin 1985; Trehan e Walsh 1988; Kremers 1989; Quintos 1995; Bohn 1998).

The methodology used to test the sustainability of the public debt in Minas Gerais will be described below. All data presented in the document were deflated by the IPCA or by the CPI (Consumer Price Index) according to the currency. In fact, these indicators are proxies for measuring the increase in prices and inflation in the period. Thus, the data in this document are in constant values. It is emphasized that because this study is interested in the monthly inflections of the various variables, it was decided not to deseasonalize the data.

The database used for the analysis was composed of data produced by the João Pinheiro Foundation (FJP), the Federal Reserve Bank of St. Louis, the Central Bank of Brazil and the IBGE, as well as data provided by SEF-MG and SEPLAG-MG. The data used in the analysis was composed of the stock and service of the internal public debt of Minas Gerais measured at constant IPCA values between the years 2003 and 2020 (laws 9.496/1997, debt from laws 7.976/1989 and 8.727/1993, among other debts); primary revenues and expenses of the state of Minas Gerais; internal debt service; the basic interest rate of the American economy (Prime) - deflated by the CPI; the chained quarterly data of Minas Gerais' GDP from 2003 to 2020; the Regional Economic Activity Index - Minas Gerais (IBCR) - BACEN, on a monthly basis for the Southeast; 12-month accumulated SELIC rate; CPI-deflated exchange rate, PIM-MG (Monthly Industrial Research of Minas Gerais - IBGE), Minas Gerais ICMS - Minas Gerais State Finance Department, Minas Gerais Exports to China[2] - MDIC, CPI-deflated exchange rate. It should be reiterated that all data were logarithmic, except for the 12-month accumulated SELIC rate, the American Prime Rate and the US$/R$ exchange rate. This decision is based on two facts: reduction of the bias effect and the imposition in the model of an adequate way to compare the behavior of the variables, since they are often in different magnitudes. Therefore, the comparison in variation rates is fairer. The monthly GDP of Minas Gerais was calculated as follows: the annual GDP of Minas Gerais was multiplied by the chained series supplied by the FJP data and, subsequently, each quarter was multiplied by the monthly value of the IBCR, since this set of data indicates the regional economic activity.[3]

In accordance with the literature reviewed, the presence of unit roots was tested by means of the ADF,[4] PP[5] and KPSS tests[6] ; besides the seasonal structural break tests (ADF and PP). The unit root test performed in this paper aimed at verifying the stationarity of the variables in question, since the non-rejection of the alternative hypothesis of unit root tests is a sine qua non condition for not rejecting the evidence of public debt sustainability, in line with previous works (Hamilton e Flavin 1985; Wilcox 1989; Trehan e Walsh 1988; Quintos 1995). In fact, as Bueno (2015) explains, stationarity is one of the central concepts for the estimation of a time series, it guarantees the validity of the statistical

2 Proxy for exports from Minas Gerais because of its high representativeness.

3 According to the Central Bank's methodological note, the IBCR-MG consists of several proxies of regional economic activities and aims to "reflect the movements of regional economic activity in a more timely manner." (BRAZIL, 2017). Thus, it composed of an amalgam of the following economic activities: Agribusiness, Manufacturing and Extractive Industry, Electricity Production and Distribution, Civil Construction, Trade, Transportation Services, Information Services, Financial Intermediation, Business Services, Public Administration, Health and Education, Market Health and Education, Household Services, Household Services. Such a composition allows the IBCR to be a very faithful proxy of the portrait of regional economic activity.

4 Augmented Dickey-Fuller.

5 Phillips-Perron.

6 Kwatikowski-Phillips-Schmidt-Shin

inference of the estimated parameters of a stochastic process. Formalizing the concept of stationarity[7] consider that any time series {yt , t ∈ Z}, where Z = {0, ± 1, ±2, ...}, will be stationary if:

1. $E|y_t|^2 < \infty$
2. $E(y_t) = \mu$, for all $t \in Z$; and
3. $E(y_t - \mu)(y_t\text{-}j - \mu) = \gamma_j$

The data were tested on level and first difference. In all the tests analyzed, Akaike's criterion was used for decision. In the ADF test, an extension of the Dickey-Fuller test, as taught (Enders 2014) its main purpose is to estimate an autoregressive model to verify the presence of unit roots in the series studied. Its hypotheses are the following:

H_0 : has unit root (not stationary)

H_1 : has no unit root (is stationary)

In case we cannot fail to reject H_0, the series will be differentiated once to eliminate the trend and unit roots of the series.[8] According to (Enders 2014), if a given historical series has a number of unit roots, the test is ineffective in assessing this context. This is one of the limitations of the test in question.

To remedy the limitation of the test mentioned in the previous paragraph, the Phillips-Perron test is applied to verify multiple structural breaks. This test makes non-parametric corrections to allow consistency even if there is dependence between lagged variables and serial correlation between errors. Thus, it is unnecessary to specify a high-order autoregressive model to purge the serial correlation from the residuals. Similarly to the previous test, the presence or not of unit roots in the analyzed series will be tested, according to the hypotheses h_0 and h_1 presented. The method for the estimation window will be the Barlett method which is the Eviews default and the bandwidth will also be the default, called Newey-West Bandwidth (Bueno 2015).

Once the presence of unit roots has been examined for each of the historical series in level and first difference, the presence of structural breaks in the variables present in the models has been verified, that is, when the values of a given historical series change in a very pronounced way, the change of level or the occurrence of a pulse is tested. Thus, the tests for structural breaks recommended by (Perron 1989) and improved by (Vogelsang and Perron 1998) are performed. Through Additive Outlier (AO) or Innovational Outlier (IO) modeling, the Phillips-Perron test detects structural breaks of short or long duration. Regarding AO, it models structural shocks without long-term reverberations, while the IO test is concerned with perennial changes in the variable's intercept. To model historical events,

7 For more information see Bueno (2015, p. 17).

8 For more information see Enders (2014) and Bueno (2015).

such as changes in legislation (complementary laws 148/2014[9] and 156/2016) and structural breaks with COVID-19 as an example, pulse and level dummies were added to the equations. Consequently, the specifications of the mining case used the IO criterion for a modeling that keeps similarities with the reality of the historical moment studied.

Once the presence of unit roots in the variables under study has been tested, this paper will estimate the public debt sustainability models. VECM (Error Correction Vector) type equations will be modeled. The main characteristic of this type of model is its economic significance, due to the short and long run components. Furthermore, this type of approach emphasizes the cointegration between variables. According to (Engle e Granger 1987) cointegration can be conceptualized:

> A series without deterministic components and having a stationary, invertible, ARMA representation, after being differentiated d times is said to be integrated of order d and is denoted as a vector xt ~ I(d) (Engle e Granger 1987, 252, translation by the authors).[10]

In this logic, the elements contained in X_t are integrated of order d, in other words, I(d); furthermore, there exists a nonzero vector, β, so that:

$$u_t = X_t'\beta \sim I(d-b), b > 0$$

Thus, all variables included in the vector must have the same order to be cointegrated. This cointegration is justified to the extent that these economic variables have similar behaviors and dynamics, in addition to receiving and generating effects among themselves, keeping a long-term equilibrium relationship. Thus, the variables present in the vector in question are stationary in the first difference and with a stochastic trend. In the long run when $X_t' \beta = 0$ the cointegrating vector β will define a perfect linear combination with X_t in which there is a common trend, without deviations. However, in the short run there will be temporary deviations, due to economic shocks, that displace the variables from the common trend (Engle e Granger 1987; Bueno 2015). This way, the cointegration theory is based on two points, the test of the residuals ut for the confirmation of the variable's stationarity; and the second point, in case ut is stationary, it will use the information obtained to better adjust the VAR model. Hence the name VECM model, since the long run equilibrium errors are incorporated to the

9 LC 148/2014 states: "Amends Complementary Law no. 101, of May 4, 2000, which establishes public finance rules aimed at fiscal management responsibility; provides for criteria of indexation of debt refinancing agreements entered into between the Federal Government, States, the Federal District, and Municipalities; and makes other provisions.

10 Translation: "A series with no deterministic component which has a stationary, invertible, ARMA representation after differencing d times, is said to be integrated of order d, denoted xt ~ I(d)".

model[11] (Bueno 2015).

Once the approach is defined, we then proceed to modeling the VECM. Inspired by the classic models in the literature already seen in section 2, the first step of the modeling will be to determine the government's budget constraint, where B_t is the government's public debt, G_t is the government's primary expenditure, T_t is the government's primary revenue, it is the interest charged on the stock of public debt; and ε_t is the statistical error.

$$B_t = (G_t - T_t) + (1 + i_t)B_{t-1} + \varepsilon_t \quad (8)$$

Subtracting B_{t-1} on both sides, the initial equation in the first difference would be redefined to the following form, where DEF is the government's primary deficit:

$$\Delta Bt = DEF + i_t * B_{t-1} + \varepsilon_t \quad (9)$$

Dividing the two sides by the GDP, the initial equation looks like this:

$$\frac{\Delta Bt}{PIB} = i_t * \frac{B_{t-1}}{PIB} + \frac{DEF}{PIB} + \varepsilon_t \quad (10)$$

In the internal debt model, the following variables were added: ICMS of Minas Gerais; PIM--MG; SELIC 12 months and exports and a series of exogenous variables to the model such as internal debt service; American Prime Basic Interest Rate deflated by the CPI, Real Dollar Exchange Rate, the changes imposed by the complementary law 148[12] and 156[13], the seasonality of the months of the year, as well as a series of dummies linked to changes in the various variables chosen.

$$\frac{\Delta Bt}{PIB} = i_t * \frac{B_{t-1}}{PIB} + \frac{DEF}{PIB} + \psi_t + \sigma_t + \theta D_t + saz + \varepsilon_t (11)$$

Where ψ_t a vector with the endogenous variables of the model that do not belong to the classical model, σ_t a vector with the exogenous variables of the model, seasonalities of the months of the year and θD_t the pulse and level dummies in the period. To measure the variables of the analyzed period, in terms of rates, the endogenous variables were taken in terms of natural logarithm. The only exception was the 12-month Accumulated Selic Rate. Thus, these variables were differentiated once,

11 For more information see (Bueno 2015; Enders 2014; Engle and Granger 1987).

12 This law amends the LRF and changes the indexation of debt refinancing contracts between the Federal Government, states, municipalities and the Federal District. In this sense, the inclusion of this variable sought to be a proxy that measures the changes that occurred in the analyzed period.

13 This law establishes a plan to help the states and the Federal District and measures to stimulate fiscal rebalancing, in addition to establishing some changes in the legislation underlying the legislative framework of Brazilian public finances.

reaching a level of stationarity so as to cointegrate according to the vector $X_t^f = (T_t, G, B, \psi_t)$, with elements of a cointegrated vector such that (1, r...,0). The representation of a cointegrated process of X_t will be:

$$A(L)\Delta X_t = -\alpha\,\beta' X_{t-1} + \theta D_t + saz + u_t (13)$$

Expanding the equation to infinity.

$$A(1)\Delta X_{t+n} = \sum_{i=1}^{n} \alpha\,\beta' X_{t+n} + \sum_{i=1}^{n} \theta D_t + \sum_{i=1}^{n} saz + u_t \ (14)$$

However, the chosen approach will only be complete after verifying the No Ponzi Games (NPG) condition, that is, this hypothesis considers that the government will not finance itself eternally by means of debt service nor by the issue of new bonds. Thus, this condition of transversality, NPG[14] must be satisfied in such a way that:

$$\lim_{n\to\infty} \rho^{\,n}\, b_{t+n} = 0 \ (15)$$

After modeling, two families of models were estimated, one with a smaller number of variables, which sought to follow an approach established in the literature, and another with a larger number of variables, aiming to incorporate information from the economic context. To choose the number of lags to be used, those with lower p-values and higher joint significance were observed through the Wald lag exclusion test. In consonance with (Lütkepohl 2005), for a VAR (Vector Autoregression) or VECM to be stationary or stable, it must have all its roots within its unit circle, except for cointegrations. If the model is not stable, some results obtained may not be valid and the public debt will not observe the budget constraint and will have an explosive path.

In the two estimated models, we tried to adapt the approach recommended by (Quintos 1995) to the case of Minas Gerais' internal public debt/GDP, since this approach aimed at measuring the sustainability of the debt, dividing it into strong and weak sustainability. As already explained in section 2, strong sustainability would be defined as a cointegration vector (1,-1) for Primary Revenues/GDP and Primary Expenditures/GDP, respectively, thus implying the fact that every increase in expenditures leads to an increase in revenues in equal number and vice-versa. The weak sustainability condition, on the other hand, has as its main premise only the cointegration between revenues and expenditures, and its cointegration is a vector of the order (1, -b), with $0< b \leq 1$. Thus, the concept of

14 This condition or assumption of the deterministic model imposes on the public debt sustainability equation that the government will not be able to finance itself by perpetually rolling over the debt via issuing new bonds, thus ensuring an optimal solution (MOURA, 2017).

strong sustainability is mixed with the concept of homogeneity and the concept of weak sustainability, and lies between the concept of weak exogeneity and homogeneity.

However, in this paper we will add to the definition of (Quintos 1995) on public debt sustainability, the concept of the Exogeneity Test elaborated and developed by (Engle, Hendry, e Richard 1983; Ericsson 1991; Johansen 1992) to the Primary Revenue/GDP variable, to verify whether it suggests signs of not having a similar dynamics to that of Primary Expenditure/GDP.

Depending on the result of this test, the variable in question, given its degree of exogeneity, which can be weak, strong and super-exogenous, in view of Granger Causality and/or invariance should be considered exogenous to the model and consequently will lend itself to distinct functions such as inference, projections and simulations. Furthermore, the Exogeneity Test may point in the direction of future unsustainability of the public debt of Minas Gerais (Ericsson 1992; Johansen 1992)[15].

After estimating the models, the Granger Causality is observed, which shows how much each variable can generate inputs in the others and what feedbacks each one receives from the others. This test indicates in which variable the first shock would have occurred, in the "Granger sense" (Enders 2014). At this point, it should be noted that a variable that has been tested as weakly exogeneous and is also not considered endogenous in the Granger sense, is conceptualized as strongly exogenous.

Next, Johansen tests were performed to define how many cointegrations would be supported by the modeling in question. In fact, this is one of the most important tests presented for the VECM model, because it provides the cointegration relations between the variables that give body to the model elaborated.[16]

The Impulse-Response test, also known as impulse-response aims to observe the time elapsed between the occurrence of a shock and the return of each variable to its previous equilibrium level, if this equilibrium is re-established at all. It is also important to note that if a variable does not exhibit any response induced by the initial shock of another variable, but is still able to induce effects on the other variables, it is considered as invariant. Returning to the concept of exogeneity, this behavior, coupled with weak exogeneity is conceptualized as super-exogenous (Engle, Hendry, e Richard 1983; Ericsson 1992; Johansen 1992).

In this follow-up, projections will be made for the variables Domestic Public Debt/GDP and Primary Revenue/GDP according to the modeling done in this work (Unrestricted Broad Model, Broad Model Strong Sustainability, Broad Model Weak Sustainability, Simpler Unrestricted Model, Simpler Model Strong Sustainability, Simpler Model Weak Sustainability). Thus, the projections made for the year 2020 will be evaluated in line with the RMSE, MAE, MAPE and Theil accuracy measures[17].

15 For more information regarding exogeneity tests see (Ericsson 1992)

16 For more information see: JOHANSEN, Søren. Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector. Econometrica, Oxford, v. 59, n. 6, p. 1551-1580, Nov. 1991. Available at: <https://www.jstor.org/stable/2938278?seq=1>. Accessed on: 16 Apr. 2020.

17 For more information see (Willmott e Matsuura 2005).

All the above measures are primarily concerned with measuring the accuracy of the models. The RMSE or standard deviation of the residuals, explains how disperse the residuals are. The MAE, in turn, is calculated from the difference between predictions and observed values divided by the number of observations, i.e., this measure of accuracy means the average absolute error of the model predictions; the MAPE is a percentage measure that aims to understand the error rate in the predictions made by the models; finally, Theil's Inequality Coefficient is a weighting of the mean squared error in relation to the sums of squares of the predicted and observed values, it penalizes disproportionate or large errors. Thus, all four accuracy measures are of the type the closer to zero the better.

In this perspective, the present work will make projections with the two model families to see which has the greater accuracy. Once we have seen which one has the greater accuracy, graphs will be plotted for the year 2019-2020 to illustrate to readers the projections made.

3.2 Data Description

This section will demonstrate the characteristics of the variables that make up the models developed in this work. To this end, two tests will be performed with the variables, one of unit roots to verify their stationarity and the second of structural breaks, to identify possible dummies and regime changes arising from economic events.

3.2.1 Unit root tests

To verify the stationarity of each variable, the existence or not of unit roots will be tested by means of the Augmented Dickey-Fuller, Phillips-Perron tests and in case of non-conclusion about the stationarity of the variable, the KPSS test will be performed. All variables were tested in level and first difference.

Table 1 contains all variables tested at level and first difference to verify or not the presence of unit roots. The table presents all variables tested in level for the three tests, ADF, PP and KPSS, except 12-month Accumulated Selic. The analyses at level revealed, for the most part, that we can no longer accept the null hypothesis of the presence of unit roots, except for LN ICMS. The PP test, at level, for variables LN Domestic Debt/GDP, Primary Revenue/GDP, LN Primary Expenditure/GDP, LN PIM-MG and LN Exports from Minas Gerais to China accept the alternative hypothesis of stationarity of the variable. In this sense, it was decided to do a tie-breaker test, KPSS, to determine if there is a possibility of non-rejection of the stationarity hypothesis for the aforementioned test. It was verified that most variables were stationary, except for the LN PIM MG. The variables tested in first difference presented a stationary behavior and accepted the null hypothesis at 1% significance level.

Table 1 - Unit root tests for the variables in the simple broad models

Variable	Unit root tests	Test statistics at level	p-value at level	Test statistic on first difference	p-value in first difference
LN Domestic Debt/GDP	ADF	-2,184782	0,2126	-5,488118	0,0000
	PP	-7,540143	0,0000	-41,62835	0,0001
	KPSS	0,7390*	0,77841		
LN Primary Revenue/ GDP	ADF	-1,899421	0,3322	-14,86112	0,0000
	PP	-8,70641	0,0000	-71,30933	0,0001
		0,7390*	1,932034		
LN Primary Expenditu-re/GDP	ADF	-1,65137	0,4545	-11,3498	0,0000
	PP	-11,79805	0,0000	-63,1562	0,0001
	KPSS	0,7390*	1,891509		
LN ICMS	ADF	-2,71341	0,0735	-7,46708	0,0000
	PP	-2,40224	0,1423	-24,5442	0,0000
	KPSS	0,7390*	1,598701		
LN PIM MG	ADF	-2,26076	0,1859	-4,33143	0,0005
	PP	-4,37836	0,0004	-21,2086	0,0000
	KPSS	0,7390*	0,443917		
Selic Accumulated 12 months	ADF	-0,96227	0,7664	-4,32891	0,0005
	PP	-1,58443	0,4888	-6,5858	0,0000
LN Domestic Debt Ser-vice/GDP	ADF	-2,356230	0,1557	-6,606498	0,0000
	PP	-15,80524	0,0000	-56,98387	0,0001
	KPSS	0,7390*	0,369886		
LN Exports from Minas Gerais to China	ADF	-2,40043	0,1429	-7,7559	0,0000
	PP	-3,64016	0,0057	-33,7303	0,0001
		0,7390*	1,15484		

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

* Asymptotic Critical Values at 1% significance level for the KPSS test.

The proof of the stationarity of the variables is an important step for the estimation of the model, since this characteristic ensures adherence to the VECM that will be estimated in the next section. Furthermore, the level of stationarity verified by the tests conjugates with what was proclaimed by (Hamilton e Flavin 1985; Trehan & Walsh 1988) is the first sign of sustainability of public debt, especially when it comes to the same level of cointegration for Public Debt/GDP, Primary Expenditure/ GDP and Primary Revenue/GDP.

