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Optimal public debt and the connection between economic growth, financial development and tax cycles: Empirical evidence for the Brazilian economy

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Summary

This monograph evaluates the effects of tax policy, based on tax cycles variables, in addition to the effects of nominal and real credit cycles on economic growth, considering monthly time series from 1996:03 to 2020:06. Based on simultaneous equation models, via GMM, the following empirical results were obtained. On the one hand, the primary surplus (%GDP) cycles respond positively to the public debt (%GDP) cycles. On the other hand, the primary surplus cycle and the cycle of the difference between the real interest rate and the growth rate have negative and positive impacts, respectively, on the public debt cycle (%GDP). In addition, the growth rate of the economy is positively affected by the credit and primary surplus cycles (%GDP) and negatively by the public debt. Finally, based on Cointegration models, the optimal level of public debt lies between 26% and 27% of GDP.

Keywords: Tax cycles, economic growth, credit cycle, GMM, Optimal level of public debt.

JEL Classification: E3, E6, O4.

1. Introduction

Endogenous growth models, such as the AK models that were introduced in the economic literature by Paul Romer (1987) and Sergio Rebelo (1991), consist of models that take into account endogenous growth in the sense that economic policies can influence the long-term growth rate. In this context, this monograph evaluates the effects of tax policy on economic growth, considering a monthly database from 1996:03 to 2020:06. Among other references on endogenous growth, one can highlight Aghion and Howitt (1998), Romer (1990 and 1994) and other relevant works.

The first objective of this paper is to perform an econometric analysis of a system of simultaneous equations based on two empirical models. The first equation refers to a tax reaction function that aims to test the hypothesis that the primary surplus adjusts in response to changes in the public debt, to ensure its sustainability. Regarding the analysis of tax reaction functions, one can highlight the works of Mendonca and Santos and Sachsida (2009) and Campos and Cysne (2019), as well as the articles by Born (1998) and Moreira (2011) that are the bases for the tax reaction model in this monograph. The second equation refers to an endogenous economic growth model to test whether the primary surplus (%GDP) cycle and the credit cycle, affect the growth of the economy.

The second objective also evaluates a system of simultaneous equations based on two empirical models, in which the first equation is based on a government budget constraint, based on the works of Buiter (1985) and Spaventa (1987), in which the public debt cycle (%GDP) is explained by the primary surplus cycle (%GDP) and by the cycle of the difference between the real interest rate and the economic growth rate. The second equation also refers to an endogenous economic growth model in which the effects of the public debt cycle (%GDP), as well as the credit cycle, on the growth of the economy are tested.

Finally, the third objective is to determine the optimal level of public debt as a proportion of GDP, to maximize the real productivity of the economy. In this context, it becomes necessary to conduct empirical tests to find out if there is a non-linear relationship between productivity and public debt. If there is and it presents an inverted U shape, one can determine the level of debt as a proportion of GDP that maximizes the economy's productivity. This is very useful information for policy makers, since they will be able to know how far the level of the debt-output ratio is from its optimal level, and whether they are below or beyond the level that maximizes the economy's productivity.

The question of public debt sustainability is directly related to the tax reaction function and the government's budget constraint. In this sense, Luporini (2006) makes a good review of the literature and analyzes the various ways of testing tax sustainability. Pereira *et al* (2009) present in detail the arithmetic of deficits and public debt, as well as the arithmetic of the debt-output ratio, which inspired the creation of the public debt cycle variable.

In models concerning the determinants of the real GDP growth rate, in addition to using *poxies* for capital and labor, as well as tax cycle variables, the credit cycle is also used as an explanatory variable in the model. In this context, a connection can be made with the literature on the nexus between financial development and economic growth. Such literature argues that financial development always leads to growth in the economy, but there is still much controversy on the topic, as Nyasha and Odhiambo (2019) point out, especially with the causal relationship of the said nexus. Thus, assuming that financial system development is positively correlated with credit market development, one can test whether, in fact, credit causes economic growth.

Tabak *et. al.* (2016) analyzes how the bank lending channel is affected by monetary authority actions based on BRICS countries (Brazil, Russia, India, China, and South Africa) over the period 2000 to 2012. The empirical results show that the effect of money supply growth on loan growth is non-linear and exhibits an inverted U-shaped curve. In this context, the results show empirical evidence that expansionary monetary policies do not increase the propensity of economic agents to systematically take higher risks in the market. After a certain level of money stock, increases in the money supply do not lead to an increase in traded credit. In short, the paper shows that changes in the money stock affect credit.

In turn, Tiryaki *et. al.* (2017) discusses the relationship between credit cycles, default and economic fluctuations, highlighting the theoretical framework of Keynesian tradition that shows the importance of credit cycles for short-term fluctuations in economic activity, which usually arise from real or monetary shocks. According to the authors, among the various theoretical currents, only those of Keynesian tradition emphasize the importance of credit for investment and, consequently, for short-term fluctuations in economic activity. Combining the considerations of Tabak *et. al.* (2016) and Tiryaki *et. al.* (2017) one observes a short-run relationship in which money affects credit, which in turn affects investment and therefore affects economic fluctuation. Such channels justify an evaluation in this monograph of the effects of the credit cycle on economic growth, because if the credit cycle affects investment in the short run, in the long run such amount of investment will affect the capital stock of the economy.

Given the above, this monograph ultimately evaluates the direct and indirect effects of tax cycles on the growth rate of the Brazilian economy with monthly data from 1996:03 to 2020:06, in addition to the effects of nominal and real credit cycles on economic growth.

2. Methodological aspects

The database described in Table 1 shows the variables used and their definitions. Real GDP is calculated by dividing GDP by the IPCA. The Real GDP growth rate (TX_GDP_REAL) is calculated by log [Real_GDP/Real_GDP (-1)]. Credit is deflated by IPCA so that the credit cycle is calculated by the difference between real credit and the HP filter of Hodrick-Prescott (1997) generating the variable (CREDIT_REAL_CYCLE). We also use the nominal credit cycle based on the HP filter (CREDIT_CYCLE). All cycle variables are calculated in the same way and are transformed into logarithmic values. The Public Sector Borrowing Requirement is multiplied by -1 to obtain the primary surplus as a proportion of GDP (SP). To this SP series a positive constant is added, in such a way that all values of the series are positive, to make the logarithmic transformation possible. In this context, we use the following tax cycles variables: i) the Primary Surplus cycle (SP_CYCLE) relative to the Federal Government and Central Bank (%GDP); and ii) the cycle of the total Federal Government and Central Bank debt - net (%GDP) defined as (DIV_T_FED_CYCLE).

Since the real change in gross fixed capital formation is quarterly, the linear interpolation method is used to convert the quarterly series into a monthly series called TX_GFCF based on the statistical software Eviews. The rate of change of hours worked in industry is calculated by log [Hours_Worked/Hours_Worked (-1)], denoted TX_HORAS_TRAB. Finally, the real interest rate denominated "**r**" is calculated by the difference between the Selic rate and the variation of the IPCA. In this way, the cycle of the difference between the real interest rate and the real product growth rate, where Tx_GDP_REAL = **"y"**, is denominated (r - y) CYCLE.

