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PUBLIC DEBT AND ECONOMIC GROWTH IN BRAZIL*

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Abstract

This paper provides new evidence on the relationship between public debt and economic growth in Brazil. We used Granger causality tests, in multivariate and bivariate analyses using respectively VEC and ARDL methodologies, and monthly data over the period 1998:1-2019:11. We find that: i) debt-to-GDP and GDP growth rate have a bi-directional Granger causality relationship; ii) debt can improve growth in the short run and becomes harmful in the long run; iii) GDP growth always reduces debt, both in the short and long run; iv) the dynamic between debt and growth in the long run is influenced by the inflation rate, exchange rate and the Emerging Markets Bond Index Plus (Embi+).

Keywords: Granger causality; Vector Autoregressive; Autoregressive Distributed Lag; government debt; economic growth; Brazil.
JEL Codes: C32; C22; H63.

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1. INTRODUCTION

Increasing government indebtedness worldwide has become a problem since the Global Financial Crisis (GFC) of 2008-2009, raising concerns related to the vulnerability of countries. In order to address this problem many researchers and policy makers argue that governments should enforce fiscal consolidations, to decrease public debt, explaining that this would result in economic growth. On the other hand, there are economists who advocate that fiscal consolidations could result in increasing debt-to-GDP ratios, moreover, this reduction in government size would affect the growth rate of the Gross Domestic Product (GDP).

Brazil, as many other countries, faces an increasing government debt burden. Between 1998 and 2020 Gross General Government Debt increased from 40% of GDP to 89%. In addition, the economic recession worsened the problem, in 2019, real GDP grew only 1.1% and in 2020 it is expected to have a significant decrease. During the period under analysis (January 1998 until November 2019) the Brazilian government had five different presidents and alternated between moments of fiscal expansions and consolidations. Furthermore, Brazil faced hyperinflation in the past, which also makes the Brazilian Central Bank strongly conservative in relation to the interest rate, indirectly implying higher costs to government debt, by increasing its debt service.

Since 1998, after the Plano Real, Brazil has followed an inflation targeting regime that includes floating exchange rate and primary surplus targets in addition to inflation targets. Therefore, macroeconomic variables such as interest rate, inflation rate, exchange rate and primary surplus may be correlated with the pattern followed by the GDP growth rate and the debt-to-GDP ratio.

In December 2016, the Brazilian Congress approved a Constitution Amendment that created a ceiling for public spending[1] that institute a new fiscal regime, since them government has tried to implement some austerity measures, aiming to implement a fiscal consolidation and control the public debt trajectory. However, the government has not

[1] Constitutional Amendment n. 95.

succeeded in reducing government debt, moreover it was not able to overcome the recession. The lack of consensus about the implications of public debt harm decision-making. The actual situation raises the question: “What is the relationship between public debt and economic growth in Brazil?”

On the one hand, public debt can adversely affect economic progress through several channels, for example, high long-term interest rates, high taxation, greater uncertainty, vulnerability to crises, etc., especially when their level exceeds a certain threshold. On the other hand, a lower level of public debt allows fiscal policy to play a more stabilizing role during economic crises and dampens, or at least does not exacerbate, economic cycles.

This paper aims to empirically investigate the dynamic between economic growth and public debt in Brazil in the period after the *Plano Real*. It will also include other variables that are related to both public debt and economic growth. The assessment is conducted through a Vector Autoregressive (VAR) model and an Autoregressive Distributed Lag (ARDL) model, applying Granger causality tests. The data frequency is monthly, over the period of January 1998 and November 2019.

Since there are just a few empirical studies applied to Brazil, this paper contributes to the literature providing empirical results using Brazilian data. Moreover, we are not aware of any other study that has analysed the interaction between debt and growth considering the interrelationships with the other variables used in this composition.

The most relevant findings of our study are summarized as follows: Debt-to-GDP ratio and GDP growth rate have a bi-directional Granger causality relationship. Debt can improve growth in the short run and become harmful the in long run. Also, GDP growth rate always reduces debt ratio, both in the short and long run. The dynamic between debt and growth in the long run is influenced by inflation rate, exchange rate and the Emerging Markets Bond Index Plus (Embi+), these variables are positively Granger caused by changes in debt-to-GDP ratio and negatively Granger cause GDP growth rate, while GDP growth rate negatively Granger causes Embi+, that in turn positively Granger causes debt.

Therefore, the negative impact of debt on growth is also indirect by changes on inflation rate, exchange rate and Embi+, while the reduction on debt ratio provoked by GDP growth rate is also indirectly, by the reduction on Embi+.

This remaining of this paper is structured as follows: section two reviews the literature related to the topic; section three presents the methodology; section four presents the data and the empirical estimation results; and section five is the conclusion.

2. Literature Review

The literature associated with the relation between public debt and economic growth is well developed, despite the lack of empirical studies applied to Brazil, for the best of our knowledge. In addition, one should consider that empirical results have mixed conclusions, divided between those which concluded that debt improves growth, therefore generates a reduction on debt-to-GDP ratio, or those which advocate that debt hurts growth. These different conclusions strengthen the idea that results are country and time specific.

Next, the empirical literature review will be divided into three different perspectives. Firstly, international studies that use Granger causality. Secondly, international studies that employed different methodologies. Lastly, studies applied to Brazil.

2.1. Literature using Granger Causality Tests

Afonso and Jalles (2014) studied the two-way causality between government spending, revenues and growth. They constructed different models applying OLS and GMM estimators and Granger causality test for one hundred fifty-five developed and developing countries, for the period of 1970 to 2010. They found weak evidence of causality from per capita GDP to expenditures. However, they have found stronger evidence supporting the reverse causality, in the short and long run. Moreover, they applied the same methodology only for OECD sub-sample countries and found stronger evidence for Granger causality from government spending to GDP in the short-run, although no significant long-run effect. The reverse relationship still holds for OECD sub-sample, in the short and long-run.[2]

Additionally, Gómez-Puig and Sosvilla-Rivero (2015) also used Granger causality for eleven EMU countries over 1980 and 2013, analysing the bi-directional relationship between debt and growth. Their study considered cross-country heterogeneity by including central and peripheral countries. Before allowing for endogenous breaks they found evidence of a positive effect of the change in debt on growth and vice versa. After allowing breaks, they found a "diabolic loop"[3] between low growth and high debt for Spain, Belgium, Greece, Italy and the Netherlands. However, they found a positive relationship from debt to growth for Austria, Finland and France. Their results somehow explain why empirical studies are not always clear and can show ambiguous conclusions, depending on the period analysed and the country considered. According to the authors, causality should be examined from a dynamic and country specific point of view.