3.2.2 Structural breaks tests

This subsection will show the structural breaks that were tested and placed as dummies for the model families in this work. Below is the table containing them:

Table 1 - Acceptance Test for Structural Breaks for Level and Pulse Dummies

Dummy	Level	In first difference
Dummy for level Exchange rate Oct/2008	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Dummy for level Exchange rate Aug/2014	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Dummy for MG export level to China mar/2010	Cannot reject the alternative hypothesis at 10% significance	Cannot reject the alternative hypothesis at 1% significance
Pulse dummy covid mar/2020*	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Pulse dummy economic crisis feb/2017*	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Pulse dummy subprime crisis mar/2008*	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Pulse dummy subprime crisis aug/2007*	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Pulse dummy Brazilian crisis Nov/2015*	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Pulse Dummy Primary Expenditure/ GDP Jan/2005	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Pulse dummy Primary Expenditure/ GDP Apr/2018	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Primary Expenditure/GDP pulse dummy jun/2019	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Pulse Dummy PIM/MG Dec/2008	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
PIM/MG level dummy nov/2019	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
PIM/MG level dummy May/2020	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Pulse dummy Primary Revenue/GDP Dec/2015	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Pulse dummy Primary Revenue/GDP Oct/2015	Cannot reject the alternative hypothesis at 10% significance	Cannot reject the alternative hypothesis at 1% significance

Dummy Pulse Swap Domestic Debt/ GDP Jan/2012	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Level dummy Complementary Law 148/2014	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance
Dummy of Level Complementary Law 156/2016	Cannot reject the alternative hypothesis	Cannot reject the alternative hypothesis at 1% significance

Source: Table prepared by the authors, based on the tests performed to model the equations

*Tests performed on the Internal Debt/GDP variable

From the analysis of table 1 above, all structural break dummies are acceptable up to 2.5% significance level. That is, all structural break tests are significant for the equations proposed here. Once this structural break test is done, we will include the dummies proposed in the next subsection in the models.

3.3 Empirical Results

As already explained in the methodology section, in this subsection, this work will estimate the VECM type equations for the broad and simple models, in addition to cointegration, Granger causality, homogeneity-exogeneity, non-conventional impulse response tests, and finally projections to the year 2020 to verify the accuracy of the modeled equations.

3.3.1 Estimating equations with 2003-2020 data base

In this subsection, the models estimated for Unrestricted Sustainability, and two restricted ones, namely Strong and Weak Sustainability, will be analyzed. To this end, it was divided into two families of models, one with a smaller number of variables, nicknamed Simpler, and another with a larger number of variables, henceforth called Broad Model. This differentiation aimed in the first case to follow the classical modeling that the seminal authors used and in the second case to bring a broad approach with more variables from the economic context of Minas Gerais.

Table 2 explains the general characteristics of all the estimated models, such as the number of lags, whether a constant is used, whether there are seasonal dummies, pulse dummies, level of economic effects, determinants of covariance residuals, log Maximum Likelihood, Akaike's and Schwarz's Information Criteria, and the number of coefficients.

Table 2 - Estimated Strong, Weak and Unrestricted Sustainability models for 5 and 3 cointegrations[18]

Error Correction Vectors	Unrestricted Broad Model (7 variables - 5 Cointegrations)	Strong Sustainability Model (7 variables - 5 Cointegrations)	Weak Sustainability Broad Model (7 variables - 5 Cointegrations)	Unrestricted Simplest Model (6 variables - 3 Cointegrations)	Simplest Strongest Sustainability Model (6 variables - 3 Cointegrations)	Simplest Model Weak Sustainability (6 variables - 3 Cointegrations)
Lag	1	1	1	1	1	1
Lag	2	2	2	2	2	2
Lag	3	3	3	3	3	3
Lag	12	12	12	12	12	12
Constant	Yes	Yes	Yes	Yes	Yes	Yes
Event Dummies	Yes	Yes	Yes	Yes	Yes	Yes
Seasonal Dummies	Yes	Yes	Yes	Yes	Yes	Yes
Determinant resid covariance (dof adj.)	2.69E-16	3.36E-16	3.36E-16	1.41E-11	1.41E-11	1.42E-11
Determinant resid covariance	1.63E-17	2.03E-17	2.03E-17	3.39E-12	3.39E-12	3.39E-12
Log likelihood	1907.056	1884.705	1.884.688	952.3265	952.3265	951.0541
Akaike information criterion	-13.77395	-13.603	-13.602.84	-6.633759	-6.633759	-6.621222
Schwarz criterion	-5.46645	-5.377111	-5.376.951	-2.080141	-2.080141	-2.067604
Number of coefficients	509	504	504	279	279	279

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

In the model with a smaller number of variables, the following were considered: LN Domestic Debt/GDP, LN Primary Revenue/GDP, LN Primary Expenditure/GDP, 12-month Accumulated Selic

18 For more information about the models, see the technical appendixes.

Rate, LN PIM-MG and LN Domestic Debt Service/GDP. This last variable presented a dynamic endogenous to this first modeling. We will see below, that in the model with a larger number of economic context variables, the service will present characteristics of exogeneity given the underlying tests of this matter, for example, the Granger-Causality Test.

In this family of models, it was noticed from table 2 that the model with the lowest Maximum Likelihood is the Weak Sustainability one, moreover, the Akaike's Tie-breaker Criterion between models, the highest negative values observed were from the Unrestricted and Strong Sustainability models, therefore indicating that these models have a better tie-breaker criterion than the previous one.

As previously mentioned, in the model with more economic context variables, Debt Service, necessarily, needed to be modeled as an exogenous variable. Furthermore, in this second model, LN Domestic Debt/GDP, LN Primary Revenue/GDP, LN Primary Expenditure/GDP, LN ICMS, 12-month Accumulated Selic Rate, LN PIM-MG and LN Exports from Minas Gerais to China were considered endogenously. And the exogenous variables were the CPI-deflated Prime Rate, the Exchange Rate and seasonal, pulse and level dummy variables for economic events.

In this model with more variables, in comparison with the previous models, better adjustments of the set of equations were found, as per technical appendix, sections 6.1.1, 6.2.1 and 6.3.1. Thus, the Akaike Criterion indicated that we should choose this family of models, since its values, in module, are greater than those of the previous models. Although the first family of models has a larger number of parameters, and therefore should suffer a cross-criteria penalty, in terms of Akaike, it remains a model of choice.

3.3.2 Cointegration test

As mentioned earlier in this paper, modeling VECM-type equations are characterized as cointegrated processes among their endogenous variables. As stated (Engle e Granger 1987):

> "The components of the vector Xt , are said to be cointegrated of order d, b, identified as Xt ~ CI (d,b), (i) if all components of X_t are I(d); (ii) there exists a vector α (≠ 0) where $z_t = \alpha' x_t$ ~I (d-b), b>0" (Engle e Granger 1987).

Therefore, testing this condition between public revenues and expenditures would guarantee, according to pioneering authors in this methodology, evidence of public debt sustainability (Trehan e Walsh 1988; Kremers 1989). However, this paper will follow a different path, testing cointegrations in larger models than those enshrined in the seminal papers reviewed. In this way, even the model with fewer variables has, on average, more variables than the classical modeling.

Next are the tests of the number of cointegrations, it is reiterated that the Akaike Information Criterion was used to define the number of cointegrations for the two families of models. The model with the highest number of cointegrations identified 5 cointegrations, being linear, intercept and trend, by Akaike's criterion. The other, on the other hand, identified 3 cointegrations, linear, intercept and trend, by the same criterion, similarly to the first.

Table 3 - VECM cointegration test Models: Unrestricted Broad, strong and weak sustainability for Akaike's information criterion

Trend of the data:	-	-	Linear	Linear	Quadratic
Rank or No. of Cointegrations	No Intercept	Intercept	Intercept	Intercept	Intercept
	No Trend			Trend	
0	-14.6476	-14.6476	-14.64495	-14.64495	-14.62939
1	-15.13083	-15.1298	-15.13336	-15.16767	-15.16138
2	-15.42779	-15.42037	-15.4173	-15.44244	-15.44599
3	-15.6595	-15.67182	-15.65243	-15.70784	-15.71887
4	-15.72764	-15.84268	-15.8269	-15.91959	-15.91993
5	-15.72628	-15.89587	-15.87886	-16.04981*	-16.03153
6	-15.60792	-15.86055	-15.85194	-16.01314	-16.00406
7	-15.4802	-15.72821	-15.72821	-15.88712	-15.88712

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

Tabela 4 - VECM cointegration test Simple Unrestricted, Strong Sustainability and Weak Sustainability models for Akaike's information criterion

Trend of the data:	-	-	Linear	Linear	Quadratic
No. of Cointegrations	No Intercept	Intercept	Intercept	Intercept	Intercept
	No Trend			Trend	
0	-6.599568	-6.599568	-6.845836	-6.845836	-6.788388
1	-6.82424	-6.840559	-7.067756	-7.197179	-7.149113

2	-7.008156	-7.020208	-7.245717	-7.409245	-7.370919
3	-7.11415	-7.181131	-7.329163	-7.520459*	-7.491508
4	-7.094592	-7.254584	-7.312534	-7.518139	-7.49871
5	-7.025164	-7.223295	-7.214693	-7.417539	-7.407945
6	-6.909249	-7.100188	-7.100188	-7.296253	-7.296253

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

After the Johansen cointegration tests, the next section will analyze the Granger-causality test for the variables in each model.

3.3.3 Granger Causality Test

The Granger Causality test aims at discovering causality among the variables in a VAR or VECM. According to (Granger 1969), the idea of the test in question is, given for example any multivariate model with p lags (equation number), one tests the null hypothesis that $\{y_t\}$ does not cause the variable $\{x_t\}$ in the Granger sense, if and only if all the coefficients of A21 (L) are equal to zero. Thus, if $\{y_t\}$ does not improve the prediction of $\{x_t\}$, we say that $\{y_t\}$ does not cause $\{x_t\}$ in the Granger sense.

$$\begin{bmatrix} x_{1t} \\ x_{2t} \\ \vdots \\ x_{nt} \end{bmatrix} = \begin{bmatrix} A_{10} \\ A_{20} \\ \vdots \\ A_{n0} \end{bmatrix} + \begin{bmatrix} A_{11}(L) & A_{21}(L) & \cdots & A_{1n}(L) \\ A_{12}(L) & A_{22}(L) & \dots & A_{2n}(L) \\ \vdots & \vdots & \ddots & \vdots \\ A_{n1}(L) & A_{n2}(L) & \dots & A_{nn}(L) \end{bmatrix} \begin{bmatrix} x_{1t-1} \\ x_{2t-1} \\ \vdots \\ x_{nt-1} \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ \vdots \\ e_{nt} \end{bmatrix} \quad (16)$$

Where x_t is a vector containing the model variables A_{i0} are the parameters of the intercept terms, and A_{ij} (L) are the polynomials of the lag operator L. However, Granger causality alone would not be sufficient to ascertain the genuine causal link necessary for the modeling proposed here. In this sense, the testing of the variable's exogeneity condition is necessary to verify its real condition in the proposed models (Engle, Hendry, e Richard 1983; Ericsson 1992). As will be seen in the next subsection, exogeneity is different from Granger-causality, because for any variable z_t to have this characteristic it is necessary that it is not affected by the contemporaneous values of y_t. In contrast, Granger-causality is limited only to the past effects of $\{y_t\}$ on $\{z_t\}$. Thus, this test would measure how much the past values of $\{y_t\}$ would help in forecasting $\{z_t\}$. (Enders 2014)[19].

19 For further explanation see (Enders 2014)

Table 5 and 6 below summarize the Granger Causality test for the variables Domestic Debt/GDP and Primary Revenue/GDP of the Unrestricted Model, and the three models estimated here are listed. Just as an example, in the broad model, the variable LN Domestic Debt/GDP is better explained by the variables Primary Revenue/GDP, Primary Expenditure/GDP, ICMS and SELIC. Furthermore, this same analysis should be repeated for the other variables. It should be emphasized that the set of variables must present a p-value at 90% significance for the entire model. In the strong and weak Sustainability models, one observes the permanence of this significance pattern for the Domestic Debt/GDP variable[20].

Table 5 - Granger Causality Test - Internal Debt/GDP Variable - Unrestricted Model

Dependent variable: D(LOG(DIV_INT_R-GDP))									
Variables/Model	**Unrestricted Broad Model**			**VECM Broad model of strong sustainability**			**VECM Broad model of weak sustainability**		
Excluded	**Chi-sq**	**df**	**Prob.**	**Chi-sq**	**df**	**Prob.**	**Chi-sq**	**df**	**Prob.**
D(LOG(REC_PRIM_PIB))	10.21082	4	0.0370	10.20146	4	0.0372	10.22011	4	0.0369
D(LOG(DESP_PRIM_PIB))	8.24774	4	0.0829	8.192218	4	0.0848	8.216941	4	0.0839
D(LOG(ICMS_R))	17.24779	4	0.0017	17.77252	4	0.0014	17.77217	4	0.0014
D(LOG(PIM_MG))	4.819745	4	0.3063	5.571376	4	0.2335	5.556133	4	0.2348
D(SELIC_RATE_R_12M)	9.193164	4	0.0564	9.420754	4	0.0514	9.437488	4	0.051
D(LOG(EXPORT_MG_R))	2.922885	4	0.5708	2.913223	4	0.5725	2.922569	4	0.5709
All	46.86997	24	0.0035	46.61951	24	0.0037	46.65926	24	0.0037

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

The Primary Revenue/GDP variable in the Broad model is not well explained by any of the broad models, as per the table below, suggesting that the variable in question should be checked for its degree of exogeneity.

20 For more information see tables 6.1.2, 6.3.2, 6.4.2, 6.5.2,6.6.2 in the Appendices section.

Table 6 - Granger Causality Test - Primary Revenue/GDP Variable - Unrestricted Model

Variáveis/Modelo	Modelo Amplo Irrestrito			VECM Modelo Amplo de sustentabilidade forte			VECM Modelo Amplo de sustentabilidade fraca		
Excluded	**Chi-sq**	**df**	**Prob.**	**Chi-sq**	**df**	**Prob.**	**Chi-sq**	**df**	**Prob.**
D(LOG(DIV_INT_R_PIB))	4.071549	4	0.3964	1.564603	4	0.8151	1.518874	4	0.8233
D(LOG(DESP_PRIM_PIB))	2.397157	4	0.6631	1.704426	4	0.7899	1.702463	4	0.7903
D(LOG(ICMS_R))	2.745232	4	0.6013	1.845651	4	0.7641	1.854001	4	0.7626
D(LOG(PIM_MG))	4.621394	4	0.3284	5.865102	4	0.2095	5.859974	4	0.2099
D(SELIC_RATE_R_12M)	1.766973	4	0.7785	2.027323	4	0.7307	2.026584	4	0.7309
D(LOG(EXPORT_MG_R))	6.099738	4	0.1918	10.19032	4	0.0373	10.1999	4	0.0372
All	29.46903	24	0.203	30.57147	24	0.1665	30.5647	24	0.1667

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

Regarding the simplest models, unrestricted, strong and weak sustainability; the Domestic Debt/GDP series presents an endogenous behavior to the modeling, as well as its counterpart in the broad model, the main variables that explain it are Primary Expenditure/GDP and Debt Service/GDP. The Primary Revenue/GDP series is endogenous to the simplest model and is mainly explained by the Domestic Debt/GDP, PIM/MG and the Domestic Public Debt Service/GDP.

Table 7 - Granger Causality Test - Internal Debt/GDP Variable - Unrestricted Model

	Simple Unrestricted Model			Simple Model Strong Sustainability			Simple Model Weak Sustainability		
Excluded	**Chi-sq**	**df**	**Prob.**	**Chi-sq**	**df**	**Prob.**	**Chi-sq**	**df**	**Prob.**
D(LOG(DESP_PRIM_PIB))	8.569405	4	0.0728	8.569405	4	0.0728	8.148421	4	0.0863
D(LOG(REC_PRIM_PIB))	6.549552	4	0.1617	6.549552	4	0.1617	6.36956	4	0.1732
D(SELIC_RATE_R_12M)	4.6175	4	0.3288	4.6175	4	0.3288	4.422545	4	0.3518
D(LOG(PIM_MG))	5.821673	4	0.2129	5.821673	4	0.2129	5.856928	4	0.2101
D(LOG(SEV_DIV_PIB))	20.15797	4	0.0005	20.15797	4	0.0005	19.7871	4	0.0006
All	45.50519	20	0.0009	45.50519	20	0.0009	44.47331	20	0.0013

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

Table 8 - Granger Causality Test - Primary Revenue/GDP Variable - Unrestricted Model

	Simple Unrestricted Model			Simple Model Strong Sustainability			Simple Model Weak Sustainability		
Excluded	**Chi-sq**	**df**	**Prob.**	**Chi-sq**	**df**	**Prob.**	**Chi-sq**	**df**	**Prob.**
D(LOG(DIV_INT_R_PIB))	15.32087	4	0.0041	15.32087	4	0.0041	15.29725	4	0.0041
D(LOG(DESP_PRIM_PIB))	6.732653	4	0.1507	6.732653	4	0.1507	6.137934	4	0.1891
D(SELIC_RATE_R_12M)	0.282031	4	0.9909	0.282031	4	0.9909	0.252376	4	0.9927
D(LOG(PIM_MG))	7.833331	4	0.0979	7.833331	4	0.0979	7.92334	4	0.0944
D(LOG(SEV_DIV_PIB))	15.71421	4	0.0034	15.71421	4	0.0034	16.00656	4	0.003
All	59.63595	20	0	59.63595	20	0	59.42331	20	0

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

3.3.4 Homogeneity-exogeneity test

This paper, as already explained in the methodology subsection, investigates the sustainability of Minas Gerais' domestic public debt/GDP through the approaches of (Quintos 1995) and the concept of exogeneity of the Primary Revenue/GDP variable in the terms of (Engle, Hendry, e Richard 1983). Hence, the idea now is to interpret the tests underlying it, especially in the weak sustainability models. However, before we proceed, we will briefly discuss the concept of homogeneity, in which the frequency distribution of the variables should be normal and their variances should behave in a homoscedastic form. These are generally necessary aspects for the verification of this condition, and they guarantee that the variables can be taken as endogenous to the model and cointegrated. Having said this, we move on to the concept of exogeneity. According to (Ericsson 1992)the testing of exogeneity allows us to know whether the removal of a variable from the model interferes or not with its explanatory power. In this sense, the author in question explains three concepts of exogeneity, namely, weak, strong and super exogeneity.

Weak exogeneity is an essential concept required to infer correlations and efficiencies from a conditional model (estimations and hypothesis testing). However, (Quintos 1995) does not use it, since at no point does it consider the $\alpha=0$. This is easily inferable, since it limits $0< \beta \leq 1$. If the condition β = 0 were accepted by the author in screen, it would be equivalent to considering α = 0 and β would transfer information to the equation, however, it would not receive feedbacks.

In terms of hypothesis testing for exogeneity where β is conceptualized as 1 and -1 for the Primary Revenue and Primary Expenditure/GDP variables, and there is an imposition that α = 0 for Primary Revenue/GDP relative to Primary Expenditure/GDP, the Likelihood Ratio test (LR) must show a high chi-square and a low p-value for there to be non-acceptance of the null hypothesis of variable exogeneity. Table 9 and 10 below, differently, point in the direction that the variable in question is weakly exogenous to the weak sustainability models presented. Conjugating this analysis with the Granger Causality test presented, one verifies that for the broad model, the variable is strongly exogenous to the model, indicating a degree of unsustainability in Minas Gerais' public debt. The model with a smaller number of variables (5 variables, 3 cointegrations), on the other hand, made explicit that, when analyzing the LR test and the Granger Causality test, it is weakly exogenous, with the Domestic Debt/GDP, LN PIM, LN Service Domestic Public Debt/GDP explaining the past movements of the variable in question.

Table 9 - LR Exogeneity/Homogeneity Test Weak Sustainability Broad Model (5 Cointegrations)

Lags 1, 2, 3 and 12; Seasonal, pulse and level dummies	
Cointegration Constraints:	
B(1,1) = 1, B(1,2) = 0, B(1,3) = 0, B(1,4) = 0, B(1,5)=0, B(2,1)= 0, B(2,2)=1, B(2,3)=0, B(2,4)=0, B(2,5)=0, B(3,1)=0, B(3,2) = 0, B(3,3)=-1, B(3,4)=0, B(3,5)=0, B(4,1)=0, B(4,2)=0, B(4,3)=0, B(4,4)=1, B(4,5)=0, B(5,1)=0, B(5,2)=0, B(5,3) =0,B(5,4)=0,B(5,5)=1,A(2,3)=0	
LR test for binding restrictions (rank = 5):	
Chi-square(1)	0.032436
Probability	0.857074

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

For more details see technical appendix.