Table 1 - Database description

Variables	Description of variables
GDP	GDP - 12 months - R$ (millions) - Banco Central do Brasil, Time Series Management System (Bacen Others/SGS) - BM12_PIBAC12.
IPCA	IPCA - general - index (Dec. 1993 = 100) - Brazilian Institute of Geography and Statistics, National System of Consumer Price Indexes (IBGE/SNIPC) - PRECOS12_IPCA12.
CREDIT	Credit operations - balance - R$ (million) - Central Bank of Brazil, Economic and Financial Notes for the Press, Monetary Policy and Credit Operations of the SFN (Bacen/Notes Press/Money) - BM12_CS12.
NFSP: PRIMARY	NFSP - Federal Government and Central Bank - primary - without exchange devaluation - accumulated 12 months - (% GDP) - Banco Central do Brasil, Financial Economic Notes for the Press, Public Finance (Bacen/Press Notes/Public Finances) - BM12_NFGFPYS12.
DIV_T_FED	Debt - total - Federal Government and Central Bank - net - (% GDP) - Central Bank of Brazil, Economic and Financial Notes for the Press, Public Finance (Bacen/Press Notes/Public Finance) - BM12_DTGFY12.
GFCF	GDP - gross fixed capital formation - real var. quarterly - (%) - Brazilian Institute of Geography and Statistics, Quarterly National Accounts System (IBGE/SCN Quarterly) - SCN104_FBKFG104.
HORAS_TRAB	Hours worked - industry - seasonal index. (Average 2006 = 100) - National Confederation of Industry - CNI12_HTRABD12.

SELIC	Interest rate - Over / Selic - (% a.m.) - Central Bank of Brazil, Bulletin, Financial and capital markets section (Bacen/Report/Ministry of Finances.) - BM12_TJOVER12.

Source: Ipea Data

In Subsection 4.1 we highlight the connections between the cycle of total federal and Central Bank (BACEN) debt as a proportion of GDP, the primary surplus cycle, the credit cycle, and economic growth, which are presented based on two systems of equations. Each system of equations consists of two equations, such that the first equation of each system is instrumented to avoid endogeneity problems.

The first system of simultaneous equations via GMM shows the econometric specifications or equations (1) and (2) that correspond to Tables 3 and 4 presented in the next section, as well as Tables 3A and 4A in the appendix. The difference between the tables presented in the text and in the appendix consists in the use of the nominal and real credit cycle. In the tables within the text the nominal credit cycle is used (Tables 3 and 4), called CREDIT_CYCLE. In the Appendix the real credit cycle (Tables 3A and 4A), called CREDIT_REAL_CYCLE, is used. In other words, all estimates made within the text are estimated with the (nominal) credit cycle instead of the actual credit cycle that is explained in the Appendix.

$$SP\_CYCLOt = Bo + B1*SP\_CYCLOt\text{-}1 + B2*DIV\_T\_FED\_CYCLOt + Vt \quad (1)$$

$$TX\_GDP\_REALt = \delta o + \delta 1*TX\_GDP\_REALt\text{-}1 + \delta 2*TX\_FBCFt + \delta 3*HORAS\_TRABT + \delta 4*SP\_ICYCLEt + \delta 5*CREDIT\_ICYCLEt + Zt \quad (2)$$

Equation 1 shows a tax policy reaction function, in which the primary surplus cycle, as a proportion of GDP (SP_CYCLE), responds to the total federal and BACEN debt cycle also as a proportion of GDP (DIV_T_FED_CYCLE), also considering the lagged dependent variable. If policy makers aim at public debt sustainability then an expansion of the primary surplus (%GDP) is expected as a response to an expansion of public debt (%GDP). Thus, we use the tax cycle variables instead of the primary surplus/GDP variation and the public debt/GDP variation.

When the values of the primary surplus/GDP series are above the HP filter trend, one can observe periods in which there is an expansion of the primary surplus/GDP and, in turn, when the values are below, there is a contraction. In the same way, when the values of the public debt/GDP series are above the HP filter, a period of expansion of the public debt/GDP is observed, otherwise, a contraction is observed.

In this context, one can formulate a tax rule of tax cycles, in which the primary surplus/GDP responds to an expansion of the public debt/GDP, also with an expansion of the primary surplus/GDP itself, with the objective of making the path of the public debt/GDP sustainable. This formulation of the tax rule makes understanding between the two variables more intuitive and is a contribution to the literature, since no such approach has been detected in the literature, unless I am mistaken. Thus, based on equation (1), assuming a tax responsibility policy, the coefficient $B2$ is expected to be positive and statistically significant.

Equation 2 of the first system via GMM, shows the traditional determinants of the economic growth rate as a function of the rate of gross fixed capital formation (TX_FBCK), which represents a *proxy* for the change in the capital stock, and the change in industry hours worked, as a *proxy* for the change in the quantum of labor in the country. In other words, the economic growth rate is a function of the change in the capital stock and the change in the quantum of labor in terms of industry hours worked. Besides the lagged dependent variable, TX_GDP_REAL (-1), the primary surplus cycle as a proportion of the GDP (SP_CYCLE) and the credit cycle (CREDIT_CYCLE) are used as a basis in an endogenous growth model and can also explain the economic growth in the period analyzed.

In the same way that tax cycles are calculated, credit cycles are calculated, whether nominal or real. When the values of the credit series are above the HP filter trend, one can observe the periods in which a credit expansion occurs and, in turn, when the values are below, a contraction is verified.

One can admit that the credit cycle can have short-run effects on the business cycle, which is not the object of investigation in this paper, just as in the long run the credit cycle can influence the economic growth rate. As is already widely known, business cycle is a short-term issue, while economic growth is a long-term issue. If the estimated coefficient of the credit cycle is positive and statistically significant based on equation 2, where $\delta 5 > 0$, then the link between economic growth and financial development is confirmed, showing that the credit cycle has a long-term effect.

Finally, we can evaluate the indirect effect of the variable DIV_T_FED_CYCLE of equation 1, on the variable TX_GDP_REAL as per equation 2. Assuming that there is tax responsibility on the part of policymakers regarding a sustainable debt trajectory, since the primary surplus cycle responds, in a directly proportional way, to the debt cycle, as per equation 1. This response of the primary surplus increase indirectly affects economic growth as per equation 2. In other words, the indirect effect is calculated by the product of the coefficients $B2*\delta4$ of equations 1 and 2 respectively (Moreira, *et al* 2016). The following scheme shows this indirect effect more clearly.

DIV_T_FED_CYCLE == SP_CYCLE == ➔➔TX_REAL_PIB

The second system of simultaneous equations via GMM shows the econometric specifications or equations (3) and (4) that correspond to Tables 5 and 6, as presented below. Similarly, in the appendix are Tables 5A and 6A which refer to the estimates with the actual credit cycle.

$$DIV\_T\_FED\_CYCLOt = Bo + B1*DIV\_T\_FED\_CYCLOt\text{-}1 + B2*SP\_CYCLOt + B3*(r - y)\_CYCLOt + Vt \quad (3)$$

$$TX\_GDP\_REALt = \delta o + \delta 1*TX\_GDP\_REALt\text{-}1 + \delta 2*TX\_FBCFt + \delta 3*HORAS\_TRABt + \delta 4*DIV\_T\_FED\_CYCLEt + \delta 5*CREDITO\_CYCLEt + Zt \quad (4)$$

It is worth noting the difference between this second system of equations (equations 3 and 4) with the previous system of simultaneous equations via GMM (equations 1 and 2). In this context, it makes more sense to first compare equations 1 and 3, which are the instrumented equations, and then equations 2 and 4, relating to economic growth.