Lai, Trang, and Kuo (2015) explored the casual relationship between government debt, GDP and inflation in France, using annual data between 1980 and 2010. After performing unit root tests, they concluded that there is no long run co-integration between these variables. Therefore, they implemented VAR models and Granger causality test to check if there is short run relationship. They found a unidirectional causality from debt to GDP and from inflation to GDP, either a bidirectional relation between inflation and government debt.

Butts (2009) studied the relationship between economic growth and short-term external debt of twenty-seven Latin American and Caribbean countries, using data from

[2] See Afonso and Alves(2016) for a complementary analyses of the possibility of Wagner´s law.

[3] The "diabolic cycle" or the link between sovereign risk and bank credit risk was one of the characteristics of the euro crisis, mainly in Spain, Greece, Ireland, Italy and Portugal. The deterioration in the quality of sovereign credit reduced the market value of banks 'participation in public debt, which in turn reduced banks' perception of solvency and reduced loans. On the other hand, the initial crisis in the banking sector, due to portfolio problems, forced governments to bail out the banking sector. The credit crisis also led to a reduction in tax revenue, thus contributing to the weakening of government solvency. (Brunnermeier et al. 2016).

1970 to 2003. He concluded by the existence of Granger causality from economic growth to short term external debt in thirteen countries.

2.2. Literature using different methodologies

Afonso and Jalles (2013) investigated the effect of government debt ratio on economic growth using a panel of one hundred fifty-five countries, over the period of 1970 and 2008. They concluded that government debt has a negative effect on growth. Moreover, they concluded that the longer the average maturity of government debt the higher the growth rate for OECD countries in the group. They also found a threshold of 59% of GDP to European countries and 79% for emerging countries.

Additionally, Afonso and Alves (2015) also used panel data techniques to analyse the effect of government debt on real per capita GDP for fourteen European countries, during the period 1970-2012. They found that debt has a negative effect on growth both in the short and in long run. Furthermore, debt service had a much more negative effect than debt on economic performance, they also found a debt threshold around 75%.

Cherif and Hasanov (2018) using a VAR model with debt feedback, analysed the impact of macroeconomic shocks on US public debt dynamics, with data from 1947 to 2015. They concluded that austerity shocks could make debt decline at a cost of lower growth, moreover, debt converged to its pre-shock path, suggesting that austerity is self-defeating. On the other hand, growth shocks could substantially reduce debt, with none of the pain associated with austerity.

Focusing on the inverted U-shape relationship between debt and growth, Reinhart and Rogoff (2010), used a data set of two hundred years for forty-four countries and found a threshold for public debt of 90%, this value is the same for advanced and for emerging markets.

However, Égert (2015) analysed a variant of Reinhart and Rogoff (2010) dataset, he employed nonlinear threshold models and concluded that this negative nonlinear relationship is not ensured, moreover, it changes across samples and different model specifications.

2.3. Studies Applied to Brazil

Gadelha (2011) investigated the relationship between GDP, public expenditure, public revenues, and government debt. He applied Granger causality in a bivariate and multivariate framework, using data over January 1997 and June 2009. Results indicated a bidirectional causality between government revenues and expenditures, concluding that there is fiscal synchronization in Brazil.

Rodrigues and Teixeira (2013) analysed the relationship between public spending and debt using Granger causality, over the period of 1950 and 2000. They concluded that public spending did not cause GDP growth, it is mostly a consequence of economic growth, supporting Wagner´s Law.

Gadelha and Divino (2008) investigated whether there is monetary dominance or fiscal dominance[4] in Brazil. They applied models of multivariate and bivariate Granger causality, with monthly data over January 1995 and December 2005. The variables analysed were interest rate, debt to GDP ratio, primary surplus to GDP ratio, real exchange rate and risk premium. They concluded that Brazil was under monetary dominance and both interest rate and primary surplus unidirectionally Granger caused the debt-to-GDP ratio.

The studies applied to Brazil did not focused on the relationship between public debt and growth, most of them emphasised the relation between government expenditures and revenues or government expenditures and growth .Therefore, this study differentiates from empirical literature applied to Brazil by analysing the relation between government debt and GDP. Moreover, aiming to find a more complete relationship we included the variables used by Gadelha and Divino (2008), that are interest rate, inflation rate, exchange rate, primary surplus and Embi+ , since these variables are also important for the changes in public debt and economic growth.

[4] In a monetary dominance the fiscal authority generates primary surplus that is enough to keep the debt-to-GDP ratio stable, therefore the monetary authority can exercise their role. On the other hand, when there is a fiscal dominance, the monetary authority needs to allow prices adjust to ensure that the current value of the outstanding government debt is equal to the actual real value of the future primary surplus. See, for instance, Afonso (2008).

3. DATA

The dataset is built with several series: the Gross Domestic General Government Debt[5] (ratio of GDP), which is called debt for simplicity throughout the text, is represented by *D*; *Y* represents the GDP[6] (growth rate), called growth throughout the text; Over Selic[7] interest rate is noted by *I*; Nominal Exchange[8] rate direct quotation (R$/US$), is noted by *E; R* represents the inflation rate[9] (percentage change; *S* represents the primary budget[10] (ratio of GDP); and Embi+[11]. We use monthly data, starting in January 1998 and ending in November 2019.

The data processing started by treating an outlier present in the series of primary surplus in September 2010, by excluding values that represented atypical revenues and expenditures, generating an elimination of 31.9 billion of the primary result for the specific period [12].

Then series of GDP and debt, which showed some seasonal component, were seasonally adjusted by the methodology Census X-13. The seasonal adjusted series and the one of primary surplus were converted to real terms, deflated by consumer index price, which considered January 1998 as the base value. Thereafter, the series were converted to annual values, by adding up the twelve rolling window values, thus they could be

[5] Source: BCB, 4502 series. Available at: https://www3.bcb.gov.br/sgspub/consultarvalores/consultarValoresSeries.do?method=consultarValores.