Table 10 - LR Exogeneity/Homogeneity Test - Simplest Model Weak Sustainability

Lags 1, 2, 3 and 12; Seasonal, pulse and level dummies	
Cointegration Constraints:	
B(1,1)= 1, B(2,2) = -1, B(3,3)= 1, B (1,2) = 0, B(1,3) = 0, B(2,1)= 0, B(2,3) = 0, B(3,1)= 0, B(3,2) = 0, A(3,2) =0	
LR test for binding restrictions (rank = 3):	
Chi-square(1)	2.544917
Probability	0.110650

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

3.3.5 Unconventional Impulse-Response Test for Debt

Impulse-response analysis sheds light on some aspects of the behaviors of variables within the modeling presented here and the information revealed by the Granger Causality test. In this sense, Impulse-response functions can explain the nature of the relationships between variables, as well as demonstrate how responsive one variable is to an exogenous shock occurring in another (Lütkepohl 2005).

In this paper, we used the non-conventional Impulse-Response test, recommended by (Issler e Lima 1997) where the responses are given as a function of the innovations of the present values of the time series and not of the system as a whole. This test aims to understand the response of a variable to a shock of one unit of a variable in the present time. In addition, the number of periods required for the impacted variable to return to its initial equilibrium is also observed.

The analysis of the Non-Conventional Impulse-Response graphs for the Unrestricted Broad Model denotes that the Domestic Public Debt/GDP has a peculiar behavior in its responses to the other variables of the model. Notably, in what concerns Primary Revenue and Expenditure/GDP, which suggests that the variable in question has taken off from what would be the conventional model related to the aforementioned variables. In other words, the Domestic Debt/GDP is much more dependent on its own dynamics, given its invariance in relation to the other variables. It is worth mentioning that the Internal Debt/GDP suffers an effect from the impact of the PIM-MG, demonstrating what seems to be an appetite for debt contraction in moments of economic euphoria.

In the broad model of strong sustainability, the Domestic Debt/GDP series receives small feedbacks from Primary Revenue and Expenditure/GDP, ICMS and Exports to China. However, it describes a quasi-invariant behavior for PIM/MG and 12 Month Selic Rate. This result shows that although the 12-month Selic is one of the indexers of the variable in question, it has a very specific behavior that is not significantly affected by the Selic. The Primary Revenue/GDP is affected mainly by the Internal Debt/GDP and the PIM/MG, besides itself. However, the variable in question is invariant regarding Primary Expenditure/GDP, that is, an increase of one unit in expenditure does not have a significant return in the increase of revenues. In this way, one can deduce that the Minas Gerais government has a certain revenue ceiling. In fact, when we look at the next impulse response graph, the reverse is not true. Thus, increases in the PIM and the ICMS generate an increase in Expenses, as well as an increase in the Domestic Public Debt/GDP.

The Broad model of weak sustainability points to a trend like previous models, that is, an invariance of the Domestic Debt/GDP vis-à-vis the other variables, except for the PIM/MG, a variable that generates some feedbacks for the series in question. In turn, Primary Revenue/GDP does not receive considerable feedbacks from the other series, only the PIM/MG has some strength in this model.

In the simplest models, in general, debt is invariant to the other variables, while Primary Revenue/GDP receives feedbacks from Domestic Debt/GDP and PIM/GDP. Finally, Primary Expenditure/GDP is affected by Domestic Debt/GDP, Primary Revenue/GDP and PIM/GM.

3.3.6 Projections and accuracy tests of the equations for the year 2020

This subsection will analyze the forecastings for each model estimated from the sustainability models for the year 2020, therefore verifying those with better adjustments to what actually happened.

According to table 10, on the following page, where the results of the accuracy tests (RMSE, MAE, MAPE and Theil) for the broad model (unrestricted, strong and weak sustainability) are presented. According to these tests, for the Internal Debt/GDP variable, the projections that were closest to reality were those of the unrestricted model. Similarly to the previous variable, for Primary Revenue/GDP the best model indicated by the tests was again the Unrestricted one. Except for Theil's test, for the Primary Expenditure/GDP variable that pointed to the Weak Sustainability model, all other tests indicated the Unrestricted model as being the most accurate. Since these three variables are the most interesting for this study, we will leave the analysis of each case to the reader's observation.

Regarding table 11, which deals with the result of the accuracy tests for the simplest model, the debt variable is more accurate in the Strong Sustainability model. The projection of the Primary Revenue/GDP variable is better explained by the Simple Unrestricted model. The Primary Expenditure/GDP variable, except for its MAPE, is better explained by the Strong Sustainability model. However, when we compare the results for the two models, the projections better adhered to the values effectively verified point to the Unrestricted Simple Model, in consonance with the variables of greatest interest (Internal Debt/GDP, Primary Revenue/GDP, Primary Expenditure/GDP).

Finally, to explain the projections made, three graphs will be plotted for the variables of interest to this study, primary revenue and domestic debt/GDP. It can be noticed that even in the purely visual analysis, the projections of the broad model are more accurate with the values for 2020, as can be seen in graphics 1 and 2.

Table 11 - Forecasting Accuracy Broad Model

Variable / Model	Unrestricted	Strong Sustainability	Weak Sustainability	Unrestricted	Strong Sustainability	Weak Sustainability	Unrestricted	Strong Sustainability	Weak Sustainability	Unrestricted	Strong Sustainability	Weak Sustainability
	RMSE			MAE			MAPE			Theil		
DESP_PRIM_PIB	0.001857	0.001887	0.001864	0.001187	0.001302	0.001289	8.500429	10.11980	10.09858	0.075426	0.075960	0.074936
DIV_INT_R_PIB	0.003922	0.004018	0.004093	0.003363	0.003465	0.003555	1.959130	2.016738	2.068672	0.011402	0.011682	0.011898
EXPORT_MG_R	1.37E+08	1.47E+08	1.35E+08	1.05E+08	1.06E+08	1.02E+08	12.49547	12.72955	12.45523	0.074746	0.078487	0.072361
ICMS_R	1.49E+08	1.76E+08	1.82E+08	1.29E+08	1.47E+08	1.52E+08	3.035271	3.474805	3.616041	0.016279	0.019190	0.019738
PIM_MG	1.308314	1.408872	1.435192	1.063520	1.080243	1.137618	1.438989	1.468772	1.536709	0.008480	0.009122	0.009290
REC_PRIM_PIB	0.000445	0.000467	0.000506	0.000346	0.000356	0.000402	3.333123	3.497114	3.965075	0.020799	0.021712	0.023532
SELIC_RATE_R_12M	1.244364	1.167321	1.234142	1.191519	1.124265	1.185005	102.8256	95.55594	101.4230	0.360798	0.343334	0.359571

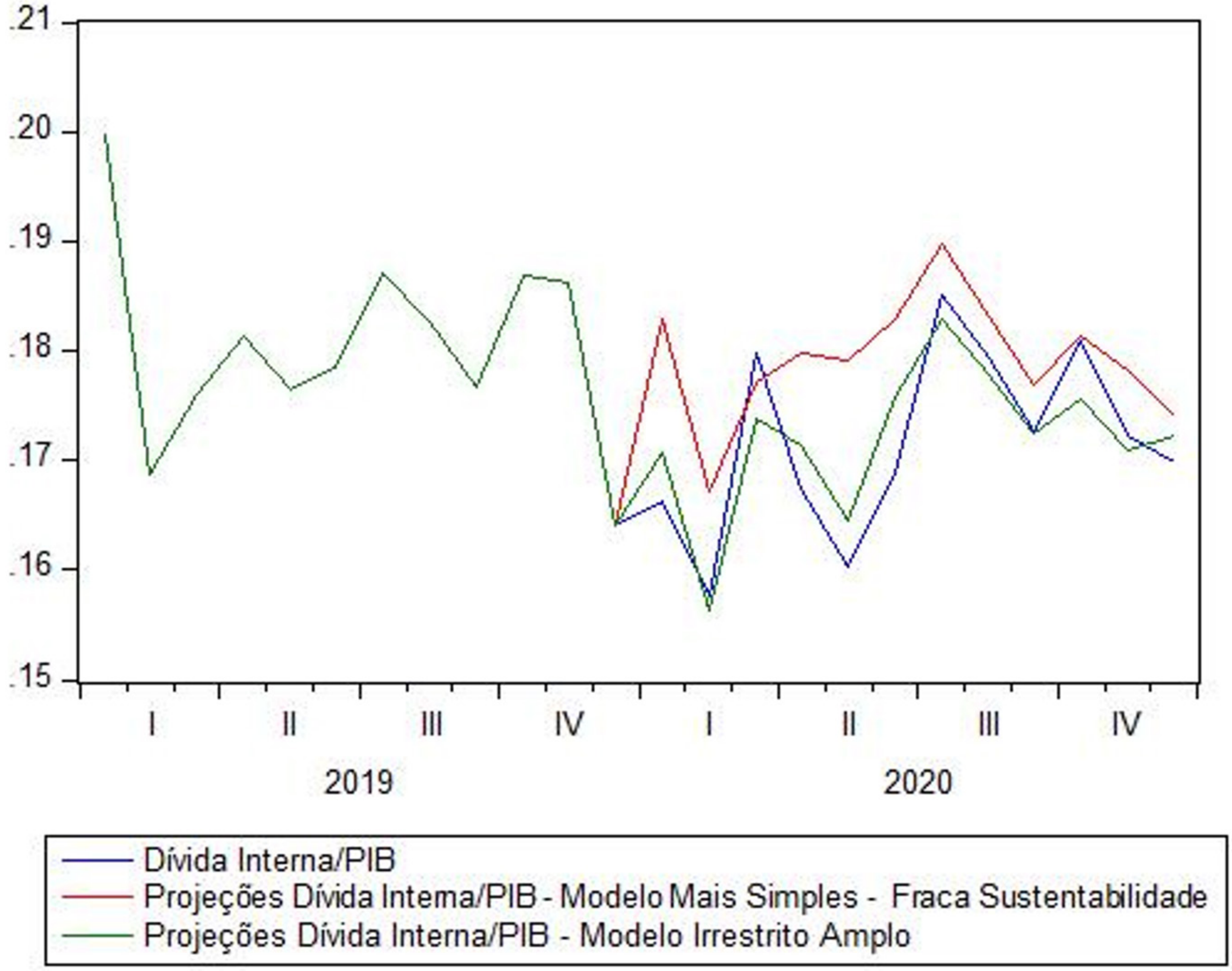
Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

Table 12 - Forecastings Accuracy Simple Model

Variable / Model	Unrestricted	Strong	Weak	Unrestricted	Strong	Weak	Unrestricted	Strong	Weak	Unrestricted	Strong	Weak
	RMSE			MAE			MAPE			Theil		
DESP_PRIM_PIB	0.003196	0.003075	0.003180	0.002734	0.002653	0.002701	23.27949	23.22502	22.98170	0.128140	0.123165	0.127482
DIV_INT_R_PIB	0.010717	0.009648	0.009933	0.009389	0.007628	0.008108	5.581816	4.567351	4.849397	0.030398	0.027518	0.028272
PIM_MG	6.240755	6.770446	6.481351	4.567009	4.967673	5.153055	6.439509	7.024416	7.130159	0.039424	0.043363	0.040783
REC_PRIM_PIB	0.000732	0.000752	0.000763	0.000585	0.000610	0.000612	5.707442	5.892679	5.710528	0.033622	0.034701	0.035490
SELIC_RATE_R_12M	1.984105	1.809242	1.913528	1.826519	1.652978	1.742623	154.2998	137.3352	144.2064	0.553396	0.537428	0.558559
SEV_DIV_PIB	0.000995	0.000902	0.000959	0.000662	0.000609	0.000655	59.98902	58.67862	62.99437	0.469101	0.405842	0.438073

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

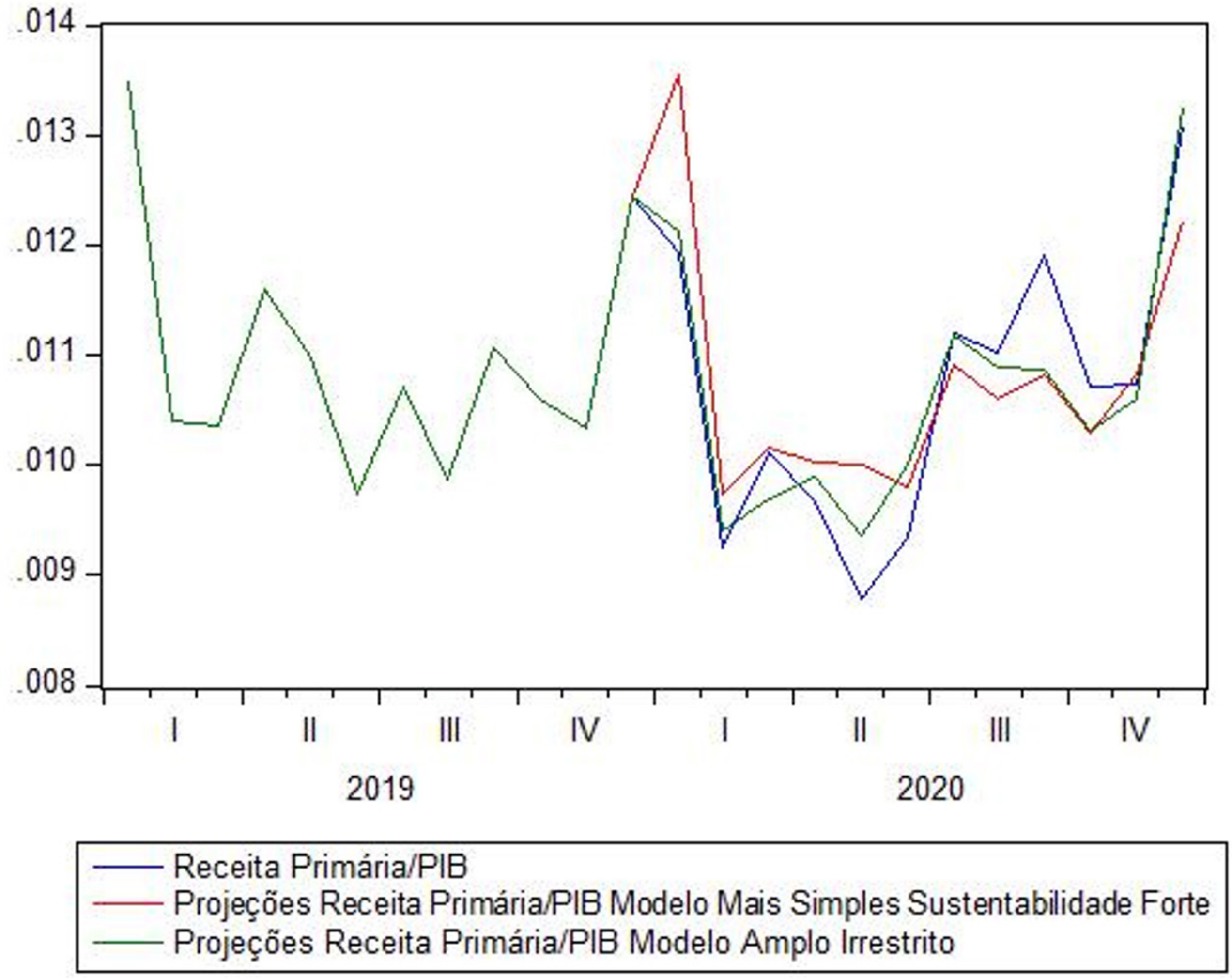
Chart 1 - Domestic Debt/GDP Projections - 2020



- Domestic Debt/GDP
- Domestic Debt/GDP Projections - Simplest Model - Poor Sustainability
- Domestic Debt/GDP Projections - Unrestricted Broad Model

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

Chart 2 - Primary Revenue/GDP Projections - 2020



- Domestic Debt/GDP
- Domestic Debt/GDP Projections - Simplest Model - Poor Sustainability
- Domestic Debt/GDP Projections - Unrestricted Broad Model

Source: Prepared by the authors with data extracted from (Brazil 2020a; Brazil 2020b; Brazil 2020c; Brazil 2020d; Minas Gerais 2020a; Minas Gerais 2020b; Minas Gerais 2020c) analyzed and compiled by the authors through tests performed in Eviews 11.

4 Conclusion

This paper sought to present, briefly, a literature review of models that investigate North American and Brazilian public debt sustainability, both related to central governments (Hamilton e Flavin 1985; Kremers 1989; Wilcox 1989; Trehan e Walsh 1988; Bohn 1991; 1998; Pastore 1994; Issler e Lima 1997; Rocha 1997). In sequence, we sought to apply the techniques of these authors to the case of the internal public debt of the state of Minas Gerais. In this sense, it was necessary to present the concepts mentioned by the authors, namely unit root tests, structural breaks tests, estimation of VAR models, cointegration test, estimation of VECM models, Granger causality test, homogeneity test, weak exogeneity through Likelihood Ratio test (LR), strong exogeneity through Granger criterion analysis; and super-exogeneity, where the impulse-response test is carried out to evaluate the existence or not of invariance of the time series studied.

From this perspective, a central theme of this work refers to the concepts proposed by (Quintos 1995) of Strong and Weak Sustainability. The first is confused with the concept of homogeneity of (Engle, Hendry, e Richard 1983; Ericsson 1992; Johansen 1992). On the other hand, the concept of Weak Sustainability is not to be confused with the concept of weak exogeneity of the authors already mentioned, because if a variable within a vector presents an $\alpha = 0$, it should necessarily be considered exogenous to the model. What actually happens, in the concept of Fifths, is that $\alpha > 0$ and the β lie between 0 and 1 ($0< \beta \leq 1$). Thus, Weak Sustainability does not have the strength of a (1,-1) vector cointegration.

Once this conceptualization was made, it was tested whether strong sustainability would prove to be consistent throughout the analyzed period or whether there would be Weak Sustainability, which would denote periods of fiscal and financial inconsistencies mixed with course correction measures, aiming at the payment of the State's financial obligations.

This research aimed to study the sustainability of Minas Gerais' public debt through the estimation of two families of models: the first with a larger number of variables, aiming to incorporate the economic environment with endogenous and exogenous variables (7 endogenous variables, 3 exogenous and 5 cointegrations) and the second, simpler (6 endogenous variables, 3 cointegrations and no exogenous variables). The results pointed out that the best fit was achieved by the broader model. It was interesting to note that in the simpler model, the variable Domestic Debt Service/GDP was endogenously located given the homogeneity/exogeneity tests. However, when estimating the broader model, this same variable had to be considered exogenous, according to its exogenous behavior in Granger terms. We emphasize that in both models several qualitative intervention variables (pulse and level dummies) were used.

With regard to the models studied, when we evaluate the behavior of the historical series Domestic Debt/GDP for the Granger Causality test, it behaves endogenously. For the Primary Revenue/GDP series, the analysis of the family of two models showed its weak exogeneity in relation to

Primary Expenditure/GDP. However, in the simplest model, the variable in question was only considered weakly exogenous due to its endogenous behavior in Granger terms and was not considered invariant for Debt/GDP, Primary Expenditure/GDP, PIM/MG.

In what concerns the broad model, the Primary Revenue/GDP, when its degree of exogeneity was tested, vis-à-vis the Primary Expenditure/GDP, it was not possible to reject the null hypothesis of weak exogeneity. When Granger is evaluated (joint probability of 0.16), strictly speaking, it should be considered as strongly exogenous to the model. However, it is important for the stability of the model, since it generates turbulence in almost all the other variables when allocated exogenously. When we analyze its behavior by means of the non-conventional impulse-response test, it responds to shocks in the Internal Debt/GDP time series, which does not allow us to take it as super-exogenous.