While equation 1 refers to a tax reaction rule, equation 3 refers to a modeling that is inspired by the arithmetic of the ratio of public debt to output, based on the government's budget constraint (Buiter, 1985 and Spaventa, 1987), as presented below:

$$\Delta b = g - \rho - \lambda m + b\,(r - y) \qquad (5)$$

where Δb is the change in the debt to real GDP ratio, g is the primary expenditure to real GDP ratio, ρ is the government revenue to real ratio, M is the nominal value of the monetary base, $\lambda = \Delta M/M$, m = M to real GDP such that λm is the seigniorage, r is the real interest rate based on Fisher's (1930) equation which is approximately the difference between the nominal interest rate (i) and the inflation rate (π), i.e., r = (i - π). Finally, y represents the growth rate of real output. Pereira et. al (2009) show in detail the development of mathematical modeling from the government's budget constraint to arrive at equation 5.

The main difference is that here we use the public debt cycle, as well as the primary surplus cycle as a proportion of GDP, instead of the variation of the public debt/GDP ratio, Budget Revenue/GDP and Primary Budget Expenditure/GDP, according to equation 5. Furthermore, since the period analyzed occurs as of 1996, with the implementation of the Real Plan that dismantled the inflationary indexation process and with the creation of an inflation targeting rule as of 1999. In this model, possible effects of the monetary base or seigniorage are not considered since this period did not present serious inflationary risks.

It should also be noted that, based on equations 3 and 5, it can be observed that when the real interest rate (r) exceeds the economic growth rate (y) and the government is unable to obtain positive primary surpluses, it is assumed, without loss of generality, that seigniorage revenues as well as privatization revenues are negligible. Such results address us to the unpleasant monetarist arithmetic of Sargent and Wallace (1981). In other words, if such conditions persist for a long period, public debt may become unsustainable.

As far as equations 2 and 4 are concerned, the difference is that in equation 4 the primary surplus cycle in equation 2 is replaced by the public debt cycle in equation 4 as the explanatory variable of economic growth.

The two simultaneous equation systems presented above can solve possible endogeneity problems with the use of the generalized method of moments (GMM) that uses instrumental variables (VI). In this context, instruments must be "good instruments" to be relevant and valid with the appropriate use of the VI method. This means that instruments should not only be correlated with endogenous regressors, but also orthogonal to the error. In this sense, we use some statistical tests for our econometric specification: the Sargan-Hansen overidentification test, in which is also known as the J-statistic, the underidentification test (Cragg and Donald, 1993) as well as the Stock-Yogo test (Stock and Yogo, 2005) to analyze the hypothesis of weak instruments.

Thus, the independence of the instrument with respect to the disturbance can only be assessed if, and only if, there is an "abundance" of instruments, that is, if the equation is overidentified. This is the case for the Sargan-Hansen test, in which the null hypothesis (Ho) states that the equation is overidentified. Thus, if Ho is not rejected, the instruments are abundant and relevant.

On the other hand, for the instruments to be valid, one can test the rank condition by means of the Cragg and Donald (1993) test, such that the null hypothesis admits that the model is underidentified or unidentified. Thus, if Ho is not accepted, the instruments are valid. In short, for the instruments to be good, the null hypothesis of the Sargan-Hansen test must be accepted and the Ho of the Cragg and Donald (1993) test must be rejected.

It should also be noted that specific problems may arise in conventional inference in relation to OLS regressions when the variables are not stationary. In this regard, Johnston and DiNardo (1997) point out the relevance of whether similar issues occur in two-stage least squares regressions. Hsiao (1997a, 1997b) analyzes this problem and concludes that inference with two-stage least squares estimators using VI remains effective, even in the case where the time series are considered non-stationary or non-cointegrated. In this context, Hsiao's conclusions are valid when GMM is applied.

The procedure of Newey and West (1987a, 1987b) is also used for all estimated models to solve the two problems of serial correlation arising from the residuals and unknown heteroscedasticity. They suggested a more general covariance estimator that is consistent in the presence of heteroscedasticity and autocorrelation of an unknown form.

To analyze the causal relationship between variables the Granger methodology is used (Hamilton, 1994). To perform the Granger causality test (1969) it is necessary, at first, to verify the stationarity tests of the time series of the respective variables to define the corresponding order of integration. In this context, ADF and Phillips-Perron unit root tests are used. Thus, one can choose the optimal number of lags to be used in the analysis. The null hypothesis (Ho) to be tested is that the coefficients of the variables evaluated are significantly equal to zero. The decision is based on the F-statistic for the joint significance of the coefficients. In this case, if Ho is not accepted, we conclude that the estimated coefficients are, statistically, different from zero and a given variable causes, in the Granger sense, another variable. In this context, we use structural analysis (SVAR) to show the impulse-response function in the appendix.

Finally, based on Cointegration models, we test whether there is an optimal level of public debt that maximizes the economy's productivity, such that:

$$Productivityet=\alpha o+\alpha i*Productivityet\text{-}i+\lambda 2*DIV\_T\_FEDt+\lambda 3*(DIV\_T\_FED)^{2t+Wt} \quad (6).$$

Productivity is defined as the ratio of the value of real GDP to hours worked, measuring output per hour worked. The coefficient αo represents the constant term, and αi are the parameters of equation 6 referring to the lagged coefficients of the dependent variable ranging from i = 1, ...2, , n. The parameters λ2 and λ3 represent, respectively, the coefficient of the ratio of total Federal Government and BACEN debt as a proportion of GDP and the coefficient of the square of the mentioned debt. In this context, it is expected that the two estimated coefficients are statistically different from zero and that $\lambda 2 > 0$ and $\lambda 3 < 0$ to generate an inverted U-shaped curve. That is, that there is a non-linear relationship between productivity and public debt as a proportion of GDP represented by a concave curve. Under these conditions, it will be possible to determine the level of public debt that maximizes the economy's real productivity, which is called the "optimal level of public debt".

In this context, cointegrating regressions are used since the variables are I (1), non-stationary, to ensure that they obtain even more robust results; which is shown in subsection 4.2. There are three efficient estimation methods. The first is the Fully Modified OLS (FMOLS) based on Phillips and Hansen (1990, 1995). The second is the Canonical Cointegrating Regression (CCR) based on Park (1992) and finally the Dynamic OLS (DOLS) based on Saikkonen (1992) and Stock and Watson (1993). The DOLS method because it is dynamic, and therefore uses the 1st difference, should be highlighted since the interest is to know the optimal level of debt, rather than the optimal level of the change in public debt. However, before estimating the two models (FMOLS and CCR), one should check, based on Johansen's (1988, 1991 and 1995) Cointegration tests, whether the two variables cointegrate. Johnston and Dinardo (1997) is also a good reference for Cointegration methods.

3. Empirical Results

The unit root tests of the time series presented in Table 2A, based on the ADF and Phillips-Perron statistics tests, show that all variables are stationary, since the null hypothesis assuming unit root is not accepted.