[6] Source: BCB, 4380 series. Available at: https://www3.bcb.gov.br/sgspub/consultarvalores/consultarValoresSeries.do?method=consultarValores.

[7] *Overnight* Selic. Source: BCB. Available at: http://www.ipeadata.gov.br.

[8] Exchange Rate - R$ / US$ - comercial - average purchase. Fonte: BCB. Disponível em: http://www.ipeadata.gov.br.

[9] Consumer Index Price (IPCA). Source: Brazilian Institute of Geography and Statistic (IBGE). Available at: : http://www.ipeadata.gov.br.

[10] Source: Brazilian National Treasury (STN). Available at: http://www.tesourotransparente.gov.br/ckan/dataset/resultado-do-tesouro-nacional.

[11] Source: JP Morgan, used as a proxy for risk. Available at: http://www.ipeadata.gov.br/Default.aspx.

[12] Regarding expenses, R$ 42.9 billion related to Petrobrás capitalization operations were excluded and, on the revenue side, R$ 74.8 billion related to petroleum transfer assignment titles by Petrobrás were also excluded. The treatment followed (Gadelha and Divino 2013).

analysed in the same bases as public debt, which is a stock variable. Lastly, the values were converted in ratios of GDP.

4. METHODOLOGY

4.1. Unit Root Tests

The Augmented Dickey Fuller (*ADF*) and Phillips-Perron (*PP*) unit root tests are the most widely applied, however they can present problems related to power and size in finite samples. Moreover, Maddala and Kim (2004) explain that structural change does affect inference on unit roots and on cointegration, being important to allow for possible breaks at the estimation stage. Therefore, the study of stationarity is going to be conducted by a new generation of tests that address these related problems. Firstly, it is applied the modified Dickey-Fuller (ADF^{GLS}) test, suggested by Elliot, Rothenberg, and Stock (1996), then the Phillips-Perron (MZ_{α}^{GLS}) suggested by Ng and Perron (2001).

Elliot, Rothenberg, and Stock (1996) proposed the use of generalized least squares (GLS) estimators instead of ordinary least squares (OLS), to purge deterministic terms presented in the regression, since OLS estimators are inefficient in the presence of heteroscedasticity. Moreover, Ng and Perron (2001) explained that distortions of size in the presence of negative moving averages, related to outliers, implicate an incorrect selection of lags by the Akaike (AIC) and Schwarz (SIC) criteria. They also proposed the use of GLS estimators in place of OLS, for the traditional PP test. Therefore, this study applies both tests, making use of the modified Akaike Criteria (MAIC) for lag selection.

However, considering economic changes during the period, we may account for structural breaks. Furthermore, the modified ADF^{GLS} and MZ_{α}^{GLS} tests still have low power in the presence of breaks. Therefore, we applied two tests with endogenous breaks. The first one is the test proposed by Saikkonen and Lütkepohl (2002), hereinafter referred to as SL. The SL test considers that the change can occur over some period, and using a level change function ($f(\theta)'\gamma$) it is possible to have a smooth transition function, which is added to the deterministic term. The general model is expressed in the following equation:

$$y_{t,} = \mu_{0,} + \mu_1 t + f(\theta)' \gamma + v_t \qquad (1)$$

where y_t is the data series, μ_0 is the intercept, μ_1 is the deterministic trend coefficient; θ and γ are unknown parameters, v_t are residuals generated by an autoregressive process, which may have a unit root. There are three possible changing functions for $f(\theta)'\gamma$: shift dummy, exponential shift and rational shift. In this study it is going to be applied the last one, rational shift, which represents a rational function in the lag operator applied to a shift dummy. In this test, deterministic trends are estimated by GLS, then they are subtracted from the original series, generating a new series. Then, an ADF test is applied for the adjusted series. Critical values were tabulated by Lanne, Lütkepohl, and Saikkonen (2002).

The second test implemented is the one proposed by Vogelsang and Perron (1998), hereinafter referred to as VP, that also allows for endogenous breaks by innovation outlier, VP similarly to SL assumes the breaks to occur gradually. Two models are used to check the stationarity hypothesis: intercept break and, trend and intercept break, both in level and in first difference. The general model is expressed in the following equation:

$$y_t = \mu_0 + \mu_1 y_{t-1} + \mu_2 t + \beta_1 D_l + \beta_2 D_p + \beta_3 D_t + \sum_{i=1}^{j} p_t \Delta y_{t-i} + \varepsilon_t \qquad (2)$$

where y_t is the data series, μ_0 is the intercept, μ_2 is the deterministic trend coefficient; β_1, β_2 and β_3 are breaking parameters to be estimated; D_l , D_p and D_t are dummy variables for the intercept break, one time break, and trend break, respectively; p_t and μ_1 are unknown parameters, Δ is the first lag operator, j is the optimum lag length to be selected by the Akaike criterion; and ε_t are i.i.d. innovations.

4.2. Growth Equations

The first specification we used to understand the interaction between variables is an estimation of the linear relationship between *D* and *Y*, which follow Afonso and Alves (2015) and Afonso and Jalles(2013), using different variables from them in the vector X_t^j , as follows:

$$Y_t = \alpha_, + \beta_1 X_t^j + \beta_2 D_t + \varepsilon_t \ , t = 1, \dots, T \qquad (3)$$

where Y_t represents the growth rate of GDP; D_t is the debt-to-GDP ratio, and ε_t is the error term. α, β_1 and β_2 are unknown parameters to be estimated. The vector X_t^j includes variables that may impact on the relation between public debt and economic growth that were described in section 3.

Next, with the inclusion of D_t^2 in the equation (3), one can check if there is some non-linear relationship. Thus, in equation (4), if β_2 is positive and β_3 is negative, we have support for the inverted U-shape relationship, meaning that we can check if debt has a positive effect on growth until some threshold:

$$Y_t = \alpha + \beta_1 X_t^j + \beta_2 D_t + \beta_3 D_t^2 + \varepsilon_t \ , t = 1, \dots, T. \qquad (4)$$

where; D_t^2 is the debt-to-GDP ratio squared, β_3 is an unknown parameter to be estimated.