Such results, when analyzed for the broad model, suggest that the Primary Expenditure/GDP is not cointegrated with the Primary Revenue/GDP, presenting signs of Poor Sustainability. In other words, this is a factor that generates concern for public managers, in the sense of constantly correcting the course of fiscal policies, with the objective of returning to a sustainable process for the Internal Debt/GDP of Minas Gerais.

As proposed in the introduction, after performing all the tests mentioned above, several estimates and projections were made based on the models analyzed. As shown in section 3.3.5, the broad models had a higher accuracy both in performance tests and in the purely visual analysis when compared to the simple models. In this sense, the Broad models, by incorporating economic context variables, provided an explanatory gain to them over the simple model family.

The cold analysis of this myriad of results denotes the pressing need for the state to implement efforts in the areas of both Revenue and Public Expenditure, since these are the manageable instruments within its reach. In this sense, the present work sought to contribute to the debate on Subnational Public Finances, notably for the case of Minas Gerais. In this way, it reviewed, adapted and applied the models in the Public Debt Sustainability bibliography to a subnational entity. It should be emphasized that, previously, the proposed modeling was only applied at the Central Government level, in the case of the VECM modeling. Certainly, this work is not intended to be definitive, and is in fact a contribution of these authors who aim to foster the debate on sustainability issues for Brazilian subnational governments. It is suggested that future studies be conducted in this field to enrich the debate and provide subnational governments with instruments that can combine theory and practice, forecast scenarios and assist public administrators in the diagnosis of public finances and in decision making.

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6 Attachments

6.1 Unrestricted Broad Model

6.1.1 Output of the Equation

Unrestricted Large Model (5 Cointegrations)							
Lags 1, 2, 3 and 12; Seasonal, pulse and level dummies							
	D(LOG(DIV_INT_R_PIB))	D(LOG(REC_PRIM_PIB))	D(LOG(DESP_PRIM_PIB))	D(LOG(ICMS_R))	D(LOG(PIM_MG))	D(SELIC_RATE_R_12M)	D(LOG(EXPORT_MG_R))
R-squared	0.843609	0.880911	0.848777	0.686968	0.905828	0.698409	0.697573
Adj. R-squared	0.767714	0.823118	0.775390	0.535055	0.860126	0.552049	0.550807
Sum sq. resids	0.092520	0.591359	3.126802	0.353120	0.087455	13.47613	6.867154
S.E. equation	0.026082	0.065941	0.151628	0.050956	0.025358	0.314784	0.224708
F-statistic	11.11541	15.24247	11.56571	4.522127	19.82061	4.771846	4.752960
Log likelihood	492.8495	304.5670	135.5348	356.9020	498.5639	-12,74746	55.68078
Akaike AIC	-4,195561	-2,340562	-0,67522	-2,856178	-4,251862	0.785689	0.111519
Schwarz SC	-3,10204	-1,247041	0.418301	-1,762657	-3,15834	1.879210	1.205041
Mean dependent	-0,000737	0.002632	0.009022	0.003512	0.000265	-0,075912	0.007373
S.D. dependent	0.054117	0.156788	0.319938	0.074729	0.067804	0.470325	0.335276

6.1.2 Granger Causality Test

VEC Granger Causality/Block Exogeneity Wald Tests

Date: 08/10/21 Time: 21:33

Sample: 2003M01 2020M12

Included observations: 203

Dependent variable: D(LOG(DIV_INT_R_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(REC_PRIM_PIB))	10.21082	4	0.0370
D(LOG(DESP_PRIM_PIB))	8.247740	4	0.0829
D(LOG(ICMS_R))	17.24779	4	0.0017
D(LOG(PIM_MG))	4.819745	4	0.3063
D(SELIC_RATE_R_12M)	9.193164	4	0.0564
D(LOG(EXPORT_MG_R))	2.922885	4	0.5708
All	46.86997	24	0.0035

Dependent variable: D(LOG(REC_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	4.071549	4	0.3964
D(LOG(DESP_PRIM_PIB))	2.397157	4	0.6631
D(LOG(ICMS_R))	2.745232	4	0.6013
D(LOG(PIM_MG))	4.621394	4	0.3284
D(SELIC_RATE_R_12M)	1.766973	4	0.7785
D(LOG(EXPORT_MG_R))	6.099738	4	0.1918
All	29.46903	24	0.2030

Dependent variable: D(LOG(DESP_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	5.476231	4	0.2418
D(LOG(REC_PRIM_PIB))	5.467321	4	0.2426
D(LOG(ICMS_R))	2.124982	4	0.7128
D(LOG(PIM_MG))	4.776786	4	0.3110
D(SELIC_RATE_R_12M)	7.130736	4	0.1291
D(LOG(EXPORT_MG_R))	0.830997	4	0.9342
All	32.33204	24	0.1190

Dependent variable: D(LOG(ICMS_R))

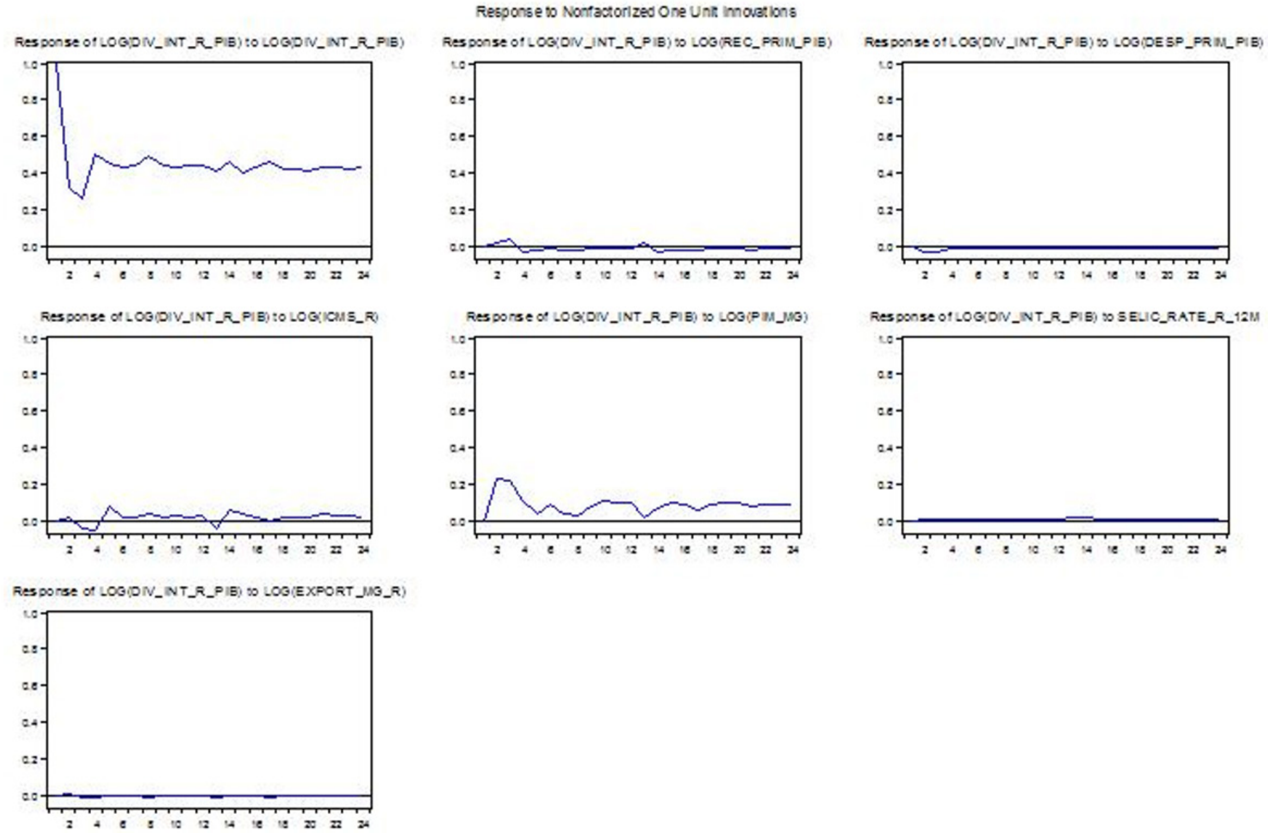
Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	1.357136	4	0.8516
D(LOG(REC_PRIM_PIB))	4.427933	4	0.3512
D(LOG(DESP_PRIM_PIB))	0.965040	4	0.9150
D(LOG(PIM_MG))	5.248196	4	0.2628
D(SELIC_RATE_R_12M)	4.035911	4	0.4012
D(LOG(EXPORT_MG_R))	5.334177	4	0.2547
All	20.76311	24	0.6527

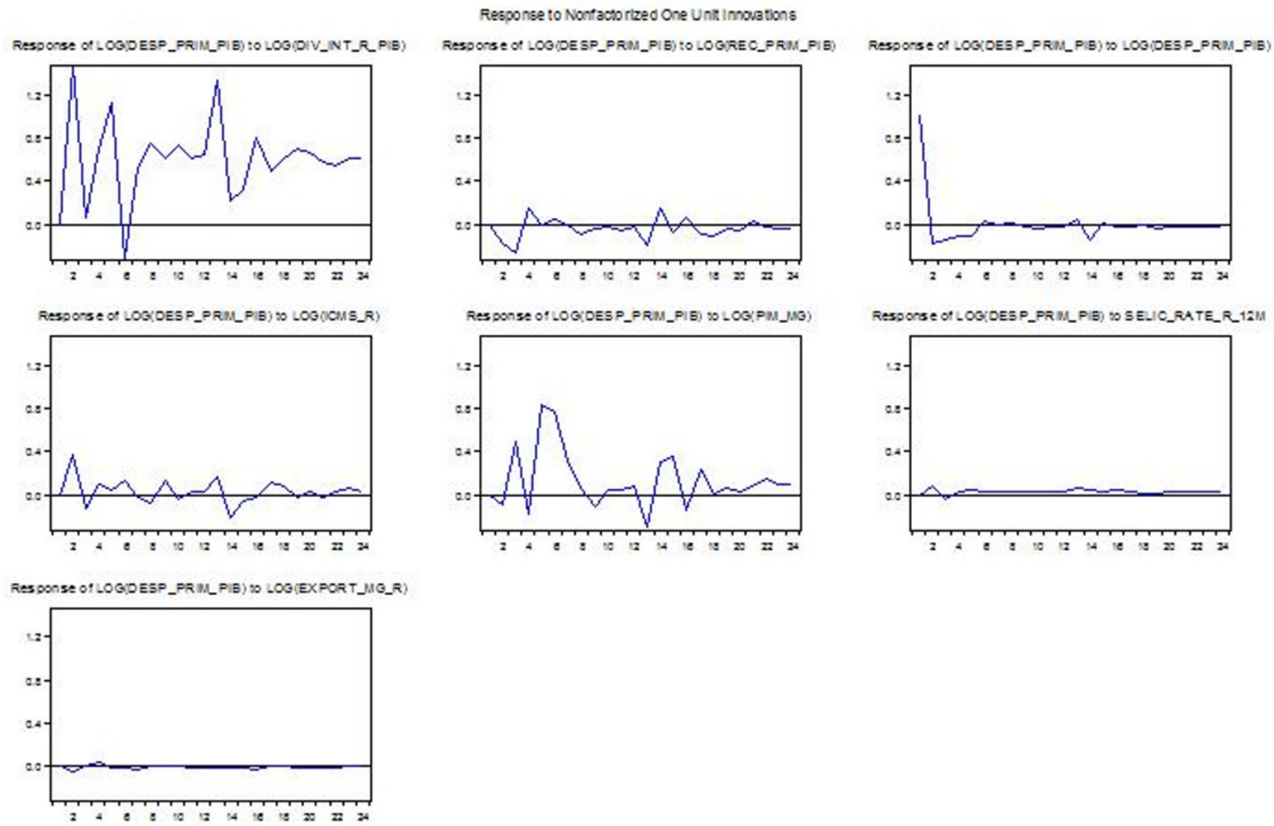
Dependent variable: D(LOG(PIM_MG))			
Excluded	**Chi-sq**	**df**	**Prob.**
D(LOG(DIV_INT_R_PIB))	39.77922	4	0.0000
D(LOG(REC_PRIM_PIB))	0.848749	4	0.9318
D(LOG(DESP_PRIM_PIB))	2.282661	4	0.6839
D(LOG(ICMS_R))	4.084172	4	0.3947
D(SELIC_RATE_R_12M)	2.243784	4	0.6910
D(LOG(EXPORT_MG_R))	4.414742	4	0.3528
All	59.27962	24	0.0001

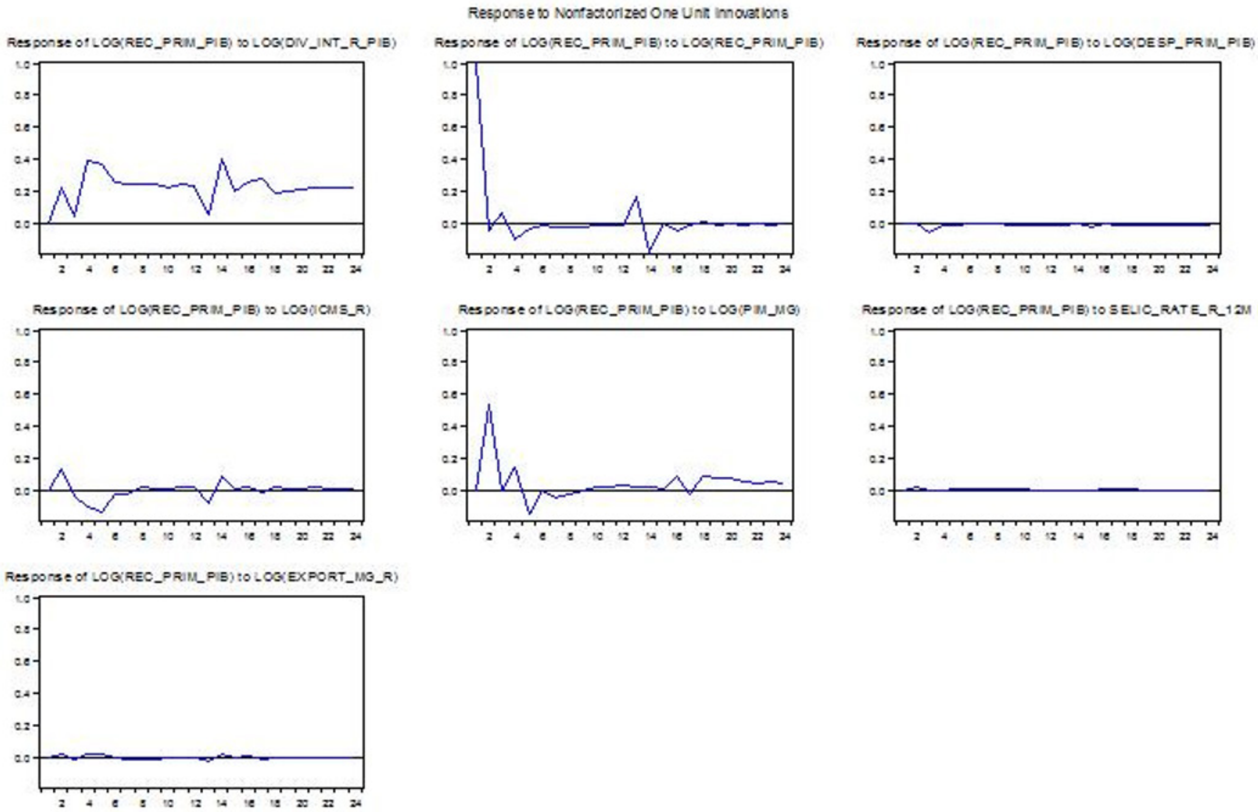
Dependent variable: D(SELIC_RATE_R_12M)			
Excluded	**Chi-sq**	**df**	**Prob.**
D(LOG(DIV_INT_R_PIB))	2.612901	4	0.6245
D(LOG(REC_PRIM_PIB))	6.896269	4	0.1415
D(LOG(DESP_PRIM_PIB))	5.204147	4	0.2670
D(LOG(ICMS_R))	8.116714	4	0.0874
D(LOG(PIM_MG))	1.156958	4	0.8851
D(LOG(EXPORT_MG_R))	1.743877	4	0.7827
All	30.60232	24	0.1656

Dependent variable: D(LOG(EXPORT_MG_R))			
Excluded	**Chi-sq**	**df**	**Prob.**
D(LOG(DIV_INT_R_PIB))	6.381395	4	0.1724
D(LOG(REC_PRIM_PIB))	1.973395	4	0.7407
D(LOG(DESP_PRIM_PIB))	1.974228	4	0.7405
D(LOG(ICMS_R))	6.819421	4	0.1457
D(LOG(PIM_MG))	9.712243	4	0.0456
D(SELIC_RATE_R_12M)	5.073137	4	0.2799
All	26.55996	24	0.3254

6.1.3 IRF Chart unconventional






Response to Nonfactorized One Unit Innovations
Response of LOG(REC_PRIM_PIB) to LOG(DIV_INT_R_PIB)
Response of LOG(REC_PRIM_PIB) to LOG(REC_PRIM_PIB)
Response of LOG(REC_PRIM_PIB) to LOG(DESP_PRIM_PIB)
Response of LOG(REC_PRIM_PIB) to LOG(ICMS_R)
Response of LOG(REC_PRIM_PIB) to LOG(PIM_MG)
Response of LOG(REC_PRIM_PIB) to SELIC_RATE_R_12M
Response of LOG(REC_PRIM_PIB) to LOG(EXPORT_MG_R)

6.2 Broad Model Strong Sustainability

6.2.1 Output of the Equation

Strong Sustainability Model (5 Cointegrations)							
Lags 1, 2, 3 and 12; Seasonal, pulse and level dummies							
Cointegration Restrictions:	B(1,1) = 1, B(1,2) = 0, B(1,3) = 0, B(1,4) = 0, B(1,5)=0, B(2,1)= 0, B(2,2)=1,B(2,3)=0,B(2,4)=0,B(2,5)=0,B(3,1)=0, B(3,2) = 0, B(3,3)=-1, B(3,4)=0, B(3,5)=0, B(4,1)=0, B(4,2)=0,B(4,3)=0,B(4,4)=1,B(4,5)=0,B(5,1)=0,B(5,2)=0,B(5,3)=0,B(5,4)=0,B(5,5)=1						
Convergence achieved after 1 iterations.							
Restrictions identify all cointegrating vectors							
Restrictions are not binding (LR test not available)							
	D(LOG(DIV_INT_R_PIB))	D(LOG(REC_PRIM_PIB))	D(LOG(DESP_PRIM_PIB))	D(LOG(ICMS_R))	D(LOG(PIM_MG))	D(SELIC_RATE_R_12M)	D(LOG(EXPORT_MG_R))
R-squared	0.843481	0.877028	0.832407	0.675910	0.905422	0.700334	0.675761
Adj. R-squared	0.767524	0.817351	0.751075	0.518631	0.859524	0.554908	0.518410
Sum sq. resids	0.092596	0.610640	3.465292	0.365595	0.087832	13.39011	7.362436
S.E. equation	0.026093	0.067007	0.159625	0.051848	0.025413	0.313778	0.232670
F-statistic	11.10463	14.69612	10.23468	4.297516	19.72674	4.815739	4.294601
Log likelihood	492.7663	301.3105	125.1020	353.3783	498.1276	-12,09749	48.61221
Akaike AIC	-4,194742	-2,308477	-0,572433	-2,821461	-4,247562	0.779286	0.181160
Schwarz SC	-3,101221	-1,214956	0.521088	-1,72794	-3,154041	1.872807	1.274682
Mean dependent	-0,000737	0.002632	0.009022	0.003512	0.000265	-0,075912	0.007373
S.D. dependent	0.054117	0.156788	0.319938	0.074729	0.067804	0.470325	0.335276

6.2.2 Granger Causality Test

VEC Granger Causality/Block Exogeneity Wald Tests

Date: 08/10/21 Time: 22:13

Sample: 2003M01 2020M12

Included observations: 203

Dependent variable: D(LOG(DIV_INT_R_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(REC_PRIM_PIB))	10.20146	4	0.0372
D(LOG(DESP_PRIM_PIB))	8.192218	4	0.0848
D(LOG(ICMS_R))	17.77252	4	0.0014
D(LOG(PIM_MG))	5.571376	4	0.2335
D(SELIC_RATE_R_12M)	9.420754	4	0.0514
D(LOG(EXPORT_MG_R))	2.913223	4	0.5725
All	46.61951	24	0.0037