Table 2: Unit root tests (H0: Unit root)

	Augmented Dickey-Fuller Statistic (ADF) Test			Phillips-Perron Statistic (P.P.) test		
Variables	Critical value: 5%	t-statistic	p-value	Critical value: 5%	Statistical Adj. t	p-value
TX_GDP_REAL	-2,871	-7,223	<0,0001	-2,871	-7,330	<0,001
CREDIT_CYCLE	-1,941	-5,087	<0,0001	-1,941	-2,502	0,012
ACTUAL_CREDIT_CYCLE	-1,941	-3,534	0,0004	-1,941	-3,768	0,0002
SP_CYCLE	-1,941	-3,544	0,0004	-1,941	-2,905	0,0037
DIV_T_FED_CYCLE	-1,941	-4,759	<0,0001	-1,941	-4,778	<0,0001
TX_FBCF	-1,941	-2,820	0,0049	-1,941	-3,415	0,0007
TX_HORAS_TRAB	-1,941	-14,190	<0,0001	-1,941	-19,040	<0,0001
(r - y) _CYCLE	-2,871	-8,242	<0,0001	-2,871	-8,3793	<0,0001
Productivity	-2,871	1,492	0,9993	-2,871	1,031	0,9969
Div_T_FED	-2,871	-1,122	0,7076	-2,871	-1,314	0,6234

Note: Constant included.

The Granger causality test, with the optimal number of 8 lags, shows, based on Table 2A, a one-sided causality of the nominal credit cycle for the output growth rate. Thus, the variable CREDIT_CYCLE temporally precedes GDP growth by 8 months. Given that, the amount of credit operations is considered by the literature as a *proxy* or instrument for the financial development of a given country (Barajas, *et al* 2013), showing that there is a link between economic growth and financial system development, then an interesting result is obtained, in which the credit cycle Granger causes output growth. This result motivates the introduction of the credit cycle variable as a determinant of economic growth in the empirical models presented.

In this context, one can assume that there is also a cycle, not of financial development itself, but of the ups and downs of the financial system. For example, the periods of expansion of credit and other financial assets that preceded the *subprime* crisis in 2007 and 2008 in the US, compared to the aftermath of the initial effect of the crisis with the Lehman Brothers bankruptcy, led the US Federal Reserve Bank (FED) to use an unprecedented policy of monetary easing in which the FED directly bought 'bad' assets of large firms to avoid the bankruptcy of not only large firms but also large banks and thus avoid a contagion effect (Longstaff, 2010).

It is worth noting that instead of using the variation in credit operations, the credit cycle is used. The advantage of using the credit cycle is that it becomes more intuitive to analyze the effects of a credit expansion or contraction on the growth rate of real output.

Table 2 A - Granger Causality Test

Null Hypothesis	Remarks	F-statistic	Prob.
TX_PIB_REAL not Granger Causes CREDIT_CYCLE	283	0,91972	0,5005
CREDIT_CYCLE not Granger Causes TX_GDP_REAL		2,36084	0,0181

Note 1: TX Real GDP=Real GDP growth rate, CREDIT_CYCLE=Cycle of credit operations. Note 2: Optimal number of 8 lags based on LT, FPE and AIC criteria.

As for the real credit cycle, the results of the Granger causality test also show that a credit expansion temporally precedes GDP growth by 8 months, just as a credit contraction precedes a fall or slowdown in economic activities, as shown in Table 2B. Thus, we use the credit cycle variable, both nominal and real, with an 8-month lag in the empirical models.

Table 2 B - Granger Causality Test

Null Hypothesis	Remarks	F-statistic	Prob.
TX_PIB_REAL not Granger Causes CREDIT_REAL_CYCLE	283	1,30477	0,2411
CREDIT_REAL_CYCLE not Granger Causes TX_PIB_REAL		5,41582	2,E-06

Note 1: TX Real GDP=Real GDP growth rate, CREDIT_CYCLE=Cycle of credit operations. Note 2: Optimal number of 8 lags based on LT, FPE and AIC criteria.

4.1 - Connection between total federal debt cycle and BACEN as a proportion of GDP, primary surplus cycle, financial development and economic growth.

Tables 3 and 4 show the first system of simultaneous equations via GMM based on equations (1) and (2) presented in the methodology section.

The empirical results presented in Table 3 show that all estimated coefficients are statistically significant at the 5% level. Furthermore, it is observed that in the analyzed period, on average, there is a positive response of the SP_CYCLE variable in relation to the DIV_T_FED_CYCLE variable, since the estimated coefficient value of the latter variable is 0.024.

One can also observe, based on the probability of the J statistic (0.113), that the instruments are adequately abundant, since the null hypothesis is not rejected. As for the Cragg-Donald F-statistic (20.99), the null hypothesis of under-identification is not accepted, considering that the Stock-Yogo critical value of 5% is 19.77, revealing that the instruments are valid. Thus, as this is a system of simultaneous equations, via Tables 3 and 4, these two tests show that the instruments can be considered good for equations 1 and 2. The instruments used are shown in the last row of Table 3.

Table 3: Dependent Variable (SP_CYCLE) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
Constant	-0,006276	0,003007	-2,087010	0,0377
SP_CYCLE (-1)	0,928979	0,012420	74,79717	0,0000
DIV_T_FED_CYCLE	0,024174	0,002146	11,26340	0,0000
Statistics				
Statistic J = 26.639	Statistical Prob. J = 0.113		Adjusted R2 = 0.821	
	Critical value Stock-Yogo (5%) = 19.77			
	Cragg-Donald F-statistic = 20.99			
Instruments				
Sp, sp(-1to-5), div_est_m(-1to-2), div_t_sp(-1to-2), credito_trab hours(-1to-2), @trend, div_int_f(-1to-3), div_t_fed_cycle(-1to-5)				

Note 1: SP CYCLE = Primary Surplus Cycle (%GDP); DIV_T_FED_CYCLE = Total Federal and Central Bank Debt - Net (%GDP)

The empirical results presented in Table 4 show that all estimated coefficients are statistically significant at the 5% level and that all coefficients show a positive sign as expected. It is worth noting that the estimated coefficient of the variation in industry hours worked in the current period is not statistically significant in the current period, but is statistically significant with a lag. This means that the variation in hours worked in the previous month affects economic growth in the following month.

From the tax policy point of view, an expansion of the tax surplus contributes to an increase in economic growth, since the estimated coefficient is positive, approximately 0.037. This result shows that a responsible tax policy, which is concerned with tax sustainability, has a positive effect on output growth.

Once the one-way causality process of credit expansion for the economic growth rate is confirmed, the empirical results show, based on Table 4, that the credit cycle with a lag of 8 months affects the real sector of the economy, contributing positively to economic growth, although the estimated coefficient is relatively small. However, one of the main contributions of this paper is to show that the credit cycle can send signals in the sense that policymakers can anticipate whether there will be an increase or decrease in the level of economic activities in the following months.

Taking into account that the credit cycle is calculated by the difference between the balance of credit operations and a trend line based on the HP Filter, values above this line show credit expansion and below, contraction. However, it is assumed that the expansion phase should present an inverted U-shape, where initially there is a first phase of credit expansion that reaches a peak and then begins to decline. Even in the credit decline phase, in this same expansionary phase, the economy will continue to expand for some time, but policy makers will already be able to foresee that later there will be a period of economic decline, where the volume of credit falls below the trend line of the HP filter. Symmetrically, the same analysis can be done considering the periods when credit is below the HP filter line, in which it should present a U-shaped curve.