4.3. Multivariate Causality

The investigation of the causality among the variables begins by estimating a Vector Autorregressive (VAR) model, following Gadelha (2011) and Gadelha and Divino (2008). The VAR model considers all variables as endogenous, which is a common characteristic in economic series, in the sense that each variable can influence and be influenced by the behaviour of other variables. The VAR in its reduced form is represented as:

$$X_t = A_0 + A_1 X_{t-1} + A_2 X_{t-2} + \dots + A_p X_{t-p} + \xi_t \qquad (5)$$

where X_t is a vector of stationary variables, p is the number of lags, A_0 is a vector of intercepts, A_i is a matrix of coefficients, and ξ_t is a vector of residuals not autocorrelated and homoscedastic. The lag selection is made by the usual lag length criteria tests, selecting the one that is considered the best for most of the test criteria results.

If the series are not stationary it is necessary to perform cointegration tests to examine if there is a long run equilibrium relation among the series. This study will perform co-integration tests following the procedures suggested by Johansen and Juselius (1990), Johansen (2002) and Johansen, Mosconi, and Nielsen (2000). The test equation is defined as follow:

$$\Delta X_t = \mu + \pi X_{t-1} + \sum_{i=1}^{p-1} \pi_i \Delta X_{t-i} + \varepsilon_t \qquad (6)$$

where X_t is a column vector, μ is a vector of constants, π and π_i represent a matrix of coefficients, p is the lag order, and ε_t is the residual not autocorrelated and homoscedastic. The matrix π is the co-integrating matrix, which represents the long run information about the relationship among the variables. The number of values of π that are statistically different from zero, provides the number of co-integration equations. Johansen proposed the use of two statistics to test for co-integration:

$$\lambda_{trace}\,(\mathrm{r}) = -T \sum_{i=r+1}^{n} (1 - \widehat{\lambda_i}\,) \qquad (7)$$

$$\lambda_{trace}\,(\mathrm{r}, \mathrm{r}+1) = -T\, ln(1 - \widehat{\lambda_{r+1}}\,) \qquad (8)$$

where $\hat{\lambda}$ are the values estimated for the matrix π , and T is the number of observations. The test follows a recursive procedure, where the null hypothesis is that there are, at least, *r* cointegrated vectors.

Engle and Granger (1987) explain that if there is co-integration among the series, there must exist a long run relationship between them. Co-integration implies that deviations from equilibrium are stationary, with finite variance. If that is the case, one should estimate a Vector Error Correction Model (VEC) using the linear combination of the series corrected by their co-integrating vector. The VEC model is represented as follow:

$$\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + \cdots + \Gamma_{p-1} \Delta X_{t-p+1} + \Pi X_{t-1} + \varepsilon_t \qquad (9)$$

where, p is the number of lags already selected in the VAR model. $\Pi = \alpha\beta'$, where β is a matrix (p x r), whose columns contain the cointegration vectors, α is a matrix (p x r) with the adjustment coefficients. The linear combinations of $\beta' X_{t-1}$ represents the r number of cointegration equations.

In addition, impulse-response functions will be generated, which allow the individual behavior of the system variables to be evaluated, in response to some shock in another model variable. With this tool, the sensitivity of variables is analysed, through simulation, to specific shocks in each period. In this way, each coefficient shows the response of its variable in the model to a specific innovation, keeping all other innovations constant in all other periods. In other words, the impulse-response function describes the path and the time lags necessary for the variables to return to their original trajectory. The cumulative effect of a change of a unit in different innovations on a variable is given by the sum of the coefficients of the impulse-response functions (Enders 2005). The impulse-response functions show the long-term effects of the series when there is a certain exogenous shock in any of the model variables. Specifically, the impulse-response function technique is a procedure that allows to trace the effects of the standard deviation of a shock related to an innovation in the present and future values of endogenous variables. This fact is transmitted by a dynamic structure of an autoregressive vector.

In order to overcome the criticism of Cholesky's ordering in multivariate model, the generalized impulse-response function (FIRG) is used. The main argument for this procedure is that the generalized impulse-response does not vary if variables are reordered in the VAR. As pointed out by Lutkenpohl (1991), the conventional method for the analysis of the impulse-response function applies the “orthogonality hypothesis”, which, therefore, makes the result depend on the ordering of the series in the estimated VAR model. Koop, Pesaran, and Potter (1996), and Pesaran and Shin (1998) developed the generalized impulse-response function to eliminate the problem of ordering variables in the VAR model. There are two potential advantages in applying this method Ewing (2003): (i) the generalized impulse-response function provides more robust results than the orthogonalized method, and (ii) because the orthogonality is not imposed, the function generalized impulse-response allows for a more accurate interpretation of the initial impact response resulting from each shock caused by one variable over the others.

If innovations in the system can be identified, another tool can be used to interpret VAR models. It is possible, in this case, to decompose the variance of the forecast error. This tool gives us the proportion of movements in a sequence that is due to shocks in itself against shocks of other variables. If the error of a variable z does not explain anything about the variance of the error of a sequence {y}, we can say that the latter is exogenous: {y} evolves independently of shocks from the errors of z and {z}.

The decomposition of the variance of the forecasting errors shows the evolution of the dynamic behavior presented by the variables of the economic system, over time, that is, it allows separating the variance of the forecasting errors for each variable into components that can be attributed by itself and by the other endogenous variables, separately presenting, in percentage terms, what effect an unanticipated shock on a given variable has on itself and on the other variables that belongs to the system.

The variance decomposition of the forecast error is an instrument used to describe the system dynamics in the VAR approach. By this method, it becomes possible to identify the proportion of the total variation of a variable produced due to each individual shock in the k variables that make up the model. In addition, it provides information on the relative importance of each innovation on the system variables.

The analysis of variance decomposition (ADV) is an instrument used to describe the dynamics of the system in the VAR approach. By this method, it is possible to identify the proportion of the total variation of a variable due to each individual shock in the k variables that make up the model. ADV provides information on the relative importance of each innovation on the system variables.

4.4. Bivariate Causality

The bivariate analysis is conducted by ARDL models, following Gadelha (2011) and Gadelha and Divino (2008). In this model both the dependent variable and the independent variables are related contemporaneously and in its lagged values. The advantages of the ARDL technique is that it accepts different lags between the variables, which allows to capture the dynamic of the system without omission of important lag lengths. However, ARDL models in a bivariate system can be affected by the omission of important variables, this problem is overcome in this study by the multivariate causality.