Dependent variable: D(LOG(REC_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	1.564603	4	0.8151
D(LOG(DESP_PRIM_PIB))	1.704426	4	0.7899
D(LOG(ICMS_R))	1.845651	4	0.7641
D(LOG(PIM_MG))	5.865102	4	0.2095
D(SELIC_RATE_R_12M)	2.027323	4	0.7307
D(LOG(EXPORT_MG_R))	10.19032	4	0.0373
All	30.57147	24	0.1665

Dependent variable: D(LOG(DESP_PRIM_PIB))			
Excluded	**Chi-sq**	**df**	**Prob.**
D(LOG(DIV_INT_R_PIB))	4.894627	4	0.2983
D(LOG(REC_PRIM_PIB))	5.310116	4	0.2569
D(LOG(ICMS_R))	5.279531	4	0.2598
D(LOG(PIM_MG))	4.648844	4	0.3253
D(SELIC_RATE_R_12M)	8.659625	4	0.0702
D(LOG(EXPORT_MG_R))	4.588289	4	0.3322
All	41.20508	24	0.0158

Dependent variable: D(LOG(ICMS_R))			
Excluded	**Chi-sq**	**df**	**Prob.**
D(LOG(DIV_INT_R_PIB))	1.779475	4	0.7762
D(LOG(REC_PRIM_PIB))	5.540055	4	0.2362
D(LOG(DESP_PRIM_PIB))	1.538494	4	0.8198
D(LOG(PIM_MG))	4.020690	4	0.4032
D(SELIC_RATE_R_12M)	2.439508	4	0.6555
D(LOG(EXPORT_MG_R))	10.01496	4	0.0402
All	22.12947	24	0.5715

Dependent variable: D(LOG(PIM_MG))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	39.15609	4	0.0000
D(LOG(REC_PRIM_PIB))	0.806375	4	0.9376
D(LOG(DESP_PRIM_PIB))	2.068948	4	0.7231
D(LOG(ICMS_R))	4.500427	4	0.3425
D(SELIC_RATE_R_12M)	1.962857	4	0.7426
D(LOG(EXPORT_MG_R))	4.440894	4	0.3496
All	54.93083	24	0.0003

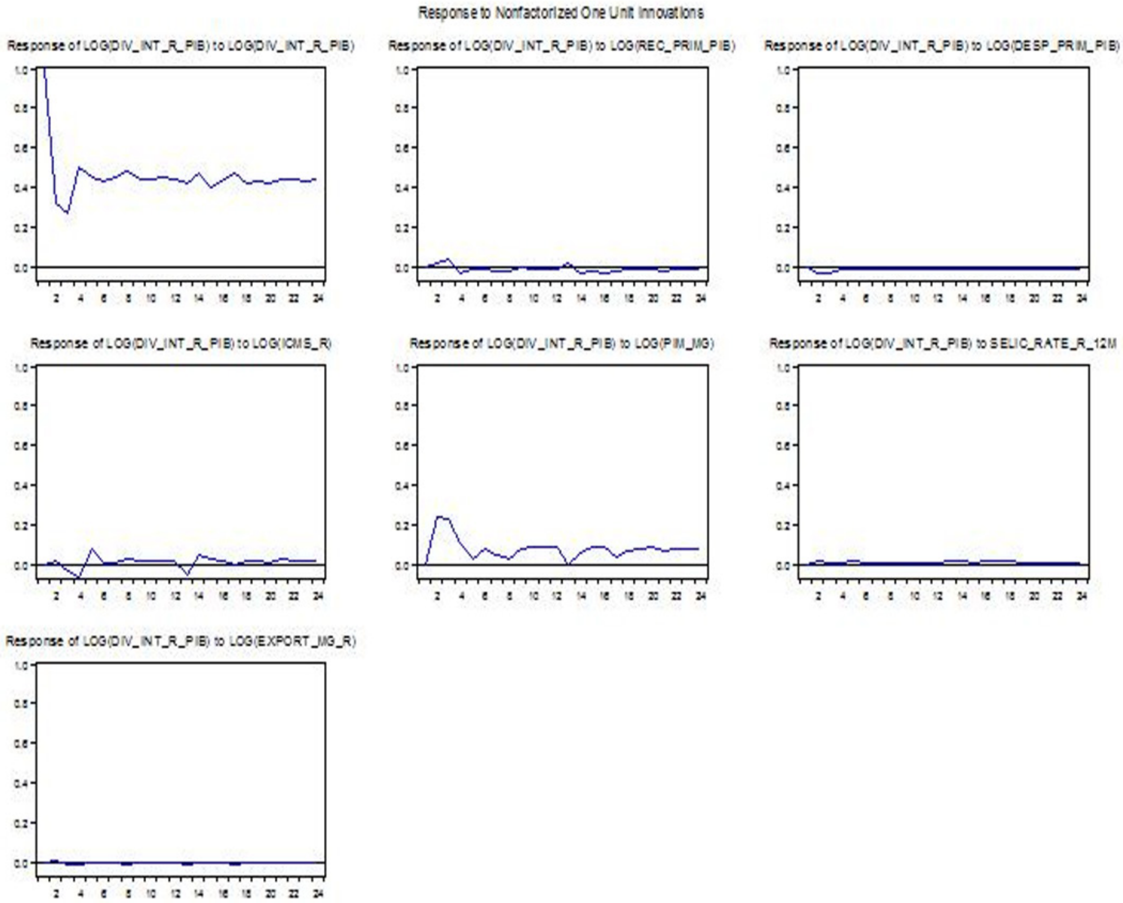
Dependent variable: D(SELIC_RATE_R_12M)

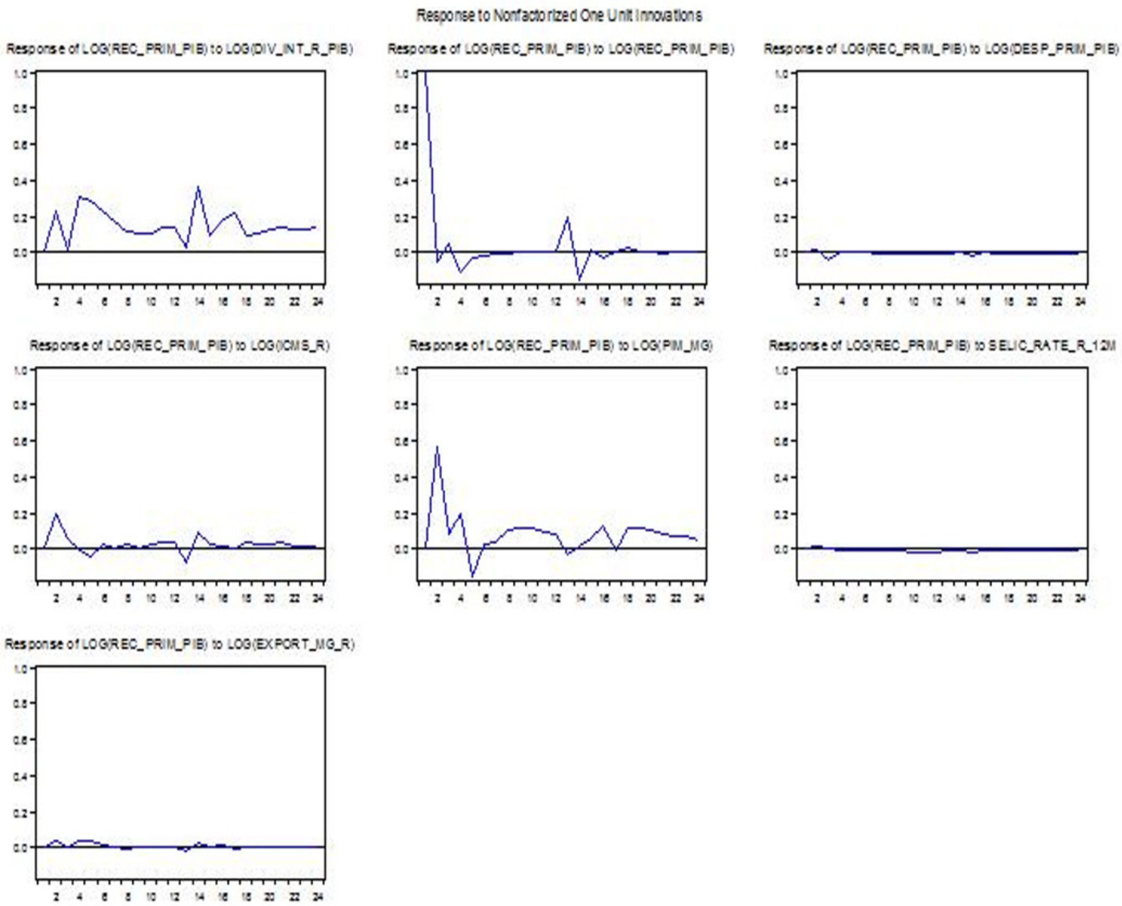
Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	2.468466	4	0.6503
D(LOG(REC_PRIM_PIB))	6.866373	4	0.1431
D(LOG(DESP_PRIM_PIB))	5.557117	4	0.2348
D(LOG(ICMS_R))	6.506574	4	0.1644
D(LOG(PIM_MG))	1.146791	4	0.8868
D(LOG(EXPORT_MG_R))	1.080120	4	0.8974
All	29.71451	24	0.1944

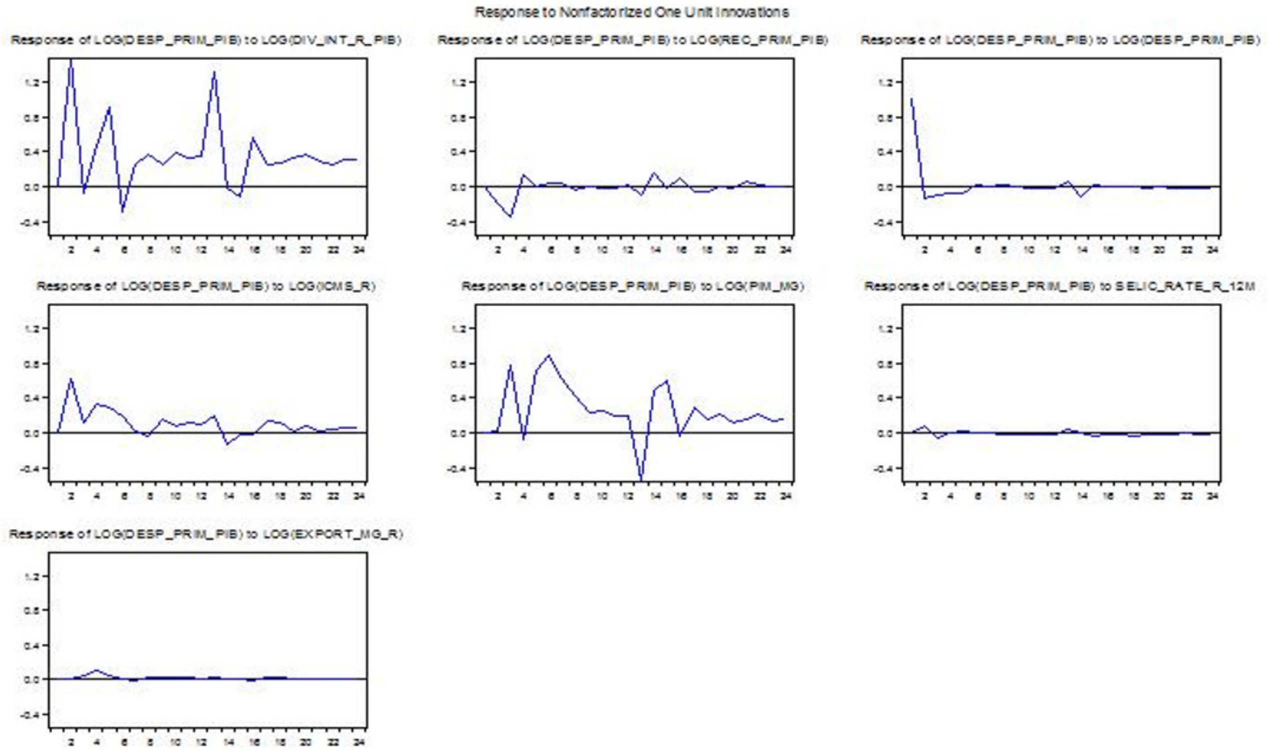
Dependent variable: D(LOG(EXPORT_MG_R))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	13.63236	4	0.0086
D(LOG(REC_PRIM_PIB))	2.436562	4	0.6560
D(LOG(DESP_PRIM_PIB))	2.967036	4	0.5634
D(LOG(ICMS_R))	4.217865	4	0.3773
D(LOG(PIM_MG))	7.942890	4	0.0937
D(SELIC_RATE_R_12M)	6.684780	4	0.1535
All	28.05635	24	0.2577

6.2.3 Non-conventional IRF Chart






Response to Nonfactorized One Unit Innovations
Response of LOG(DESP_PRIM_PIB) to LOG(DIV_INT_R_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(REC_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(DESP_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(ICMS_R)
Response of LOG(DESP_PRIM_PIB) to LOG(PIM_MG)
Response of LOG(DESP_PRIM_PIB) to SELIC_RATE_R_12M
Response of LOG(DESP_PRIM_PIB) to LOG(EXPORT_MG_R)

6.3 Broad Model Weak Sustainability

6.3.1 Output of the Equation

Weak Sustainability Model (5 Cointegrations)							
Lags 1, 2, 3 and 12; Seasonal, pulse and level dummies							
Cointegration Restrictions:	B(1,1) = 1, B(1,2) = 0, B(1,3) = 0, B(1,4) = 0, B(1,5)=0, B(2,1)= 0, B(2,2)=1,B(2,3)=0,B(2,4)=0,B(2,5)=0,B(3,1)=0, B(3,2) = 0, B(3,3)=-1, B(3,4)=0, B(3,5)=0, B(4,1)=0, B(4,2)=0,B(4,3)=0,B(4,4)=1,B(4,5)=0,B(5,1)=0,B(5,2)=0,B(5,3)=0,B(5,4)=0,B(5,5)=1,A(2,3)=0						
Convergence achieved after 83 iterations.							
Restrictions identify all cointegrating vectors							
LR test for binding restrictions (rank = 5):							
Chi-square(1)	0.032436						
Probability	0.857074						
	D(LOG(DIV_INT_R_PIB))	D(LOG(REC_PRIM_PIB))	D(LOG(DESP_PRIM_PIB))	D(LOG(ICMS_R))	D(LOG(PIM_MG))	D(SELIC_RATE_R_12M)	D(LOG(EXPORT_MG_R))
R-squared	0.843510	0.877070	0.832397	0.675925	0.905437	0.700361	0.675767
Adj. R-squared	0.767566	0.817413	0.751061	0.518654	0.859546	0.554948	0.518418
Sum sq. resids	0.092579	0.610431	3.465.487	0.365577	0.087818	1.338.890	7.362.307
S.E. equation	0.026091	0.066996	0.159629	0.051847	0.025411	0.313764	0.232668
F-statistic	1.110.701	1.470.187	1.023.399	4.297.825	1.973.016	4.816.356	4.294.713
Log likelihood	4.927.847	3.013.453	1.250.963	3.533.832	4.981.435	-1.208.839	4.861.400
Akaike AIC	-4.194.923	-2.308.820	-0.572377	-2.821.509	-4.247.720	0.779196	0.181143
Schwarz SC	-3.101.402	-1.215.299	0.521144	-1.727.988	-3.154.198	1.872.717	1.274.664
Mean dependent	-0.000737	0.002632	0.009022	0.003512	0.000265	-0.075912	0.007373
S.D. dependent	0.054117	0.156788	0.319938	0.074729	0.067804	0.470325	0.335276

6.3.2 Granger Causality Test

VEC Granger Causality/Block Exogeneity Wald Tests

Date: 08/10/21 Time: 22:07

Sample: 2003M01 2020M12

Included observations: 203

Dependent variable: D(LOG(DIV_INT_R_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(REC_PRIM_PIB))	10.20146	4	0.0372
D(LOG(DESP_PRIM_PIB))	8.192218	4	0.0848
D(LOG(ICMS_R))	17.77252	4	0.0014
D(LOG(PIM_MG))	5.571376	4	0.2335
D(SELIC_RATE_R_12M)	9.420754	4	0.0514
D(LOG(EXPORT_MG_R))	2.913223	4	0.5725
All	46.61951	24	0.0037

Dependent variable: D(LOG(REC_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	1.564603	4	0.8151
D(LOG(DESP_PRIM_PIB))	1.704426	4	0.7899
D(LOG(ICMS_R))	1.845651	4	0.7641
D(LOG(PIM_MG))	5.865102	4	0.2095
D(SELIC_RATE_R_12M)	2.027323	4	0.7307
D(LOG(EXPORT_MG_R))	10.19032	4	0.0373
All	30.57147	24	0.1665

Dependent variable: D(LOG(DESP_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	4.894627	4	0.2983
D(LOG(REC_PRIM_PIB))	5.310116	4	0.2569
D(LOG(ICMS_R))	5.279531	4	0.2598
D(LOG(PIM_MG))	4.648844	4	0.3253
D(SELIC_RATE_R_12M)	8.659625	4	0.0702
D(LOG(EXPORT_MG_R))	4.588289	4	0.3322
All	41.20508	24	0.0158

Dependent variable: D(LOG(ICMS_R))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	1.779475	4	0.7762
D(LOG(REC_PRIM_PIB))	5.540055	4	0.2362
D(LOG(DESP_PRIM_PIB))	1.538494	4	0.8198
D(LOG(PIM_MG))	4.020690	4	0.4032
D(SELIC_RATE_R_12M)	2.439508	4	0.6555
D(LOG(EXPORT_MG_R))	10.01496	4	0.0402
All	22.12947	24	0.5715

Dependent variable: D(LOG(PIM_MG))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	39.15609	4	0.0000
D(LOG(REC_PRIM_PIB))	0.806375	4	0.9376
D(LOG(DESP_PRIM_PIB))	2.068948	4	0.7231
D(LOG(ICMS_R))	4.500427	4	0.3425
D(SELIC_RATE_R_12M)	1.962857	4	0.7426
D(LOG(EXPORT_MG_R))	4.440894	4	0.3496
All	54.93083	24	0.0003

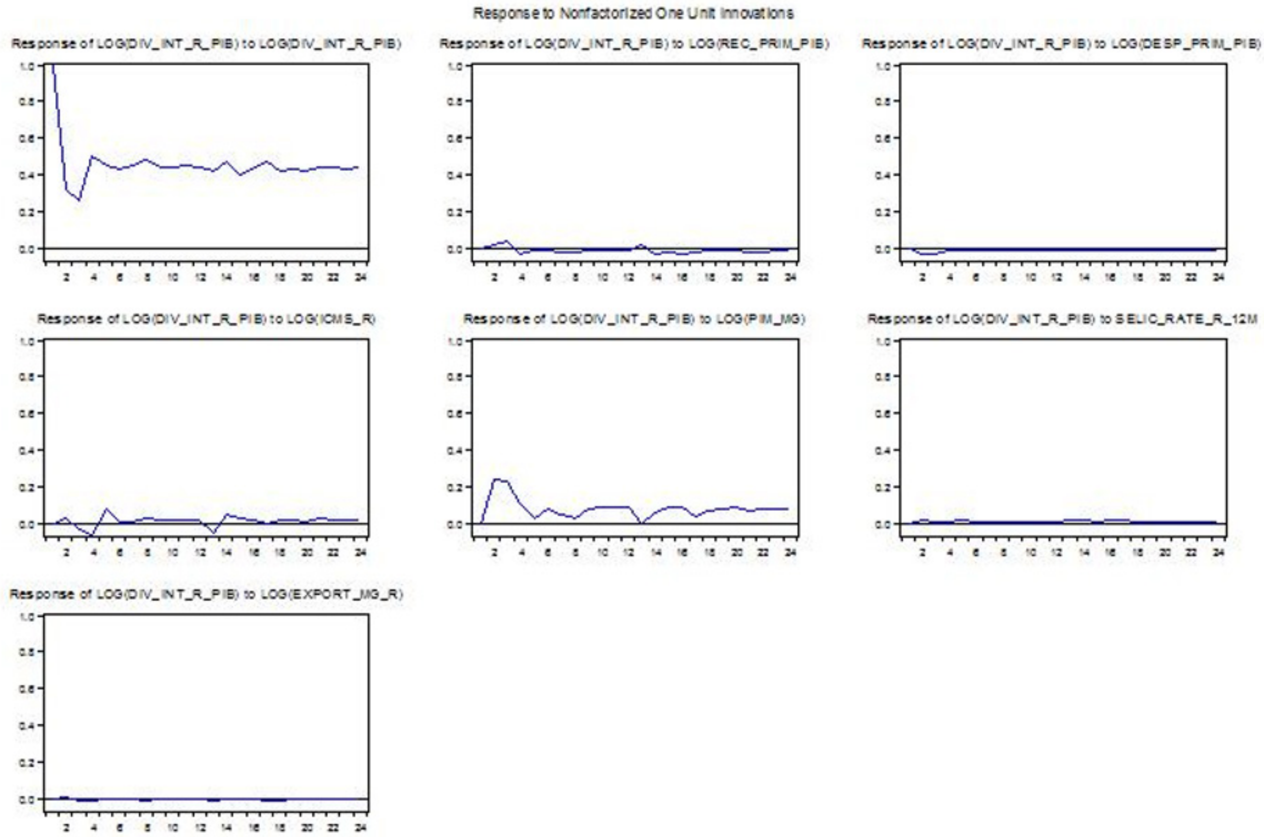
Dependent variable: D(SELIC_RATE_R_12M)