Based on Table 3, it is observed that a 1% increase in the variable DIV_T_FED_CYCLE increases the variable SP_CYCLE by 0.024174%. In turn, based on Table 4, a 1% increase in the SP_CYCLE variable generates a 0.036958% increment in the TX_GDP_REAL variable. Therefore, the final effect of the response function of the tax rule on economic growth is given by the product between 0.024174*0.036958, which equals 0.00089. In other words, when the primary surplus cycle responds to a given increase in the debt cycle, with an increase in the difference between primary revenues and expenditures, then this tax behavior will reflect positively on economic growth.

Table 4: Dependent Variable (TX_GDP_REAL) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
Constant	0,061390	0,006305	9,737063	< 0,0001
TX_REAL_GDP (-1)	0,752686	0,023629	31,85379	< 0,0001
TX_FBCF	0,008376	0,001160	7,218123	< 0,0001
TX_TRAB_HOURS (-1)	0,101615	0,010602	9,584749	< 0,0001
SP_CYCLE	0,036958	0,017194	2,149470	0,0323
CYCLE_CREDIT (-8)	4,21E-06	7,22E-07	5,834239	< 0,0001

Note 1: Adjusted R2 = 0.256; Note 2: TX Real GDP = Real GDP growth rate; TX_FBCF= Real change in gross fixed capital formation (%); TX WORKING HOURS=Change in industry hours worked; SP CYCLE = Primary surplus cycle (%GDP); CREDIT CYCLE=Cycle of credit operations.

Next, the second system of simultaneous equations is evaluated via GMM according to equations (3) and (4), based on the empirical models presented in Tables 5 and 6.

Based on Table 5, except for the estimated coefficient of the constant term, the other estimated coefficients are statistically significant at the 1% level. The coefficient of the government debt cycle as a percentage of lagged GDP, DIV_T_FED_CYCLE (-1), has an estimated coefficient of approximately 0.857 which is less than 1, suggesting, *ceteris paribus*, that the debt is not explosive. As expected, an expansion of the primary surplus cycle (%GDP) contributes to a reduction in the debt cycle expansion, based on the estimated negative coefficient worth 0.365. Furthermore, also as expected, if the cycle of the difference between the real interest rate and the economic growth rate is positive, the explosive character of the debt cycle is confirmed, considering everything else constant. The estimated coefficient of the variable (r - y) _CYCLE is positive in the value of approximately 0.233.

These results are like those expected based on the models of Buiter (1985) and Spaventa (1987), with due differences. It should also be noted that this second system of equations also shows that the instruments are good, according to the J-Statistic and Cragg-Donald F-Statistic.

Table 5: Dependent Variable (DIV_T_FED_CYCLE) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
Constant	0,003253	0,024483	0,132870	0,8943
DIV_T_FED_CYCLE (-1)	0,856644	0,014457	59,25574	0,0000
SP_CYCLE	-0,365189	0,050119	-7,286464	0,0000
(r - y) _CYCLE	0,232876	0,066356	3,509465	0,0005
Statistics				
Statistic J = 16.335	Statistical Prob. J = 0.176		Adjusted R2 = 0.769	
	Critical value Stock-Yogo (5%) = 18.73			
	Cragg-Donald F-statistic = 25.877			
Instruments				
div_t_fed(-1to-3); sp(-1to-3); r_y(-1to-3); @trend; @trend*div_t_fed r_y_ciclo (-1); div_t_fed_ciclo (-2); credito_ciclo; tx_pib_real				

Note 1: DIV_T_FED_CYCLE = Total Federal Government and Bacen Debt - Net (%GDP); SP CYCLE = Primary Surplus Cycle (%GDP); (r - y) _CYCLE = Cycle of the difference between real interest rate and real GDP growth rate.

Table 6 shows that, except for the estimated coefficient of the variable DIV_T_FED_CYCLE, all the others are statistically significant at the 5% level and present positive signs, as expected. Note that these empirical results are like the results presented in Table 4. The estimated coefficient of the federal debt cycle variable (%GDP) shows a negative sign, but the coefficient is marginally significant with a p-value of 0.0749. This result shows that an increase in the debt cycle contributes marginally to the reduction in economic growth.

Table 6: Dependent Variable (TX_Real_GDP) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
Constant	0,077751	0,022686	3,427197	0,0007
TX_Real_GDP (-1)	0,520879	0,045553	11,43462	< 0,0001
TX_FBCF	0,024799	0,004209	5,892510	< 0,0001
TX_TRAB_HOURS (-1)	0,079814	0,034551	2,310022	0,0212
CYCLE_CREDIT (-8)	4,19E-06	8,98E-07	4,667319	< 0,0001
DIV_T_FED_CYCLE	-0,030781	0,017253	-1,784155	0,0749

Note 1: Adjusted R2 = 0.409, Note 2: TX Real GDP = Real GDP growth rate; TX_FBCF= Real change in gross fixed capital formation (%); TX HOURS WORKED=Change in hours worked in industry; CREDIT CYCLE=Cycle of credit operations; DIV_T_FED_CYCLE = Total Federal and Central Bank Debt - Net (%GDP).

Considering Table 5, it is observed that a 1% increase in the SP_CYCLE variable reduces the DIV_T_FED_CYCLE variable by 0.365189%. In turn, based on Table 6, a

1% increase in the variable DIV_T_FED_CYCLE generates a reduction of 0.030781% in the variable TX_GDP_REAL. Therefore, the final effect of the budget constraint on economic growth is given by the product between (-0.365189) *(-0.030781), which equals 0.01124. In other words, when there is an increase in the primary surplus over the debt cycle, with an increase in the difference between primary revenues and expenditures as a proportion of GDP, then this tax behavior will ultimately reflect positively on economic growth. Thus, the indirect effect of the primary surplus cycle on economic growth, via the debt cycle, contributes to increased economic growth.

One can also evaluate the indirect effect of the (r-y) _CYCLE variable on the real GDP growth rate of the Brazilian economy by means of the public debt cycle variable. In this case, if a non-virtuous cycle occurs in which the real interest rate is higher than the economic growth rate, based on Table 5, a 1% increase in the (r - y) _CYCLE variable will also lead to an increase on the DIV_T_FED_CYCLE variable of 0.232876. Similarly, a 1% increase in the debt cycle, based on Table 6, generates a 0.030781% reduction on economic growth. Thus, the indirect effect of the (r - y) _CYCLE variable on economic growth corresponds to the product between (0.232876) *(-0.030781) which is equivalent to the reduction of the TX_Real_GDP variable by 0.0072.

Finally, it can be pointed out that the two simultaneous equation systems using the real credit cycle, estimated via GMM, which are presented in the Appendix, show similar results to the systems presented in subitem 4.1, which uses the credit cycle in nominal values.