The Error Correction Model (ECM) in a bivariate relationship can be derived as follows:

$$Y_t = \mu + \beta_1 X_t + e_t \tag{10}$$

where Y_t and X_t are vectors respectively of the dependent variable and the independent, e_t is the error term.

Solving for e_t we find the cointegration equation for X_t and Y_t. The ECMs for both variables are respectively:

$$\Delta X_t = \mu_x + \alpha_x e_{x,t-1} + \sum_{i=1}^{p} \alpha_{11} \Delta X_{t-i} + \sum_{i=1}^{q} \alpha_{12} \Delta Y_{t-i} + \varepsilon_{xt} \tag{11}$$

$$\Delta Y_t = \mu_y + \alpha_y e_{y,t-1} + \sum_{i=1}^{l} \alpha_{21} \Delta Y_{t-i} + \sum_{i=1}^{m} \alpha_{22} \Delta X_{t-i} + \varepsilon_{yt} \tag{12}$$

where ε_{xt} and ε_{yt} are uncorrelated residuals, $e_{x,t-1}$ and $e_{y,t-1}$ are estimated parameters for the lagged residual, that came from the solution of equation (10), the parameters α_x and α_y from equations (11) and (12) measures the speed of adjustment of X_t and Y_t respectively in direction to the long-run equilibrium. *p*, *q*, *l* and *m* are the optimal lags. The parameters $\alpha_{11}, \alpha_{21}, \alpha_{12}$ and α_{22} represents the short-run relationship.

In equations (11) and (12), the null hypothesis $H_0: \alpha_{12} = 0$ and $\alpha_x = 0$ means that ΔY_t does not Granger cause ΔX_t, on the other hand, the alternative hypothesis $H_1: \alpha_{12} \neq 0$ and $\alpha_x \neq 0$ means that ΔY_t Granger cause ΔX_t. Similarly, $H_0: \alpha_{22} = 0$ and $\alpha_y = 0$ means that ΔX_t does not Granger cause ΔY_t, on the other hand, the alternative hypothesis $H_1: \alpha_{22} \neq 0$ and $\alpha_y \neq 0$ means that ΔX_t Granger cause ΔY_t.

4. Empirical Analysis

4.1. Unit Root Tests and Data Analysis

Table I reports the results of ADF^{GLS} and MZ_{α}^{GLS} unit root tests applied to the series in level and in first differences. Results show that the primary surplus, the debt, the exchange rate and the Embi+ are stationary in first difference; the inflation rate is stationary in level; the GDP growth rate and the interest rate are not stationary in neither of these tests. These results were expected because of the presence of structural changes, which are represented by breaks in the series, also graphically noticeable in Figure 1.

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TABLE I

UNIT ROOT TESTS WITHOUT STRUCTURAL BREAK

Variable	Test Equation	ADF^{GLS}	Lags	MZ_{α}^{GLS}	Lags
Y	Intercept	-1.290721	12	-1.34344	12
	Trend and Intercept	-1.753942	12	-1.85005	12
	D(Intercept)	-0.801839	11	-0.11943	11
	D(Trend and Intercept)	-2.115286	11	-0.84698	11
D	Intercept	1.328672	2	1.37089	2
	Trend and Intercept	-0.952966	2	-0.96881	2
	D(Intercept)	-3.598385***	12	-1.70343*	12
	D(Trend and Intercept)	-3.593600***	9	-2.32025	9
S	Intercept	-0.746254	12	-0.78408	12
	Trend and Intercept	-0.779822	12	-0.84424	12
	D(Intercept)	-3.265969***	10	-2.18834**	10
	D(Trend and Intercept)	-3.501313***	10	-2.42425	10
E	Intercept	0.267376	3	0.23421	3
	Trend and Intercept	-1.475775	2	-1.50945	2
	D(Intercept)	-6.005510***	4	-6.16886***	4
	D(Trend and Intercept)	-5.987699***	4	-6.11966***	4
I	Intercept	0.786271	12	0.87691	12
	Trend and Intercept	-1.185095	12	-1.16717	12
	D(Intercept)	-0.329988	11	0.07366	11
	D(Trend and Intercept)	-1.688773	11	-0.23832	11
Embi+	Intercept	-1.766380*	7	-1.65462*	7
	Trend and Intercept	-2.393650*	7	-2.33533	7
	D(Intercept)	-2.698043***	11	-1.36413	11
	D(Trend and Intercept)	-4.847106***	11	-3.01152**	11
R	Intercept	-3.256705***	8	-2.68997***	8
	Trend and Intercept	-4.319382***	8	-3.89828***	8
	D(Intercept)	-0.811405	11	0.53147	11
	D(Trend and Intercept)	-2.126006	12	-0.60669	12

Source: Authors Elaboration.
Note: *, ** and *** denotes significance at 10%, 5% and 1% levels; D() denotes tests in first difference. The critical values for ADF^{GLS} are: (i) model with constant: -3,46 (1%); -2,87 (5%); e -2,57 (10%); (ii) model with constant and tendency: -3,99 (1%); -3,42 (5%); -3,13 (10%). The critical values for MZ_{α}^{GLS} are: (i) model with constant: -2,58 (1%); -1,98(5%); e -1,62 (10%); (ii) model with constant and tendency: -3,42 (1%); -2,91 (5%); -2,62 (10%). "Y" represents the GDP growth rate; "D" represents the

public debt; “S” represents the primary surplus; “E” represents the exchange rate; “I” represents the interest rate; “Embi+” represents the Embi+; and “R” represents the inflation rate.