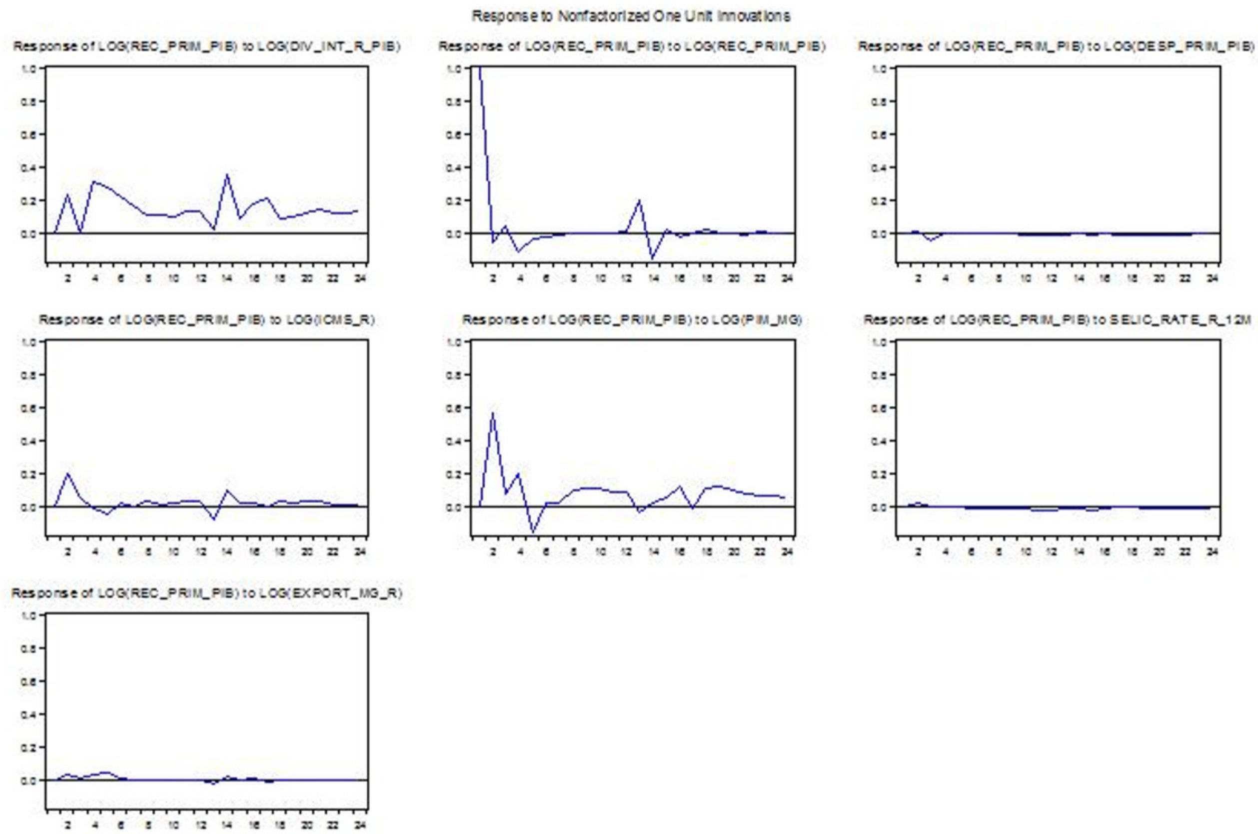
Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	2.468466	4	0.6503
D(LOG(REC_PRIM_PIB))	6.866373	4	0.1431
D(LOG(DESP_PRIM_PIB))	5.557117	4	0.2348
D(LOG(ICMS_R))	6.506574	4	0.1644
D(LOG(PIM_MG))	1.146791	4	0.8868
D(LOG(EXPORT_MG_R))	1.080120	4	0.8974
All	29.71451	24	0.1944

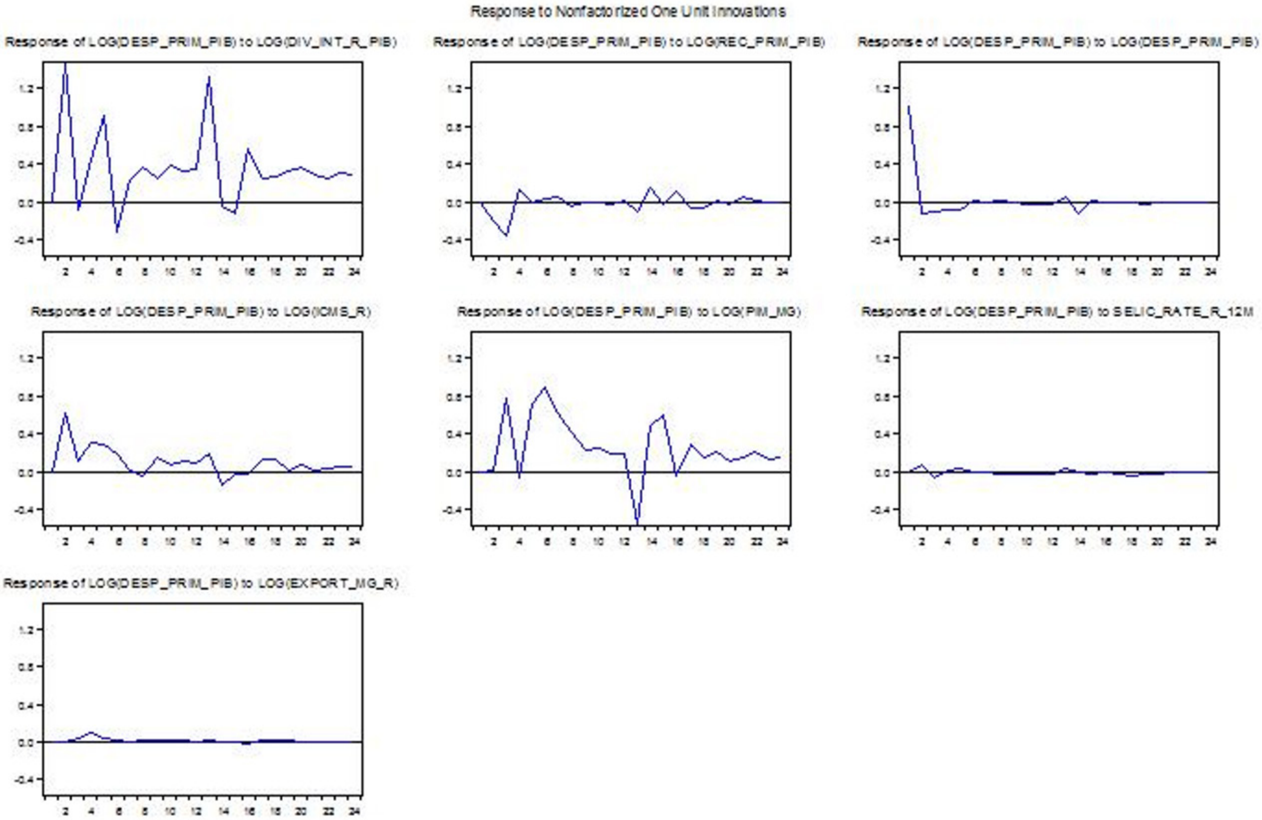
Dependent variable: D(LOG(EXPORT_MG_R))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	13.63236	4	0.0086
D(LOG(REC_PRIM_PIB))	2.436562	4	0.6560
D(LOG(DESP_PRIM_PIB))	2.967036	4	0.5634
D(LOG(ICMS_R))	4.217865	4	0.3773
D(LOG(PIM_MG))	7.942890	4	0.0937
D(SELIC_RATE_R_12M)	6.684780	4	0.1535
All	28.05635	24	0.2577

6.3.3 Non-conventional IRF Chart







Response to Nonfactorized One Unit Innovations
Response of LOG(DESP_PRIM_PIB) to LOG(DIV_INT_R_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(REC_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(DESP_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(ICMS_R)
Response of LOG(DESP_PRIM_PIB) to LOG(PIM_MG)
Response of LOG(DESP_PRIM_PIB) to SELIC_RATE_R_12M
Response of LOG(DESP_PRIM_PIB) to LOG(EXPORT_MG_R)

6.4 Unrestricted Simplest Model

6.4.1 Output of the equation

Unrestricted Simplest Model						
Lags 1, 2, 3 and 12; Seasonal, pulse and level dummies						
	D(LOG(DIV_INT_R_PIB))	D(LOG(-DESP_PRIM_PIB))	D(LOG(REC_PRIM_PIB))	D(SELIC_RATE_R_12M)	D(LOG(PIM_MG))	D(LOG(SEV_DIV_PIB))
R-squared	0.701416	0.743673	0.789038	0.642784	0.821823	0.764479
Adj. R-squared	0.623038	0.676387	0.733661	0.549015	0.775052	0.702655
Sum sq. resids	0.176641	5.300029	1.047572	15.96165	0.165467	151.1991
S.E. equation	0.033227	0.182003	0.080916	0.315849	0.032159	0.972108
F-statistic	8.949113	11.05244	14.24835	6.854953	17.57104	12.36538
Log likelihood	427.2100	81.97308	246.5287	-2.992.831	433.8424	-2.581.418
Akaike AIC	-3.785.320	-0.383971	-2.005.209	0.718506	-3.850.665	2.966914
Schwarz SC	-3.083.508	0.317841	-1.303.397	1.420318	-3.148.852	3.668726
Mean dependent	-0.000737	0.009022	0.002632	-0.075912	0.000265	0.008833
S.D. dependent	0.054117	0.319938	0.156788	0.470325	0.067804	1.782726

6.4.2 Granger Causality Test

VEC Granger Causality/Block Exogeneity Wald Tests

Date: 09/13/21 Time: 17:50

Sample: 2003M01 2020M12

Included observations: 203

Dependent variable: D(LOG(DIV_INT_R_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DESP_PRIM_PIB))	8.569405	4	0.0728
D(LOG(REC_PRIM_PIB))	6.549552	4	0.1617
D(SELIC_RATE_R_12M)	4.617500	4	0.3288
D(LOG(PIM_MG))	5.821673	4	0.2129
D(LOG(SEV_DIV_PIB))	20.15797	4	0.0005
All	45.50519	20	0.0009

Dependent variable: D(LOG(DESP_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	3.346547	4	0.5016
D(LOG(REC_PRIM_PIB))	6.730723	4	0.1508
D(SELIC_RATE_R_12M)	7.855370	4	0.0970
D(LOG(PIM_MG))	16.15413	4	0.0028
D(LOG(SEV_DIV_PIB))	0.971123	4	0.9141
All	42.62839	20	0.0023

Dependent variable: D(LOG(REC_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	15.32087	4	0.0041
D(LOG(DESP_PRIM_PIB))	6.732653	4	0.1507
D(SELIC_RATE_R_12M)	0.282031	4	0.9909
D(LOG(PIM_MG))	7.833331	4	0.0979
D(LOG(SEV_DIV_PIB))	15.71421	4	0.0034
All	59.63595	20	0.0000

Dependent variable: D(SELIC_RATE_R_12M)

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	3.576302	4	0.4664
D(LOG(DESP_PRIM_PIB))	7.938516	4	0.0939
D(LOG(REC_PRIM_PIB))	10.37478	4	0.0346
D(LOG(PIM_MG))	1.588034	4	0.8109
D(LOG(SEV_DIV_PIB))	2.889039	4	0.5766
All	26.33708	20	0.1550

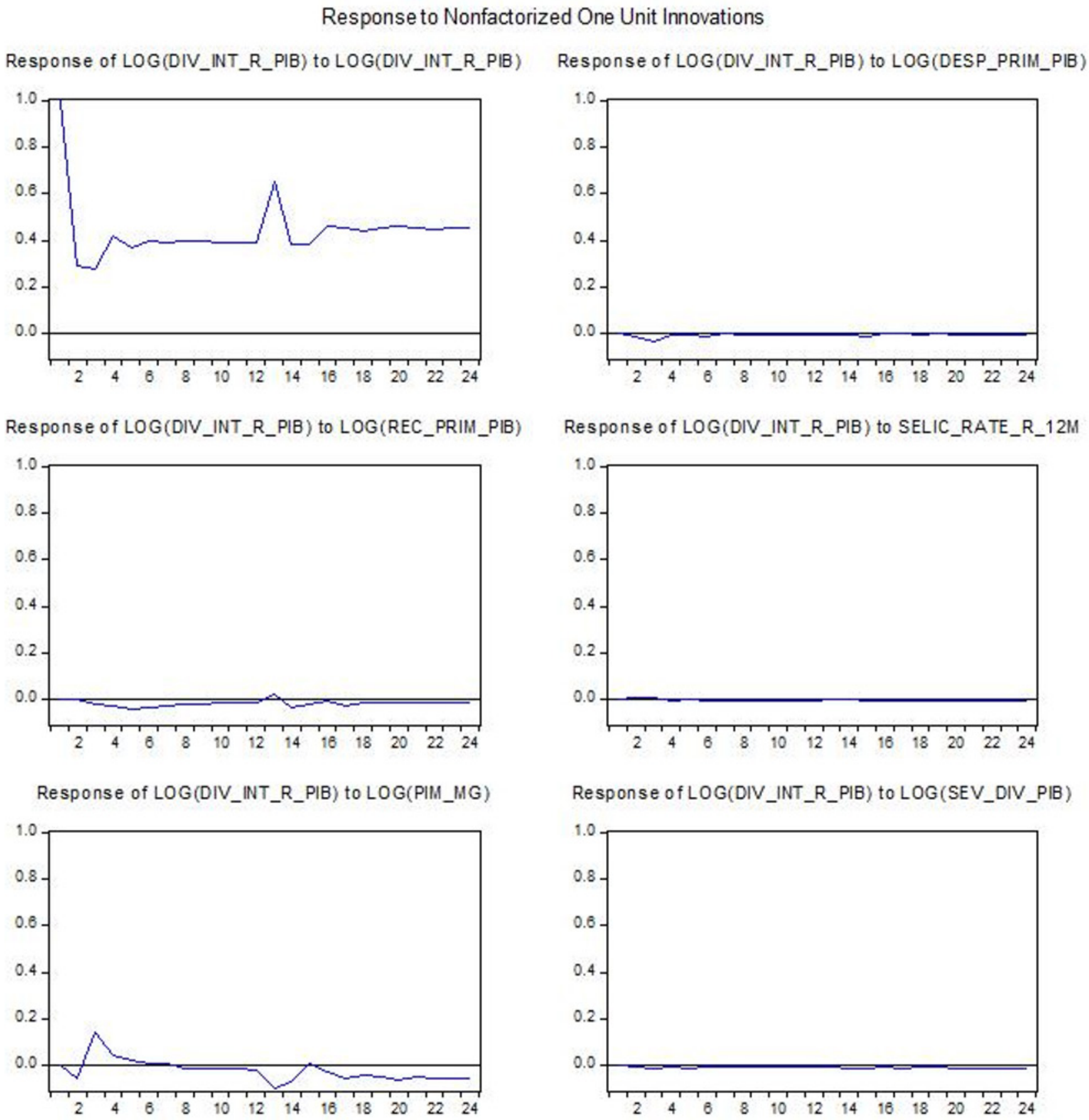
Dependent variable: D(LOG(PIM_MG))

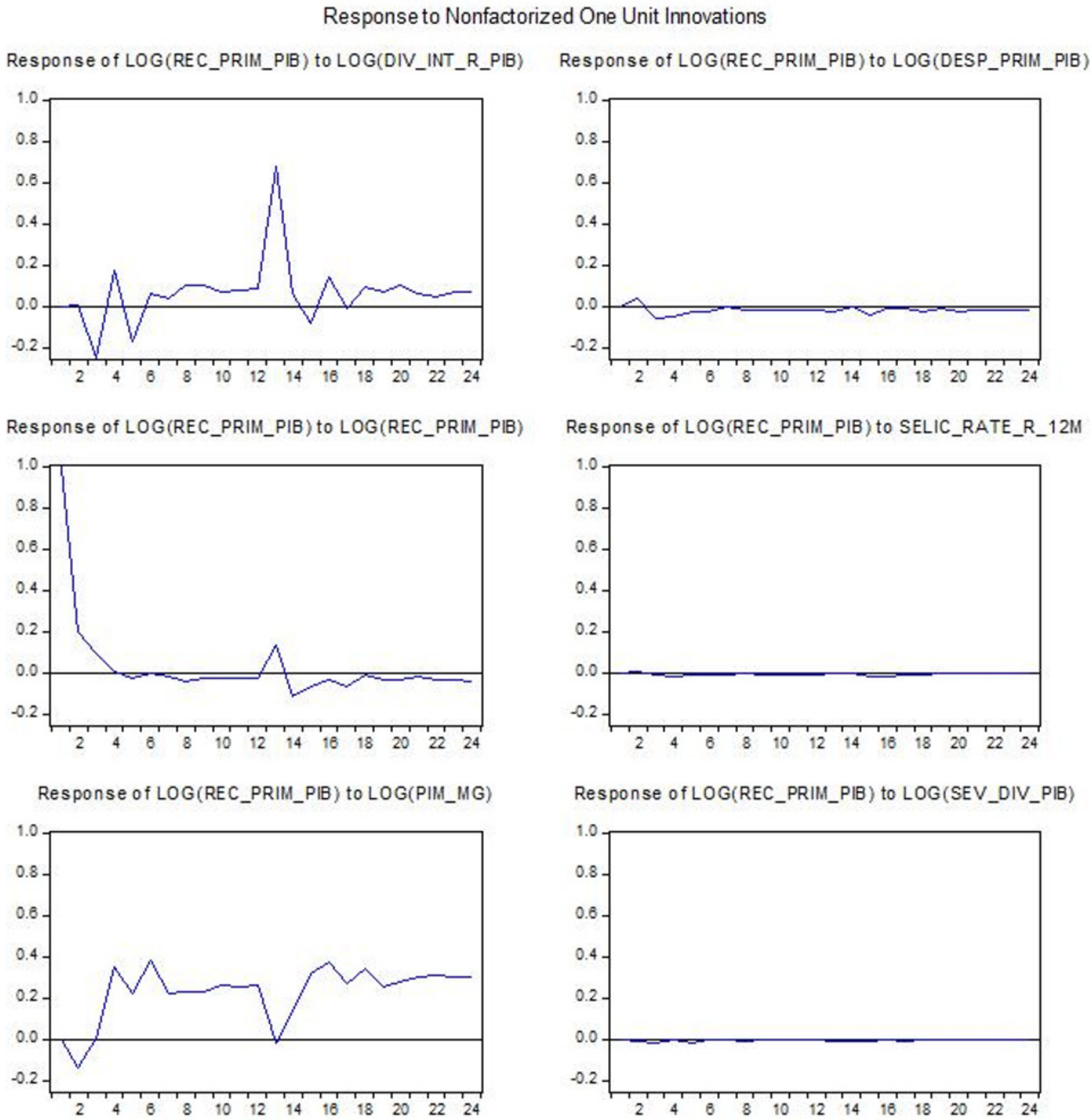
Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	67.01150	4	0.0000
D(LOG(DESP_PRIM_PIB))	17.44247	4	0.0016
D(LOG(REC_PRIM_PIB))	5.172899	4	0.2700
D(SELIC_RATE_R_12M)	3.785381	4	0.4358
D(LOG(SEV_DIV_PIB))	2.426254	4	0.6579
All	114.9154	20	0.0000