4.2 - Optimum level of public debt

What is the level of total federal and Bacen debt as a proportion of GDP that maximizes the real productivity of the Brazilian economy in the period under consideration? To answer this question, we test equation 6 presented below:

$$Productivityet = \alpha o + \alpha i * Productivityet\text{-}i + \lambda 2 * \mathrm{DIV\_T\_FEDt} + \lambda 3 * (DIV\_T\_FED)^{2t+Wt}$$

The variables Productivity and DIV_T_FED are not stationary, as they do not reject the null hypothesis of unit root, according to Table 2. Thus, it is initially necessary to perform cointegration tests. Considering that, based on the VAR estimation, an optimal number of 3 lags was obtained, according to the selection criteria LR, FPE, AIC, SC and HQ, the trace and maximum eigenvalue tests were performed, which indicate that there is an integration equation at the 5% significance level that rejects the hypothesis of 0 cointegrations and accepts the hypothesis of 1 Cointegration relation, considering the two variables of interest, productivity of DIV_T_FED (Johansen, 1988).

Since both variables cointegrate and therefore the regressions are not spurious, we use the FMOLS (Table 7) and CCR (Table 8) methods presented in section 2 regarding methodological aspects to calculate the optimal public debt level (%GDP).

Table 7: Dependent Variable (Productivity) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
	Method: Fully Modified Least Squares (FMOLS)			
Productivity (-1)	0,723369	0,044341	16,31377	< 0,0001
Productivity (-2)	-0,132728	0,053987	-2,458497	0,0146
Productivity (-3)	0,304046	0,053810	5,650395	0,0000
DIV_T_FED	0,058685	0,023940	2,451350	0,0148
$(DIV\_T\_FED)^2$	-0,001097	0,000433	-2,534606	0,0118
Constant	0,071183	0,240907	0,295481	0,7678
@trend	-0,001295	0,000950	-1,362538	0,1741
$(@Trend)^2$	2,00E-05	6,15E-06	3,247970	0,0013

Note: Adjusted R2 = 0.992

Deriving equation 6 with respect to the variable of interest DIV_T_FED and equaling zero, we obtain the first-order condition (CPO) for finding the value of public debt (%GDP) that maximizes the real productivity of the economy, as expressed below:

$$0.058685 - 2*(0.001097)*DIV\_T\_FED = 0, \rightarrow DIV\_T\_FED)^* = 26.748$$

Based on the empirical results presented in Table 7, it can be observed that the estimated coefficients of the variables DIV_T_FED and $(DIV\_T\_FED)^2$ are statistically different from zero and have positive and negative signs respectively, indicating that there is a non-linear relationship between productivity and public debt (%GDP). Under these conditions, a concave curve is obtained, in the form of an inverted U, in which, as the debt increases from low levels of indebtedness onwards, productivity increases in a decreasing manner until it reaches a maximum point, which determines the optimal level of public debt (%GDP) that maximizes the economy's productivity. For debt levels above the optimal level of 26.75% of GDP, successive increases in public debt (%GDP) lead to a reduction in the economy's productivity. Hence the importance of the policymaker having to be concerned not only with the sustainability of the debt-GDP ratio in the medium and long run, but also with a search for improving the economy's productivity. As the economy tends to approach the optimal level of public debt (%GDP), this means that productivity tends to increase and, therefore, there tends to be an increase in the real product which, in turn, given the level of debt, the debt-GDP ratio tends to reduce, creating a virtuous circle in the economy.

Table 8 estimates the same equation 6, but with the CCR Cointegration method. The empirical results are like those presented in Table 7. In the same way, one can deduce the first order conditions, which reveal the optimal level of public debt at 26.48% of GDP, a result very close to the one presented in Table 7, amounting to 26.75% of GDP. It is also worth noting that tax policy makers may use control variables on the right-hand side of the equation to refine the estimates of the optimal public debt level.

Table 8: Dependent Variable (Productivity) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
	Method: Canonical Cointegrating Regression (CCR)			
Productivity (-1)	1,227120	0,269013	4,561573	< 0,0001
Productivity (-2)	-1,435404	0,727636	-1,972695	0,0495
Productivity (-3)	1,111249	0,588938	1,886871	0,0602
DIV_T_FED	0,051529	0,024481	2,104878	0,0362
$(DIV\_T\_FED)^2$	-0,000973	0,000443	-2,193486	0,0291
Constant	0,111234	0,238721	0,465959	0,6416
@trend	-0,001104	0,000968	-1,140432	0,2551
$(@Trend)^2$	1,83E-05	6,29E-06	2,912626	0,0039

Note: Adjusted R2 = 0.983

4. A brief discussion of the empirical results

One of the contributions of this monograph concerns the creation of tax cycle variables associated with the primary surplus cycle, as a proportion of GDP, and with the total debt cycle of the Federal Government and BACEN also as a proportion of GDP. Such variables are deduced from the trend generated by the Hodrick-Prescott HP filter, a procedure usual in the literature as, for example, in the "construction" of the output gap, as well as of the unemployment cycle.

Tax cycles are more intuitive for understanding the effects of these cycles on the rate of economic growth. For example, on the one hand, a tax policy rule is used in which the primary surplus (%GDP) should respond positively to the expansion of public debt, that is, it should respond to public debt values above the HP filter. In this case, it is considered that the debt expansion should be responded with an expansion of the primary surplus cycle (%GDP), that is, with an increase in the difference between total revenues and public expenditures, without accounting for interests (%GDP). On the other hand, inspired by the government's budget constraint, the effects of the primary surplus cycle (%GDP) and of the cycle of the difference between the real interest rate and the real output growth rate on the public debt cycle are evaluated. Unless I am mistaken, such approaches are new to the literature or are little used.

Other little used approaches refer to the evaluation of the indirect effects of these variables, via a system of simultaneous equations. For example, the initial effect of the tax rule in the first equation, where the tax surplus cycle responds to changes in the public debt cycle and that, in turn, this same response of the tax rule indirectly affects a second equation, which, in this case, is the equation of the economy's growth rate.

It is also worth noting that the real and nominal credit cycles temporally precede the economic growth rate according to the Granger causality test. Furthermore, the empirical results show that there is a directly proportional relation between the nominal and real credit cycles, with a time lag of 8 months, and the economic growth rate. These results are also very interesting because they reveal some aspects that draw attention, among which are:

i) The empirical results show that not only does the real credit cycle affect economic growth, but that the nominal credit cycle also positively affects economic growth. Thus, we find that there is empirical evidence that the real GDP growth rate is affected by both real and nominal factors;
ii) The results are in line with the literature on the nexus between financial development and economic growth by showing that credit cycles affect economic growth;
iii) Since the credit cycle temporally precedes the evolution of the economic growth rate with a lag of 8 months, in this case, policy makers can better

predict the evolution of the economy's growth rate by controlling for other determinants of growth;

iv) The empirical results also suggest that economic growth is affected by short-term phenomena that are more associated with business cycles, which is not the object of study in this paper, since business cycles are also associated with the credit cycle;

v) Finally, if the evolution of the credit balance in the economy depends on monetary policy in the sense that money creation affects credit, then the results also suggest that economic growth is determined by real factors as well as monetary factors.

Finally, one other contribution that may be very relevant for the formulators of economic policy is the estimation of the public debt level as a proportion of the GDP, which maximizes the productivity of the Brazilian economy in the evaluated period. In this way, this paper is not only highlighting the relevant discussion on public debt sustainability, which in fact is the most important issue for tax policy, but it is emphasizing that one can go beyond. In other words, once, by hypothesis, the public debt is on a sustainable path, a tax adjustment can be sought that aims at reaching its optimal level, and nothing prevents this goal from also having tolerance deviations above or below the optimal level. In this sense, it is possible to consider implementing a tax policy rule for the public debt, considering that the conditions for the sustainability of the debt are already assured.