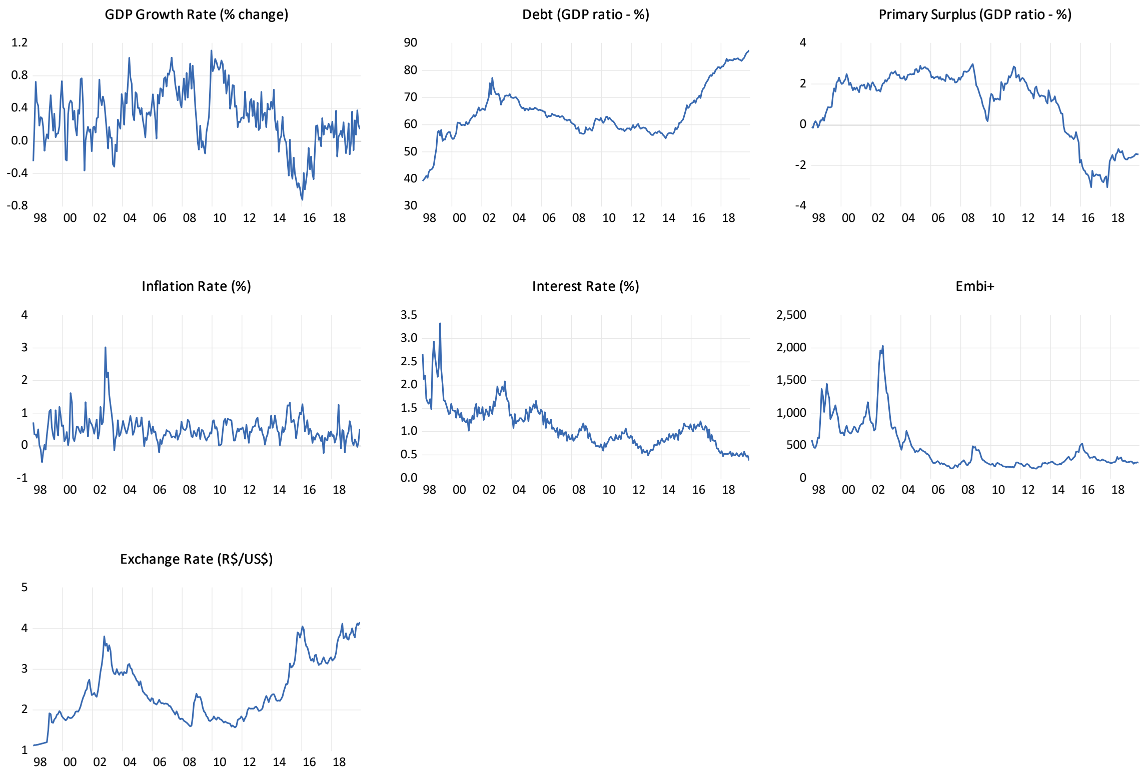

FIGURE 1 – Treated Series.

Therefore, the analysis was improved using unit root tests with structural endogenous breaks, presented in Table II. Both SL and VP tests reached the same conclusions, in which series of the GDP growth rate, the interest rate, the Embi+ and the inflation rate are stationary in level. However, the debt, the primary surplus and the exchange rate are stationary in first difference.

Most of the selected break dates occurred between September 1998 and April 1999. During this period multiple changes occurred in the economic policy, the most relevant one was the switch from foreign exchange anchor to inflation target policies, which came out with sharp exchange rate devaluation and strong control of interest rates to achieve the inflation target.

Another important break selected by the tests were from October and November 2002. This period reflects a crisis of external confidence related to the election of President Lula, which is called Lula´s effect. The third important break is related to the economic recession that hit Brazil in the second quarterly of 2014, which may explain the breaks that appeared between April 2014 and December 2015.

TABLE II

UNIT ROOT TESTS WITH STRUCTURAL BREAK

Variable	Test Equation	SL - Rational Shift			VP- Innovational Outilier		
		Date	t- statistic	Lags	Date	t- statistic	lags
Y	Intercept	2001 M6	-3.0334**	2	1998 M10	-5.672562***	0
	Trend and Intercept	2001 M6	-2.4919	2	1999 M01	-5.715973***	0
	D(Intercept)	2001 M6	-6.0331***	2	1999 M03	-21.14956***	0
	D(Trend and Intercept)	2001 M6	-4.7778***	2	1998 M06	-21.55431***	0
D	Intercept	1999 M1	-0.6209	2	2015 M06	-4.317432	5
	Trend and Intercept	1999 M1	-0.9221	2	2012 M07	-4.445778	4
	D(Intercept)	1999 M1	-7.3233***	2	1999 M01	-19.23603***	0
	D(Trend and Intercept)	1999 M1	-4.2197***	2	1999 M01	-19.51033***	0
S	Intercept	2015 M12	-0.6352	2	2014 M04	-4.450804	12
	Trend and Intercept	2015 M12	-1.3882	2	2015 M10	-3.372438	12
	D(Intercept)	2015 M12	-8.4324***	2	1998 M 06	-15.33422***	0
	D(Trend and Intercept)	2015 M12	-9.0591***	2	1998 M08	-15.30660***	0
E	Intercept	2002 M10	-0.8667	2	2005 M03	-3.381711	1
	Trend and Intercept	2002 M10	-1.1440	2	2008 M12	-3.554331	4
	D(Intercept)	2002 M10	-3.5849***	2	2002 M10	-13.11930***	0
	D(Trend and Intercept)	2002 M10	-4.8238***	2	2003 M04	-12.38337***	0
I	Intercept	1998 M9	-3.1055**	2	1999 M03	-7.773424***	0
	Trend and Intercept	1998 M9	-0.8898**	2	1999 M03	-8.281520***	0
	D(Intercept)	1998 M9	-10.1937***	2	1999 M03	-23.45007***	0
	D(Trend and Intercept)	1998 M9	-5.1881***	2	1998 M11	-23.66599***	0
Embi+	Intercept	2002 M11	-3.0340**	2	2002 M10	-6.100274***	3
	Trend and Intercept	2002 M11	-2.2273	2	2002 M10	-6.285755***	3
	D(Intercept)	2002 M11	-3.8811***	2	1998 M09	-11.87812***	0
	D(Trend and Intercept)	2002 M11	-4.8686***	2	1998 M09	-12.23370***	0
R	Intercept	2002 M11	-4.2899***	2	1998 M11	-8.149452***	0
	Trend and Intercept	2002 M11	-4.7562***	2	1998 M11	-8.074853***	0
	D(Intercept)	2002 M11	-4.9046***	2	1998 M08	-18.22486***	0
	D(Trend and Intercept)	2002 M11	-5.4818***	2	1999 M04	-18.34451***	0

Source: Authors Elaboration.