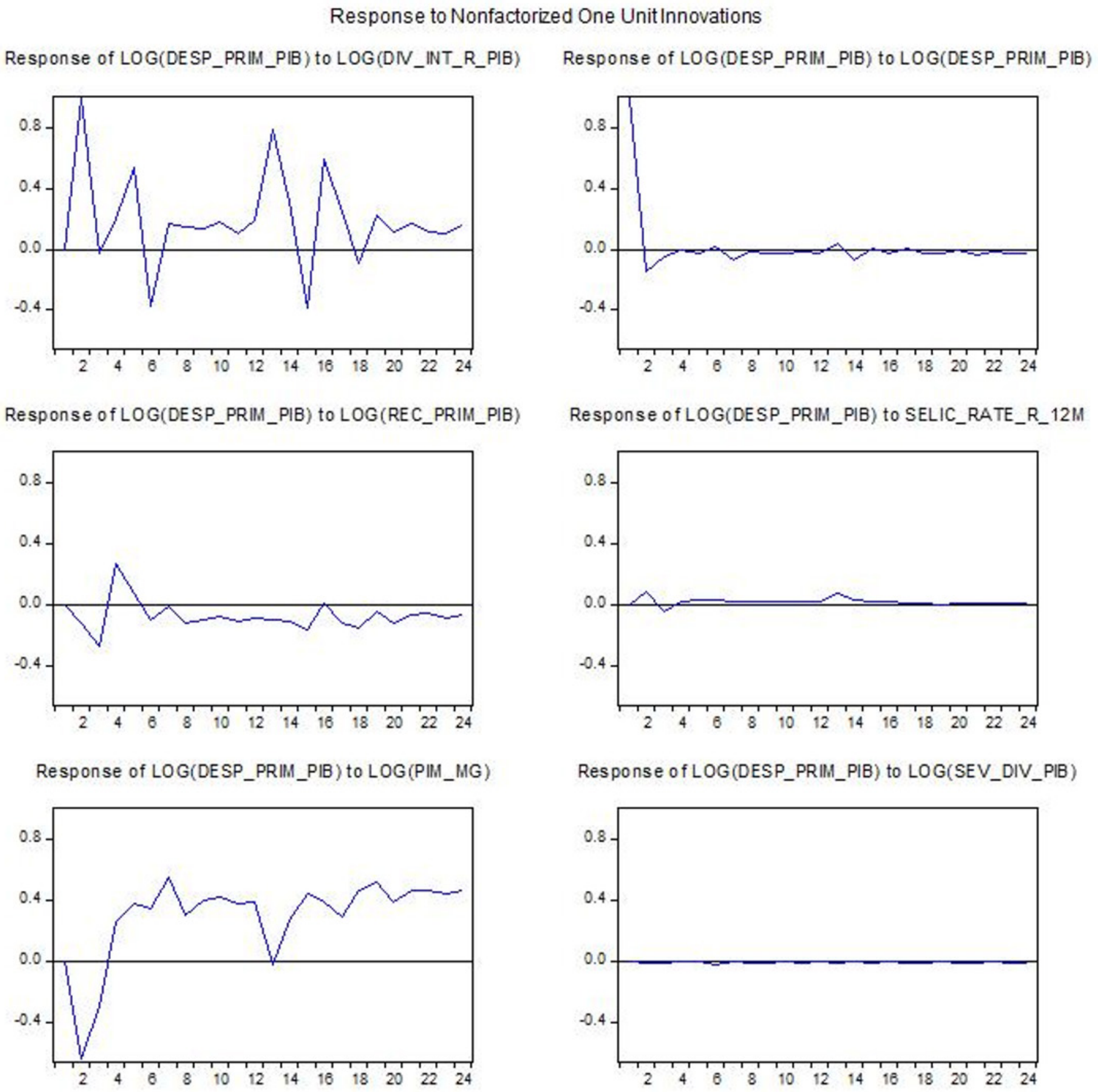
Dependent variable: D(LOG(SEV_DIV_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	16.09150	4	0.0029
D(LOG(DESP_PRIM_PIB))	4.223563	4	0.3766
D(LOG(REC_PRIM_PIB))	2.488722	4	0.6467
D(SELIC_RATE_R_12M)	8.980616	4	0.0616
D(LOG(PIM_MG))	5.610361	4	0.2302
All	38.96672	20	0.0067

6.4.3 IRF Chart unconventional




Response to Nonfactorized One Unit Innovations
Response of LOG(REC_PRIM_PIB) to LOG(DIV_INT_R_PIB)
Response of LOG(REC_PRIM_PIB) to LOG(DESP_PRIM_PIB)
Response of LOG(REC_PRIM_PIB) to LOG(REC_PRIM_PIB)
Response of LOG(REC_PRIM_PIB) to SELIC_RATE_R_12M
Response of LOG(REC_PRIM_PIB) to LOG(PIM_MG)
Response of LOG(REC_PRIM_PIB) to LOG(SEV_DIV_PIB)


Response to Nonfactorized One Unit Innovations
Response of LOG(DESP_PRIM_PIB) to LOG(DIV_INT_R_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(DESP_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(REC_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to SELIC_RATE_R_12M
Response of LOG(DESP_PRIM_PIB) to LOG(PIM_MG)
Response of LOG(DESP_PRIM_PIB) to LOG(SEV_DIV_PIB)
0.8
0.4
0.0
-0.4
2 4 6 8 10 12 14 16 18 20 22 24

6.5 Simple Model Strong Sustainability

6.5.1 Model Output

Simpler Model Stronger Sustainability						
Lags 1, 2, 3 and 12; Seasonal, pulse and level dummies						
Cointegration Restrictions:						
B(1,1)= 1, B(2,2) = -1, B(3,3)= 1, B (1,2) = 0, B(1,3) = 0, B(2,1)= 0, B(2,3) = 0, B(3,1)= 0, B(3,2) = 0						
Convergence achieved after 1 iterations.						
Restrictions identify all cointegrating vectors						
Restrictions are not binding (LR test not available)						
	D(LOG(DIV_INT_R_PIB))	D(LOG(-DESP_PRIM_PIB))	D(LOG(REC_PRIM_PIB))	D(SELIC_RATE_R_12M)	D(LOG(PIM_MG))	D(LOG(SEV_DIV_PIB))
R-squared	0.701416	0.743673	0.789038	0.642784	0.821823	0.764479
Adj. R-squared	0.623038	0.676387	0.733661	0.549015	0.775052	0.702655
Sum sq. resids	0.176641	5.300.029	1.047.572	1.596.165	0.165467	1.511.991
S.E. equation	0.033227	0.182003	0.080916	0.315849	0.032159	0.972108
F-statistic	8.949.113	1.105.244	1.424.835	6.854.953	1.757.104	1.236.538
Log likelihood	4.272.100	8.197.308	2.465.287	-2.992.831	4.338.424	-2.581.418
Akaike AIC	-3.785.320	-0.383971	-2.005.209	0.718506	-3.850.665	2.966.914
Schwarz SC	-3.083.508	0.317841	-1.303.397	1.420.318	-3.148.852	3.668.726
Mean dependent	-0.000737	0.009022	0.002632	-0.075912	0.000265	0.008833
S.D. dependent	0.054117	0.319938	0.156788	0.470325	0.067804	1.782.726

6.5.2 Granger Causality Test

VEC Granger Causality/Block Exogeneity Wald Tests

Date: 09/13/21 Time: 17:52

Sample: 2003M01 2020M12

Included observations: 203

Dependent variable: D(LOG(DIV_INT_R_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DESP_PRIM_PIB))	8.569405	4	0.0728
D(LOG(REC_PRIM_PIB))	6.549552	4	0.1617
D(SELIC_RATE_R_12M)	4.617500	4	0.3288
D(LOG(PIM_MG))	5.821673	4	0.2129
D(LOG(SEV_DIV_PIB))	20.15797	4	0.0005
All	45.50519	20	0.0009

Dependent variable: D(LOG(DESP_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	3.346547	4	0.5016
D(LOG(REC_PRIM_PIB))	6.730723	4	0.1508
D(SELIC_RATE_R_12M)	7.855370	4	0.0970
D(LOG(PIM_MG))	16.15413	4	0.0028
D(LOG(SEV_DIV_PIB))	0.971123	4	0.9141
All	42.62839	20	0.0023

Dependent variable: D(LOG(REC_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	15.32087	4	0.0041
D(LOG(DESP_PRIM_PIB))	6.732653	4	0.1507
D(SELIC_RATE_R_12M)	0.282031	4	0.9909
D(LOG(PIM_MG))	7.833331	4	0.0979
D(LOG(SEV_DIV_PIB))	15.71421	4	0.0034
All	59.63595	20	0.0000

Dependent variable: D(SELIC_RATE_R_12M)

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	3.576302	4	0.4664
D(LOG(DESP_PRIM_PIB))	7.938516	4	0.0939
D(LOG(REC_PRIM_PIB))	10.37478	4	0.0346
D(LOG(PIM_MG))	1.588034	4	0.8109
D(LOG(SEV_DIV_PIB))	2.889039	4	0.5766
All	26.33708	20	0.1550

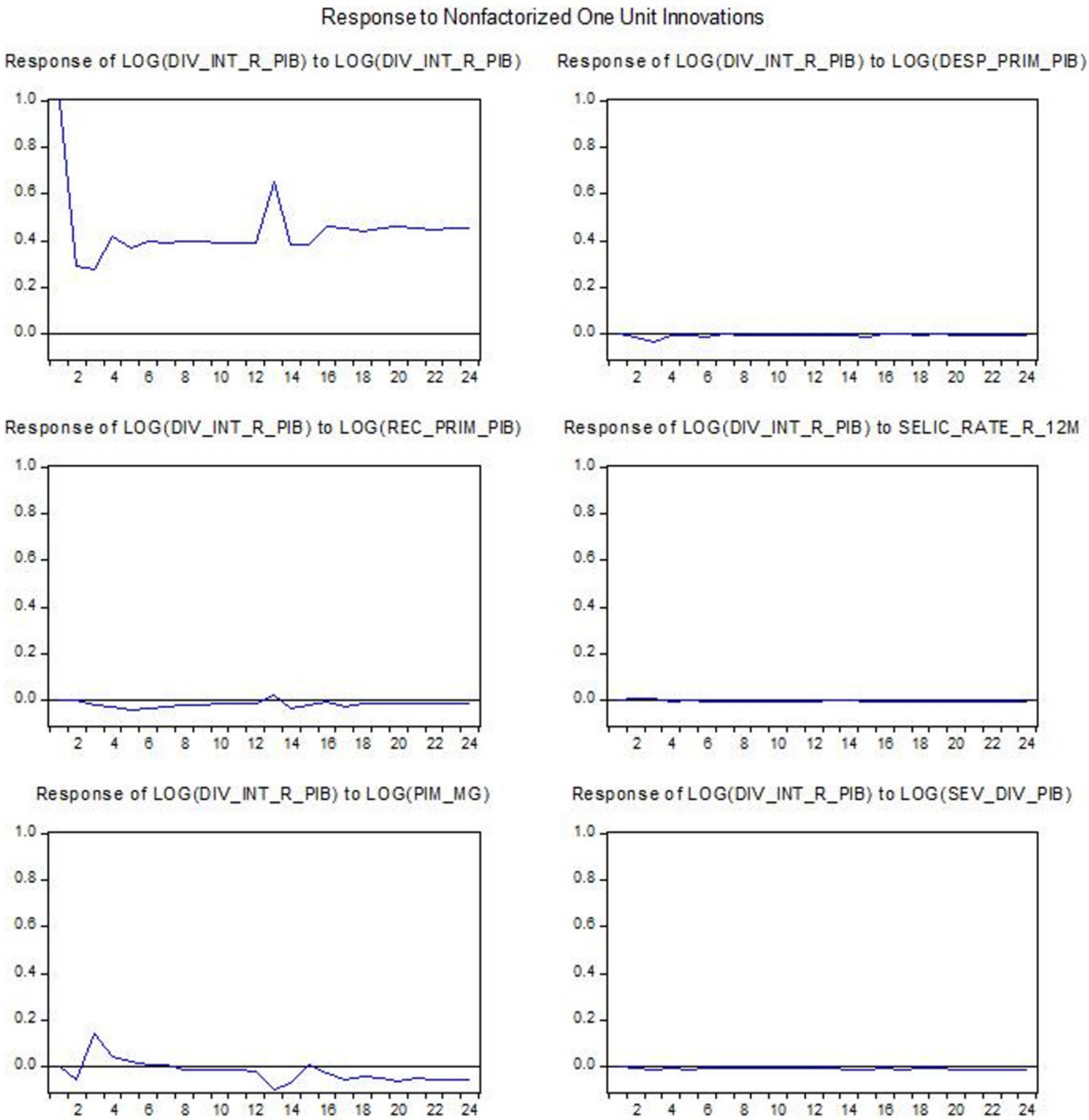
Dependent variable: D(LOG(PIM_MG))

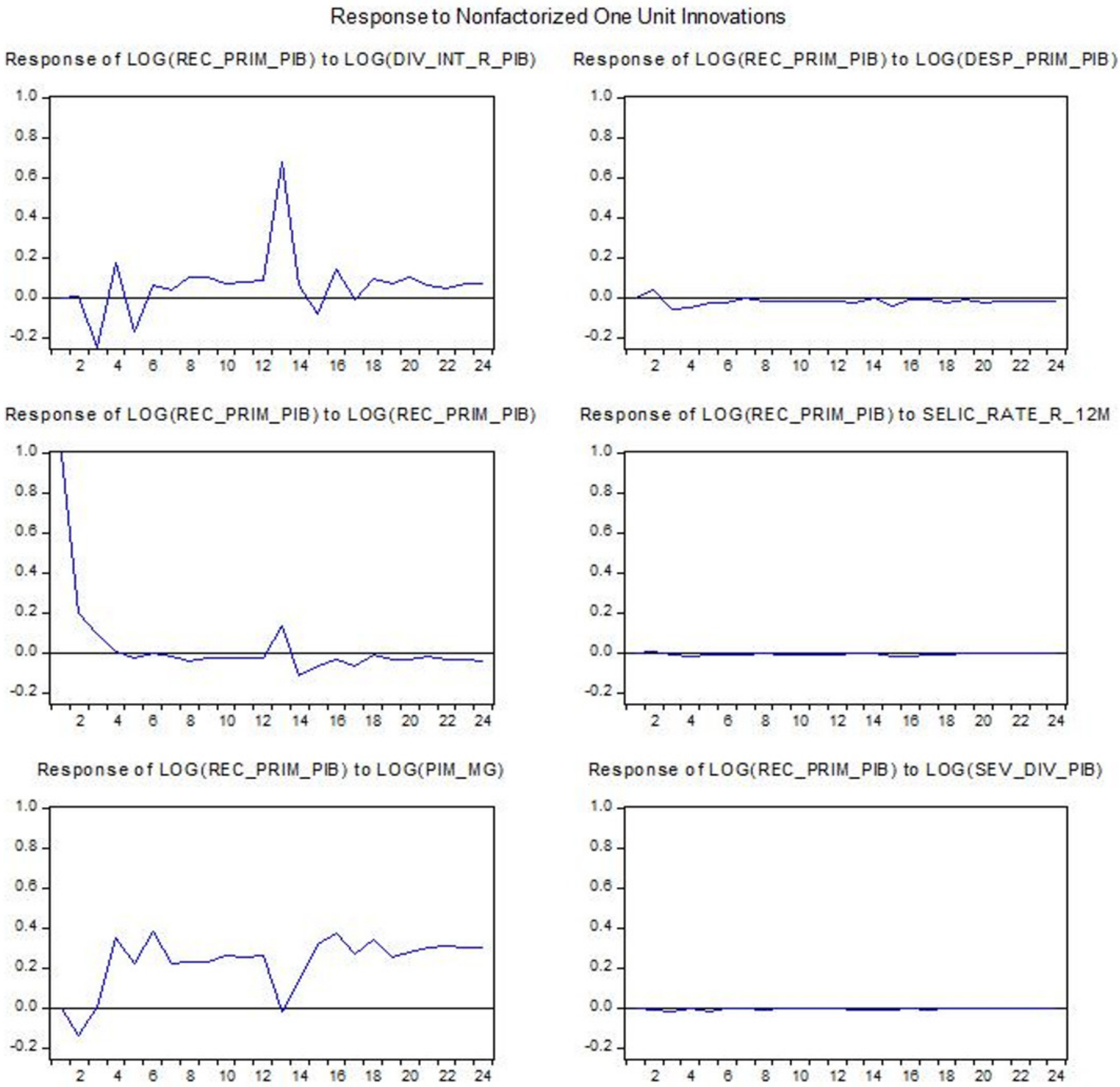
Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	67.01150	4	0.0000
D(LOG(DESP_PRIM_PIB))	17.44247	4	0.0016
D(LOG(REC_PRIM_PIB))	5.172899	4	0.2700
D(SELIC_RATE_R_12M)	3.785381	4	0.4358
D(LOG(SEV_DIV_PIB))	2.426254	4	0.6579
All	114.9154	20	0.0000

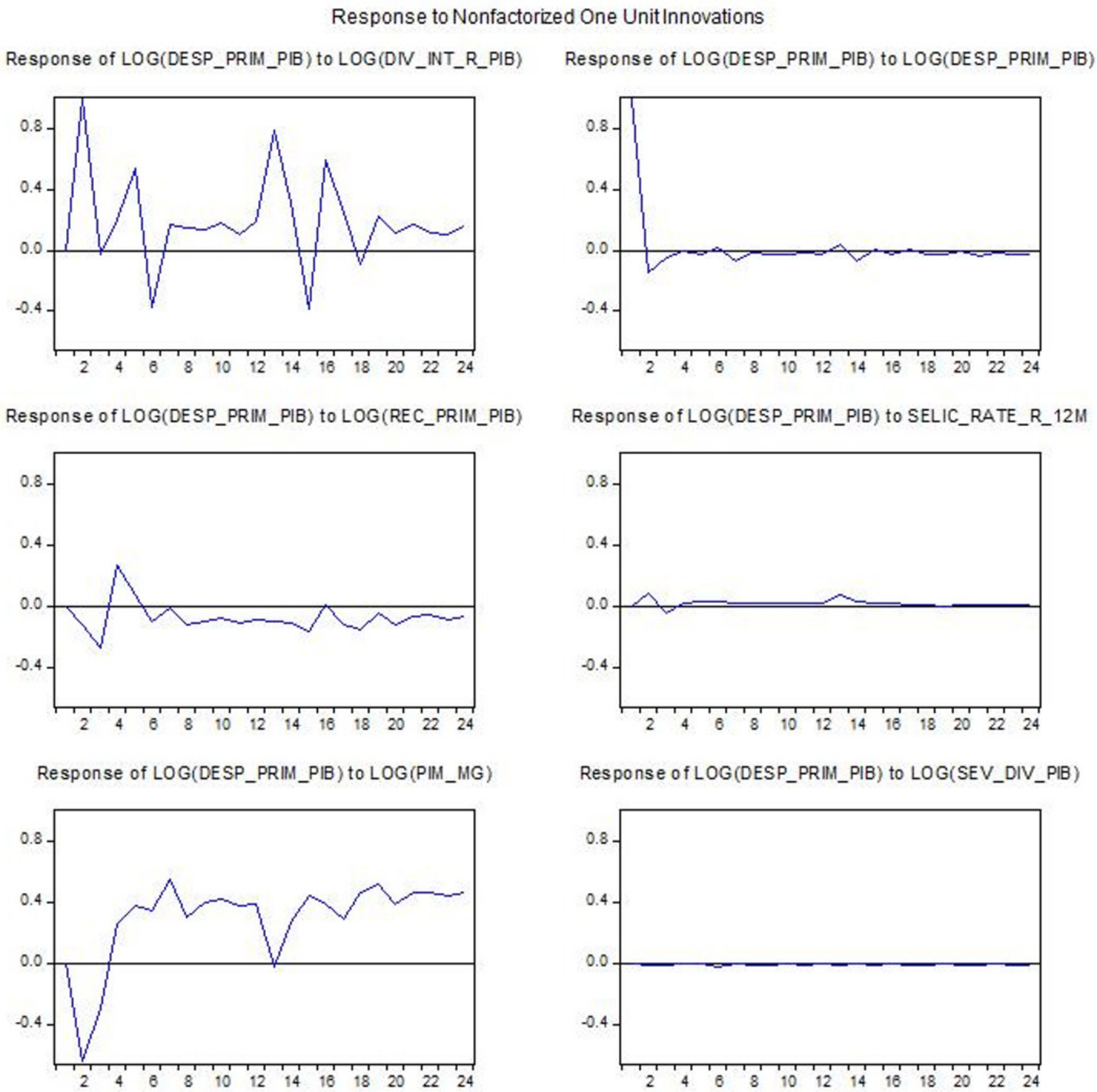
Dependent variable: D(LOG(SEV_DIV_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	16.09150	4	0.0029
D(LOG(DESP_PRIM_PIB))	4.223563	4	0.3766
D(LOG(REC_PRIM_PIB))	2.488722	4	0.6467
D(SELIC_RATE_R_12M)	8.980616	4	0.0616
D(LOG(PIM_MG))	5.610361	4	0.2302
All	38.96672	20	0.0067