5. Final considerations

This monograph ultimately evaluates the direct and indirect effects of tax cycles on the growth rate of the Brazilian economy, with monthly data from 1996:03 to 2020:06, in addition to the effects of nominal and real credit cycles on economic growth. If we implicitly work with endogenous growth models, and using systems of simultaneous equations estimated via GMM, we can find some interesting empirical results and even some contributions to the literature.

The empirical results show that the credit cycle, whether real or nominal, temporally precedes the growth rate of the economy, based on Granger causality tests. In other words, the credit cycle affects economic growth unidirectionally. Moreover, there is empirical evidence based on the simultaneous equation systems via GMM that the real and nominal credit cycle positively affect the real GDP growth rate with a lag of 8 months.

The first system of simultaneous equations via GMM shows two equations, in which the first refers to a tax policy rule and the second to an economic growth equation. In this context we observe that the tax rule, in which the primary surplus (%GDP) cycle responds positively to the public debt (%GDP) cycle, is confirmed based on the empirical models. In other words, if there is an expansion of debt (%GDP) the primary surplus as a proportion of GDP also increases. In addition, there is an indirect effect of the response of the primary surplus (%GDP) expansion on economic growth, which shows that increases in the proportion of tax surpluses positively affect the growth rate of real GDP. Similarly, there is also empirical evidence of a direct and positive effect of the expansion of the primary surplus on economic growth, generating a virtuous circle.

On the other hand, the second system of simultaneous equations via GMM shows two equations, in which the first refers to an adjustment of the government budget constraint and the second to an economic growth equation. The first equation shows empirical evidence that, *ceteris paribus*, if on the one hand the cycle of the primary surplus as a proportion of GDP reduces the expansion of public debt, on the other hand the increase in the cycle of the difference between the real interest rate and the growth rate of the economy contributes to increase the debt. Thus, if the real interest rate is higher than the rate of economic growth, the public debt, as a proportion of the GDP, tends to increase. Therefore, two effects with opposite signs are observed, which can result in a total net positive or negative effect. Based on the elasticities, there is empirical evidence that, in module, the elasticity of the primary surplus cycle is higher than that of the cycle of the difference between real interest rates and economic growth. In other words, the net effect of the elasticities is negative, which helps to avoid an explosive public debt trajectory.

Regarding the economic growth equation, we point out that there is empirical evidence that an increase in the expansion of public debt (%GDP) has a negative effect on economic growth, that is, the direct effect of an expansionary debt cycle results in lower growth and, consequently, lower welfare. As for the indirect effects, via public debt (%GDP), if on the one hand an increase in the primary surplus expansion cycle positively affects economic growth, on the other hand, an increase in the difference between the real interest rate and the economy's growth rate reduces economic growth.

Finally, based on Cointegration models, it became possible to determine the level of the total public debt of the Federal Government and BACEN (%GDP) that maximizes the real productivity of the Brazilian economy in the period analyzed. In this case, the empirical results, based on the FMOLS and CCR models, show that the optimal level of public debt in the two models is 26.75% of GDP and 26.48% of GDP, respectively. For comparison, in December 2019, the debt percentage was 42.41% of GDP and in June 2020, it was equivalent to 43.63% of GDP.

Considering the above, these results reveal a "red alert" regarding the trajectory of tax policy, which is being aggravated by the restrictive measures in the economy, resulting from the exogenous shock due to Covid-19.

It should also be noted that there are some limitations in this study, such as, for example, the use of the number of hours worked in industry, which is used as a *proxy* for the labor production factor, because it does not reflect the other sectors of the economy. In this case, one of the possibilities would be to use the economically active population (EAP), but this series is not available monthly. On the other hand, a linear interpolation of the variations in the quarterly gross fixed capital formation at constant values is used.

References

AGHION, P; HOWITT, P. (1998). Endogenous growth theory. Cambridge, MA: MIT Press, 1998.

Barajas, Adolfo; Chami, Ralph e Yousefi, Seyed Reza (2013). The Finance and Growth Nexus Re-Examined: Do All Countries Benefit Equally? IMF Working Paper, WP/13/130, May.

Born, H. (1998). The behavior of U.S. public debt and deficits, The Quarterly Journal of Economics, volume 113, issue 3, Cambridge, MIT.

Buiter, W. (1985). A Guide to Public Sector and Deficits, Economic Policy, Nº 1, pp. 13 - 79.

Campos, E. Lima e Cysne, R. Penha (2019). An alert on the recent fall of the tax reaction in Brazil. Brazilian Journal of Political Economy, vol. 39, nº 2 (155), pp. 253-262, April-June.

Cragg JG, Donald SG. (1993). Testing Identifiability and Specification in Instrumental Variables Models. Econometric Theory 9: pp. 222-240.

Fisher, I. (1930). The theory of interest, New York, Macmillan.

Granger, C. W. J. (1969). Investigating Causal Relations by Econometric Models and Cross-Spectral Methods, Econometrics, 37, pp. 424-438.

Hamilton, James D. (1994). Time Series Analysis. Princeton University Press, Princeton, New Jersey.

Hsiao, C. (1997a). Statistical properties of the two-stage least squares estimator under co-integration. The Review of Economic Studies, volume 64, issue 3, N.Y.

Hsiao, C. (1997b). Co-integration and dynamic simultaneous equations models, Econometrics, vol. 65, Nº 3, N.Y., The Econometric Society.

Hodrick, R. J. e Prescott, E. C. (1997). Postwar U.S. Business Cycles: An Empirical Investigation. Journal of Money, Credit and Banking, Vol. 29, Nº 1, Feb., pp. 1-16.

Johnston, J e Dinardo, J. (1997). Econometric Methods, New York, McGrall-Hill.

Johansen, Søren (1988). Statistical Analysis of Cointegration Vectors. Journal of Economic Dynamic and Control. Vol. 12, Issues 2-3, June-September, Pages 231-254.

Johansen, Søren (1991). "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models". Econometrics. Vol. 59 (6), November: pp. 1551-1580.

Johansen, Søren (1995). Likelihood-based Inference in Cointegrated Vector Autoregressive Models. Oxford. University Press.

Longstaff, Francis A. (2010). The subprime credit crisis and contagion in financial markets. Journal of Financial Economics, Issue 3, Vol. 97, September, pp. 436-450

Luporini, V. (2006). Concepts of tax sustainability, TD No. 189, Universidade Federal Fluminense, May.

Mendonça, Mário J. Cardoso, Santos, Cláudio Hamilton M. and Sachsida, Adolfo (2009). Revisiting the Tax Reaction Function in Post-Real Brazil: A Regime Change Approach. Economic Studues, São Paulo, v. 39, n. 4, p. 873-894, October - December.

Moreira, Tito B. S. (2011). Brazil: an empirical study on tax policy transmission. CEPAL Review Nº 103, April, pp. 187 - 2005.

Moreira, Tito B. S.; Benjamin Miranda Tabak ; Mendonça, Mario Jorge C. ; Sachsida , Adolfo(2016) . An Evaluation of the Non-Neutrality of Money. Plos One, v. 11, p. e0145710, 2016.