Note: *, ** and *** denotes significance at 10%, 5% and 1% levels; D() denotes tests in first difference. The critical values for SL are: (i) model with constant: -3,48 (1%); -2,88 (5%); e -2,58 (10%); (ii) model with constant and tendency: -3,55 (1%); -3,03 (5%); -2,76 (10%). The critical values for VP are: (i) model with constant: -5,34 (1%); -4,86 (5%); e -4,60 (10%); (ii) model with constant and tendency: -5,72 (1%); -5,18 (5%); -4,89 (10%). "Y" represents the GDP growth rate; "D" represents the public debt; "S" represents the primary surplus; "E" represents the exchange rate; "I" represents the interest rate; "Embi+" represents the Embi+; and "R" represents the inflation rate.

Therefore, we used three dummies variables, considering the breaking dates that appeared in the unit root tests and the information presented in Pastore *et al* (2020)[13], which reinforces the breaks pointed in the unit root tests. These dummies take the values "1" for the specific period when some event occurred, and "0" otherwise. The dummies used were: *dexchangerate, dlula* and *dcrisis*. The first one selected the period of January 1998 until March 1999, and it is related to changes in economic policy due to exchange rate depreciation; the second one selected the period between June 2002 and April 2003, which is related to Lula´s effect; the last is referred to the period between April 2014 and December 2016, a period of a strong economic recession.

4.2. Growth Equation

We estimated five different static models: where model 1, model 2, model 3 and model 4 are applications of equation (3) and model 5 checks the possibility of non-linear relationship, as presented in the equation (4). We also used the dummies *exchangerate, dlula,* and *dcrisis* which appeared to be statically significant at 10% level in all the models. Results are presented in Table III.

In model 1 we found significantly positive coefficients for debt and primary surplus, significantly negative coefficient for interest rate, exchange rate and Emib+. Results shows that debt has a positive impact on the GDP growth rate.

In model 2 we checked if the Brazilian Constitution amendment, which imposed a ceiling for government expenditure, representing a fiscal consolidation, had some impact on the debt and growth relationship. For this analysis we used a dummy represented as *dconsolidation,* which receives the value "1" if in the period considered there was a government consolidation, and "0" otherwise. However, *dconsolidation* showed not to be

[13] Report presented by the Business Cycle Dating Committee (CODACE), which presents the most relevant changes in economic cycles of Brazilian economy.

significant at the 10% level. Moreover, the results for the coefficients are very similar to those found on Model 1.

In model 3 and 4 we analysed if the relationship between debt and growth changes when the debt ratio is "high" or "low". In model 3, we used a dummy which received the value "1" if in the period government debt was placed in between 30% and 60% of GDP, and "0" otherwise, represented by *d3060*. In model 4 we used a dummy which received the value "1" if in the period government debt was placed in between 60% and 90%, and "0" otherwise, represented by *d6090*. These values followed those applied by Reinhart and Rogoff (2010). The dummies used presented significant coefficients. Although the results of the debt coefficient were almost the same in both models, our results also suggest that growth intensifies when debt is in between 60% and 90%, since *d6090* has a positive coefficient and *d3060* has a negative coefficient. These results are in line with the findings of Reinhart and Rogoff (2010), they showed that growth rates in Brazil are larger when debt ratio is between 60% and 90%, and smaller when it is above 90% or in between 30% and 60%. They also found a threshold of 90% for debt-to-GDP ratio for advanced and emerging countries, including Brazil.

Previous results lead us to estimate Model 5, where we followed equation (4), to check for the possibility of an inverted U-shape relationship between debt and growth. However, β_3 was not statistically significant. Therefore, we could not confirm the threshold pointed by Reinhart and Rogoff (2010).

In every model the results of the coefficients were very similar, all of them showed debt presenting positive impacts on growth; interest rate, exchange rate and Embi+ presented negative impact; and primary surplus presented positive coefficients. In none of the models inflation presented significant coefficients. The results of this section do not consider the possibility of lagged effects, as well as the possible interaction between the dependent and independent variables, which are going to be analysed in sections 4.3 and 4.4.

TABLE III

GROWTH EQUATIONS

Dependent Variable: GDP growth rate	Model 1	Model 2	Model 3#	Model 4#	Model 5
D	0.024870***	0.024935***	0.015705***	0.015705***	0.026825***
	(0.002983)	(0.003012)	(0.005455)	(0.005455)	(0.004447)
I	-0.132585**	-0.133311**	-0.186561***	-0.186561***	-0.147985**
	(0.0277)	(0.060150)	(0.065033)	(0.065033)	(0.065337)
R	0.042858	0.041920	0.044768	0.044768	0.036706
	(0.047159)	(0.047558)	(0.047371)	(0.047371)	(0.048343)
E	-0.361561***	-0.359584***	-0.330204***	-0.330204***	-0.340081***
	(0.046593)	(0.048054)	(0.056359)	(0.056359)	(0.059040)
Embi+	-0.000225***	-0.000226***	-0.000196**	-0.000196**	-0.000223***
	(8.17E-05)	(8.23E-05)	(8.14E-05)	(8.14E-05)	(8.19E-05)
S	0.094739***	0.091282***	0.083513***	0.083513***	0.091210***
	(0.017846)	(0.026777)	(0.018371)	(0.018371)	(0.018832)
D^2					-3.30E-05
					(5.56E-05)
dconsolidation		-0.019213			
		(0.110776)			
d3060			-0.132068***		
			(0.045517)		
d6090				0.132068***	
				(0.045517)	
dexchangerate	-0.318553**	-0.317802**	-0.310161**	-0.310161**	-0.331970**
	(0.126371)	(0.126687)	(0.124783)	(0.124783)	(0.128535)
dlula	0.349224***	0.348969***	0.341118***	0.341118***	0.359455***
	(0.111081)	(0.111303)	(0.110715)	(0.110715)	(0.112550)
dcrisis	-0.124405*	-0.129998*	-0.183423**	-0.183423**	-0.150904*
	(0.069517)	(0.076753)	(0.078511)	(0.078511)	(0.082689)
R-squared	0.559782	0.559834	0.574379	0.574379	0.560394

Source: Authors Elaboration.
Note: Models are estimated by OLS*, ** and *** denotes significance at 10%, 5% and 1% levels; std. error in between (), # represents models estimated with constant. "D" represents the public debt; "I" represents the interest rate; "R" represents the inflation rate; "E" represents the exchange rate; "Embi+"

represents the Embi+; “S” represents the primary surplus; “D^2” represents the public debt squared; *dconsolidation, d3060, d6090, dexchangerate, dlula* and *dcrises* represent dummy variables.