6.5.3 IRF Chart unconventional




Response to Nonfactorized One Unit Innovations
Response of LOG(REC_PRIM_PIB) to LOG(DIV_INT_R_PIB)
Response of LOG(REC_PRIM_PIB) to LOG(DESP_PRIM_PIB)
Response of LOG(REC_PRIM_PIB) to LOG(REC_PRIM_PIB)
Response of LOG(REC_PRIM_PIB) to SELIC_RATE_R_12M
Response of LOG(REC_PRIM_PIB) to LOG(PIM_MG)
Response of LOG(REC_PRIM_PIB) to LOG(SEV_DIV_PIB)



Response to Nonfactorized One Unit Innovations
Response of LOG(DESP_PRIM_PIB) to LOG(DIV_INT_R_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(DESP_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(REC_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to SELIC_RATE_R_12M
Response of LOG(DESP_PRIM_PIB) to LOG(PIM_MG)
Response of LOG(DESP_PRIM_PIB) to LOG(SEV_DIV_PIB)
0.8
0.4
0.0
-0.4
2 4 6 8 10 12 14 16 18 20 22 24

6.6 Simple Model Weak Sustainability

6.6.1 Model Outputs

Model Simpler Sustainability Weak						
Lags 1, 2, 3 and 12; Seasonal, pulse and level dummies						
Cointegration Restrictions: B(1,1)= 1, B(2,2) = -1, B(3,3)= 1, B (1,2) = 0, B(1,3) = 0, B(2,1)= 0, B(2,3) = 0, B(3,1)= 0, B(3,2) = 0, A(3,2) =0						
Convergence achieved after 15 iterations.						
Restrictions identify all cointegrating vectors						
LR test for binding restrictions (rank = 3):						
Chi-square(1)	2.544917					
Probability	0.110650					
Error Correction:	D(LOG(DIV_INT_R_PIB))	D(LOG(-DESP_PRIM_PIB))	D(LOG(REC_PRIM_PIB))	D(SELIC_RATE_R_12M)	D(LOG(PIM_MG))	D(LOG(SEV_DIV_PIB))
R-squared	0.699628	0.742432	0.788037	0.642742	0.821906	0.765744
Adj. R-squared	0.620781	0.674820	0.732397	0.548961	0.775157	0.704252
Sum sq. resids	0.177698	5.325693	1.052543	15.96354	0.165390	150.3871
S.E. equation	0.033326	0.182443	0.081107	0.315867	0.032151	0.969494
F-statistic	8.873181	10.98082	14.16306	6.853695	17.58104	12.45272
Log likelihood	426.6042	81.48279	246.0481	-2.994.028	433.8899	-2.575.952
Akaike AIC	-3.779.351	-0.379141	-2.000.474	0.718623	-3.851.132	2.961529
Schwarz SC	-3.077.539	0.322671	-1.298.662	1.420436	-3.149.320	3.663341
Mean dependent	-0.000737	0.009022	0.002632	-0.075912	0.000265	0.008833
S.D. dependent	0.054117	0.319938	0.156788	0.470325	0.067804	1.782726

6.6.2 Granger Causality Test

VEC Granger Causality/Block Exogeneity Wald Tests

Date: 08/10/21 Time: 22:41

Sample: 2003M01 2020M12

Included observations: 203

Dependent variable: D(LOG(DIV_INT_R_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DESP_PRIM_PIB))	8.148421	4	0.0863
D(LOG(REC_PRIM_PIB))	6.369560	4	0.1732
D(SELIC_RATE_R_12M)	4.422545	4	0.3518
D(LOG(PIM_MG))	5.856928	4	0.2101
D(LOG(SEV_DIV_PIB))	19.78710	4	0.0006
All	44.47331	20	0.0013

Dependent variable: D(LOG(DESP_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	3.426989	4	0.4891
D(LOG(REC_PRIM_PIB))	6.767554	4	0.1487
D(SELIC_RATE_R_12M)	7.743707	4	0.1014
D(LOG(PIM_MG))	16.10711	4	0.0029
D(LOG(SEV_DIV_PIB))	0.968785	4	0.9145
All	43.78811	20	0.0016

Dependent variable: D(LOG(REC_PRIM_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	15.29725	4	0.0041
D(LOG(DESP_PRIM_PIB))	6.137934	4	0.1891
D(SELIC_RATE_R_12M)	0.252376	4	0.9927
D(LOG(PIM_MG))	7.923340	4	0.0944
D(LOG(SEV_DIV_PIB))	16.00656	4	0.0030
All	59.42331	20	0.0000

Dependent variable: D(SELIC_RATE_R_12M)

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	3.623791	4	0.4593
D(LOG(DESP_PRIM_PIB))	7.926631	4	0.0943
D(LOG(REC_PRIM_PIB))	10.49474	4	0.0329
D(LOG(PIM_MG))	1.711949	4	0.7885
D(LOG(SEV_DIV_PIB))	2.852537	4	0.5828
All	26.41554	20	0.1525

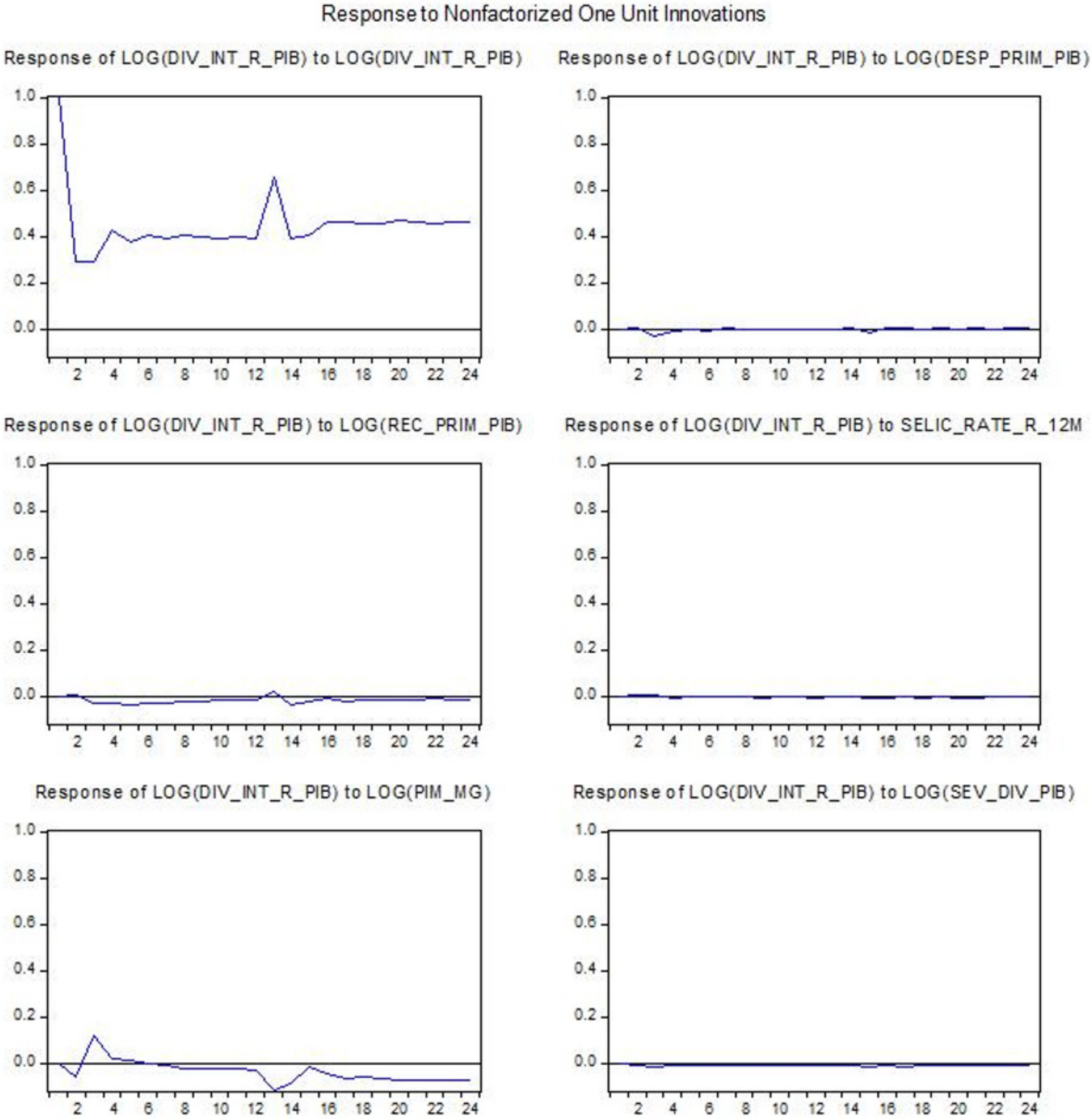
Dependent variable: D(LOG(PIM_MG))

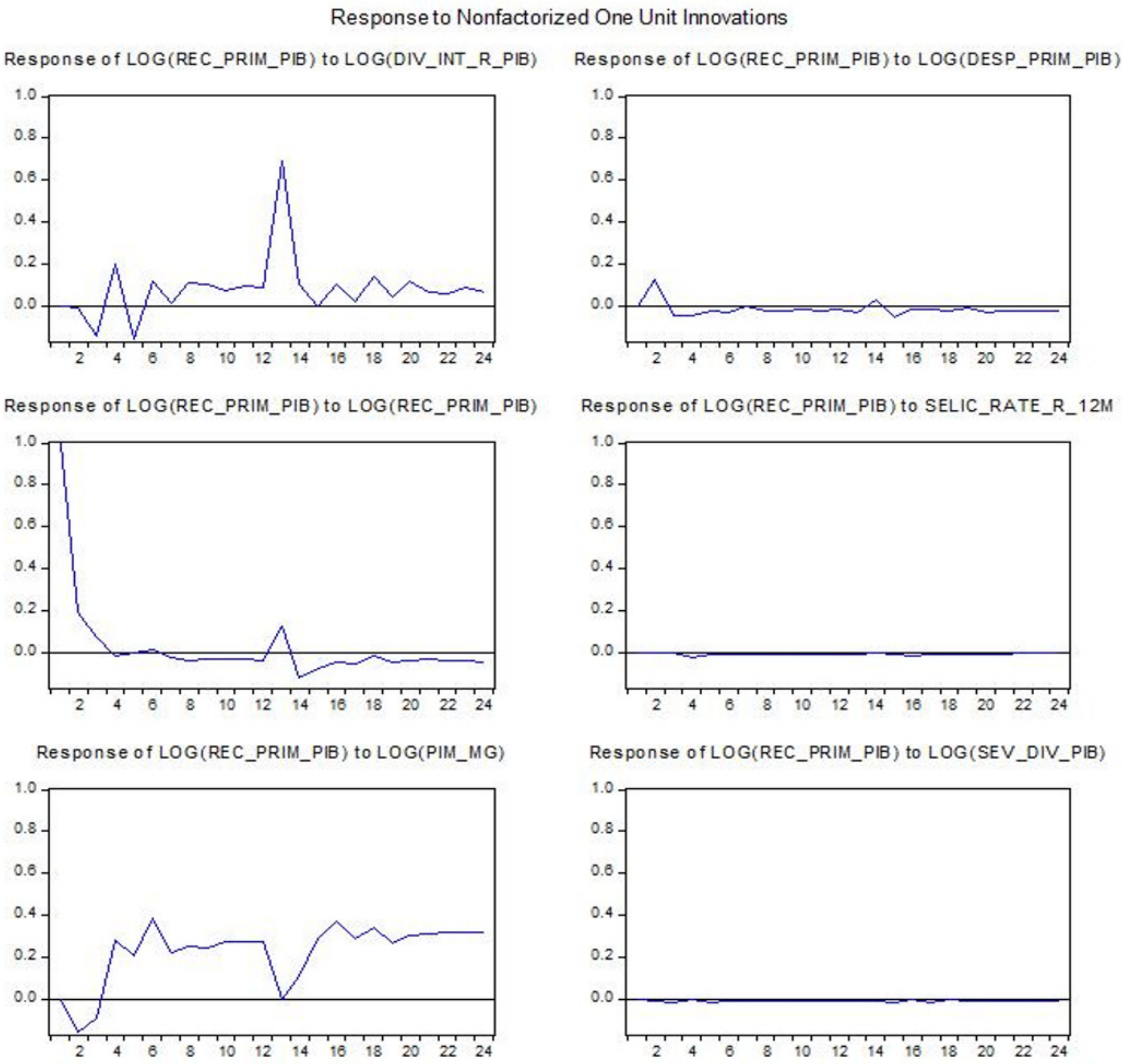
Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	68.08488	4	0.0000
D(LOG(DESP_PRIM_PIB))	17.37288	4	0.0016
D(LOG(REC_PRIM_PIB))	5.124568	4	0.2748
D(SELIC_RATE_R_12M)	3.797938	4	0.4340
D(LOG(SEV_DIV_PIB))	2.676672	4	0.6133
All	116.6638	20	0.0000

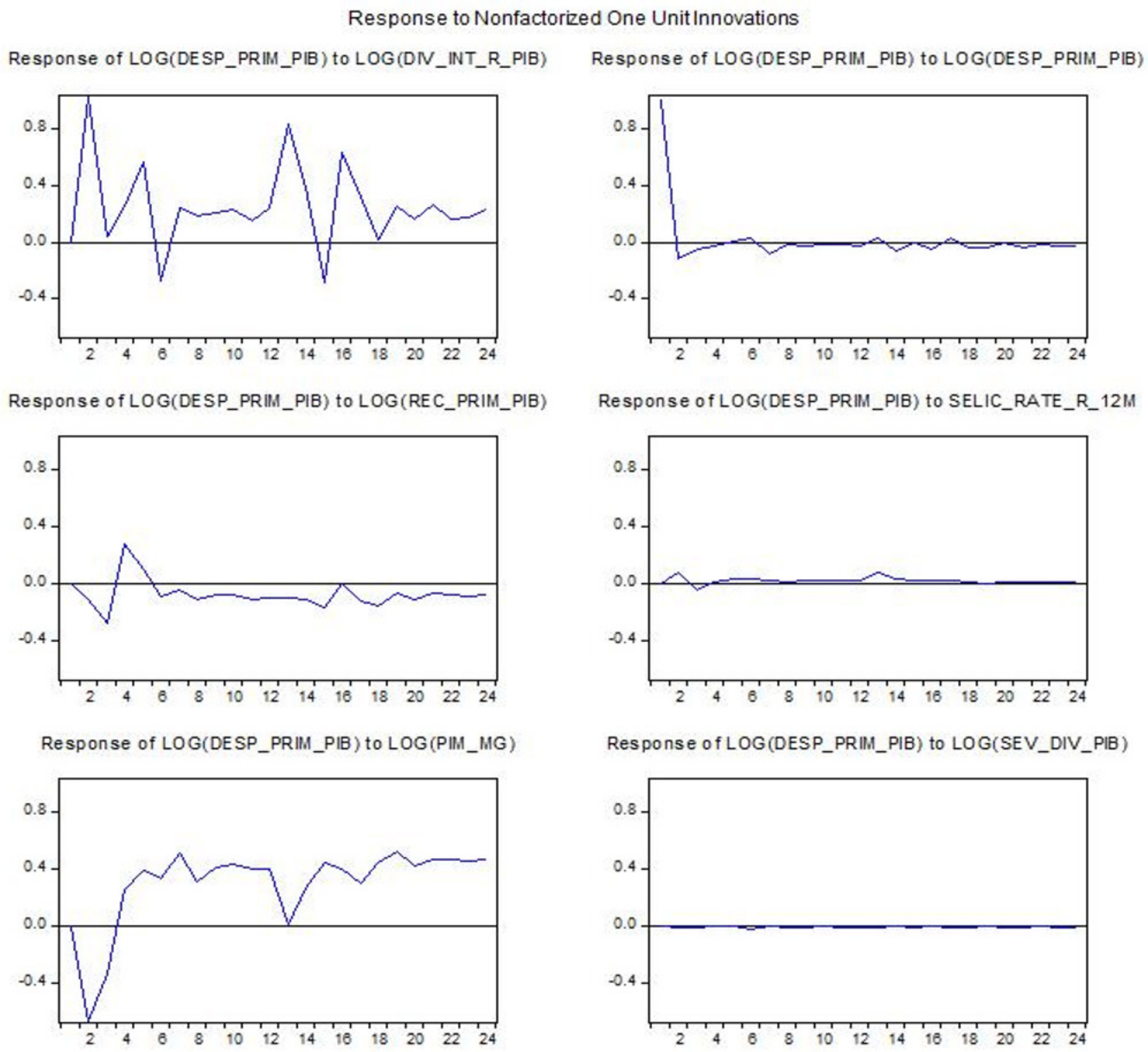
Dependent variable: D(LOG(SEV_DIV_PIB))

Excluded	Chi-sq	df	Prob.
D(LOG(DIV_INT_R_PIB))	15.64549	4	0.0035
D(LOG(DESP_PRIM_PIB))	4.503888	4	0.3421
D(LOG(REC_PRIM_PIB))	2.427110	4	0.6577
D(SELIC_RATE_R_12M)	9.097894	4	0.0587
D(LOG(PIM_MG))	5.370831	4	0.2513
All	39.20361	20	0.0063

6.6.3 IRF Chart unconventional




Response to Nonfactorized One Unit Innovations
Response of LOG(REC_PRIM_PIB) to LOG(DIV_INT_R_PIB)
Response of LOG(REC_PRIM_PIB) to LOG(DESP_PRIM_PIB)
Response of LOG(REC_PRIM_PIB) to LOG(REC_PRIM_PIB)
Response of LOG(REC_PRIM_PIB) to SELIC_RATE_R_12M
Response of LOG(REC_PRIM_PIB) to LOG(PIM_MG)
Response of LOG(REC_PRIM_PIB) to LOG(SEV_DIV_PIB)


Response to Nonfactorized One Unit Innovations
Response of LOG(DESP_PRIM_PIB) to LOG(DIV_INT_R_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(DESP_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to LOG(REC_PRIM_PIB)
Response of LOG(DESP_PRIM_PIB) to SELIC_RATE_R_12M
Response of LOG(DESP_PRIM_PIB) to LOG(PIM_MG)
Response of LOG(DESP_PRIM_PIB) to LOG(SEV_DIV_PIB)
0.8
0.4
0.0
-0.4
2 4 6 8 10 12 14 16 18 20 22 24