Newey W, West K. (1987a). Hypothesis testing with efficient method of moments estimation. International Economics Review, 28, pp. 777-787.

Newey W, West K. (1987b). A simple positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrics, 55, pp. 703-708.

Nyasha, S. e Odhiambo, N. (2018). Financial Development and Economic Growth Nexus: Revisionist Approach. Economic Notes, Review of Banking, Finance and Monetary Economics, Vol. 47, No 1, pp. 223 - 229.

Park, Joon Y. (1992). Canonical cointegrating regression. Econometrics 60, pp. 119-143.

Pereira, Paulo T., Afonso, A., Arcanjo, M. and Santos, Jose C. G. (2009). Economia e Finanças Públicas, 3ª Edição, Escolar Editora, pp. 520 - 542.

Phillips, Peter C.B., Hansen, Bruce E. (1990). Statistical interference in instrumental variable regression with I (Aghion et al., 1999) process. Rev. Econ. Stud. 57, pp. 99-125.

Phillips, Peter C.B., Hansen, Bruce E. (1995). Fully modified least squares and vector autoregression. Econometrics 63 (5), pp. 1023-1078.

Rebelo, Sergio T. (1991). Long-Run Policy Analysis and Long-Run Growth, Journal of Political Economy, 99(3), 500-521.

Romer, Paul M. (1987). Crazy Explanations for the Productivity Slowdown. In NBER, 1994, Macroeconomics Annual. Edited by Stanley Fisher. Cambridge, Mass. MIT Press.

ROMER, Paul M. (1990). Endogenous Technological change. The journal of Political Economy, The University of Chicago Press, v. 98, p.S71- S102.

Romer, Paul M. (1994). The Origins of Endogenous Growth, The Journal of Economic Perspectives, Vol. 8,Nº1 (Winter), pp. 3-22.

Saikkonen, Pentti (1992). "Estimation and Testing of Cointegrated Systems by an Autoregressive Approximation," Econometric Theory, 8, 1-27.

Sargent, T. J. e Wallace N. (1981). Some unpleasant monetarist arithmetic, Federal Reserve Bank of Minneapolis Quarterly Review, volume 5, No 3.

Spaventa, L. (1987). The Growth of Public Debt, IMF Staff Papers, No 34, Vol 2, pp. 374 - 399.

Stock JH, Yogo M. (2005). Testing for Weak Instruments in Linear IV Regression. In Identification and Inference for Econometric Models: Essays in Honor of Thomas Rothenberg, ed. Andrews DW, Stock JH, pp. 80-108. Cambridge University Press.

Stock, James H., Watson, Mark (1993). A simple estimation of cointegrating vectors in higher order integrated system. Econometrics 61, 783-820, (1993).

Tabak, Benjamin M.; Moreira, Tito B. S.; Fazio, D. M.; Cavalcanti, A. L. C.; Cunha, G. H. M. (2016). Monetary Expansion and the Banking Lending Channel. Plos One, v. 11.

Tiryaki, G. F.; Gavazza, I. O.; Andrade, C. M.; Mota, A. L. (2017). Credit cycles, defaults and the economic fluctuations in Brazil. Journal of Contemporary Economics, Vol. 21, No. 1, pp. 1-33, Jan-Apr.

Appendix

Table 3A: Dependent Variable (SP_CYCLE) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
Constant	-0,011837	0,007184	-1,647808	0,1000
SP_CYCLE (-1)	0,983706	0,023247	42,31601	< 0,0001
DIV_T_FED_CYCLE	0,023728	0,004986	4,758529	< 0,0001
Statistics				
Statistic J = 27.85	Statistical Prob. J = 0.112		Adjusted R2 = 0.707	
	Critical value Stock-Yogo (5%) = 20.60			
	Cragg-Donald F-statistic = 38.64			
Instruments				
Sp, sp(-1to-5), div_est_m(-1to-2), div_t_fed(-1to-2), credito_trab hours(-1to-2), @trend, div_int_f(-1to-3), div_t_fed_cycle(-1to-5).				

Note 1: SP CYCLE = Primary Surplus Cycle (%GDP); DIV_T_FED_CYCLE = Total Federal and Central Bank Debt - Net (%GDP)

Table 4A: Dependent Variable (TX_GDP_REAL) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
Constant	0.051842	0.013991	3.705506	0.0002
TX_REAL_GDP (-1)	0.778244	0.057390	13.56065	< 0.0001
TX_FBCF	0.009664	0.003019	3.200763	0.0014
TX_HORAS_TRAB	0.023546	0.011185	2.105156	0.0357
SP_CYCLE	0.042956	0.021126	2.033339	0.0425
ACTUAL_CREDIT _CYCLE (-8)	0.005916	0.002261	2.616625	0.0091

Note 1: Adjusted R2 = 0.464; Note 2: TX Real GDP = Real GDP growth rate; TX_FBCF= Real variation of gross fixed capital formation (%); TX WORKING HOURS=Variation of industry working hours; SP CYCLE = Primary surplus cycle (%GDP); CREDIT_REAL_CYCLE=Cycle of real credit operations.

Table 5A: Dependent Variable (DIV_T_FED_CYCLE) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
Constant	0.009678	0.026860	0.360308	0.7188
DIV_T_FED_CYCLE (-1)	0.857071	0.015361	55.79592	< 0.0001
SP_CYCLE	-0.334893	0.055840	-5.997339	< 0.0001
(r - y) _CYCLE	0.204952	0.069145	2.964082	0.0032
Statistics				
Statistic J = 16.335	Statistical Prob. J = 0.176		Adjusted R2 = 0.766	
	Critical value Stock-Yogo (5%) = 18.73			
	Cragg-Donald F-statistic = 25.877			
Instruments				
div_t_fed(-1to-3); sp(-1to-3); r_y(-1to-3); @trend; @trend*div_t_fed, r_y_ciclo(-1); div_t_fed_ciclo (-2); credito_ciclo; tx_pib_real				

Note 1: DIV_T_FED_CYCLE = Total Federal Government and Bacen Debt - Net (%GDP); SP CYCLE = Primary Surplus Cycle (%GDP); (r - y) _CYCLE = Cycle of the difference between real interest rate and real GDP growth rate.

Table 6A: Dependent Variable (TX_Real_GDP) - Period 1996:03 to 2020:06

Variables	Coefficient	Standard Deviation	t-statistic	Probability
Constant	0.100456	0.027206	3.692354	0.0002
TX_Real_GDP (-1)	0.504155	0.057219	8.811020	< 0.0001
TX_FBCF	0.019062	0.004493	4.243044	< 0.0001
TX_TRAB_HOURS (-1)	0.157288	0.039389	3.993203	0.0001
ACTUAL_CREDIT_CYCLE (-8)	0.008186	0.004050	2.021266	0.0437
DIV_T_FED_CYCLE	-0.057422	0.019656	-2.921414	0.0036

Note 1: Adjusted R2 = 0.166; Note 2: TX Real GDP = Real GDP growth rate; TX_FBCF= Real change in gross fixed capital formation (%); TX HOURS WORKED=Change in industry hours worked; CREDIT_REAL_CYCLE=Cycle of real credit operations; DIV_T_FED_CYCLE = Total Federal Government and Bacen Debt - Net (%GDP).