4.3.Multivariate Causality

Since half of the series became stationary only after the first difference, the next step was testing for cointegration. We applied the lag length criteria to the VAR of the series and the decision was to select the lag pointed by most of the criteria’s results, which was seven. This value was selected by Final Prediction Error (FPE) and Akaike Information Criteria (AIC).

Firstly, we applied the Johansen Trace and Max-Eigenvalues tests, and results are reported in Table IV. They suggest a long run relationship between the variables, as we do not reject the null hypothesis for the presence of co-integration vector after the fourth rank. Since it is known that the series have breaks, we selected the breaks that appeared the most on the unit root test, then applied Johansen cointegration tests using these breaks. Three pairs of dates were selected for the application of the test with breaks, they are: January 1999 and November 2002, results are presented in Table V; January 1999 and December 2015; and November 2002 and December 2015[14]. All of them came to the same conclusion: the existence of a long run relationship between the variables with five co-integrating vectors. The presence of co-integration denotes that the multivariate analysis should be conducted using a VEC.

[14] Results of the last two tests are not presented for reasons of parsimony.

TABLE IV

JOHANSEN COINTEGRATION TESTE WITHOUT STRUCTURAL BREAK

Rank	λ^{trace}	P-value	$\lambda^{Eigenvalue}$	P-value
r = 0	250.11***	0.00	80.82***	0.00
r ≤ 1	169.29***	0.00	56.71***	0.00
r ≤ 2	112.58***	0.00	51.26***	0.00
r ≤ 3	61.316***	0.00	36.69***	0.00
r ≤ 4	24.62	0.18	14.38	0.33
r ≤ 5	10.24	0.26	10.18	0.20
r ≤ 6	0.05	0.82	0.05	0.82

Source: Authors Elaboration.
Note: *** denotes the rejection of the null hypo paper at 1% level of significance.

TABLE V

JOHANSEN COINTEGRATION TESTE WITH STRUCTURAL BREAK

Rank	LR	P-Value	90%	95%	99%
r = 0	325.12***	0.0000	155.31	160.86	171.61
r ≤ 1	218.79***	0.0000	121.52	126.47	136.11
r ≤ 2	135.43***	0.0000	91.65	96.00	104.53
r ≤ 3	86.33***	0.0009	65.79	69.55	76.97
r ≤ 4	55.08***	0.0064	43.98	47.15	53.48
r ≤ 5	24.68	0.1426	26.11	28.68	33.93
r ≤ 6	6.35	0.5306	12.24	14.25	18.57

Source: Authors Elaboration.
Note: *** denotes the rejection of the null hypo paper at 1% level of significance.
Breaks date used: January 1999 and December 2002.

We estimated the VEC with five co-integration vectors, we used only *exchangerate*, *dlula* and *dcrisis*, we did not use *dconsolidation*, *d3060* and *d6090* since they were not significant for most of the vectors. Table VI presents the results of the Granger Causality based on the VEC. In addition, we also tested the significance of the coefficients of the co-integration equations in the VEC by employing a χ^2 Wald Test. If the null hypothesis is rejected, we can validate the results of Granger causality, moreover we can follow the strategy of analysing all the variables as endogenous in the system. We rejected the null hypothesis for all the coefficients 1% of significance, results are presented in Table VII.

TABLE VI

GRANGER CAUSALITY TEST

Independent Variable	Dependent Variable: Y	D	I	R	E	Embi+	S	Direction of causality
Y	-	18.97380*** (0.0042)	14.53546** (0.0242)	10.96128* (0.0896)	9.820979 (0.1324)	16.47853** (0.0114)	3.821981 (0.7008)	Y ⟶ D; Y ⟶ I; Y ⟶ R; Y ⟶ Embi+
D	11.92609* (0.0636)	-	26.46606*** (0.0002)	21.87011*** (0.0013)	11.40607* (0.0766)	13.32261** (0.0382)	5.725819 (0.4546)	D ⟶ Y; D ⟶ I; D ⟶ R D ⟶ E; D ⟶ Embi+
I	6.819487 (0.3379)	27.62727*** (0.0001)	-	27.06110*** (0.0001)	8.311279 (0.2162)	23.70176*** (0.0006)	6.525338 (0.3670)	I ⟶ D; I ⟶ R; I ⟶ Embi+
R	6.532053 (0.3663)	30.84374*** (0.0000)	16.02811** (0.0136)	-	19.41075*** (0.0035)	14.47299** (0.0248)	6.533845 (0.3661)	R ⟶ D; R ⟶ I; R ⟶ E; R ⟶ Embi+
E	12.36035* (0.0544)	9.346753 (0.1550)	33.94641*** (0.0000)	10.94679* (0.0900)	-	12.50652* (0.0516)	8.922092 (0.1780)	E ⟶ Y; E ⟶ I; E ⟶ R; E ⟶ Embi+
Embi+	2.133336 (0.9070)	41.60621*** (0.0000)	43.68729*** (0.0000)	25.21548*** (0.0003)	15.43095** (0.0172)	-	3.891478 0.6914	Embi+ ⟶ D; Embi+ ⟶ I; Embi+ ⟶ R; Embi+ ⟶ E
S	15.53278** (0.0165)	3.176217 (0.7864)	6.516320 (0.3679)	7.688136 (0.2619)	2.334217 (0.8865)	4.296242 (0.6367)	-	S ⟶ Y

Source: Authors Elaboration.
Note: *, ** and *** denotes significance at 10%, 5% and 1% levels; variables between () are the p-values; all the other values are the Chi-square of the Granger Causality. "Y" represents the GDP growth rate; "D" represents the public debt; "I" represents the interest rate; "R" represents the inflation rate; "E" represents the exchange rate; "Embi+" represents the Embi+ and "S" represents the primary surplus.

TABLE VII

SIGNIFICANCE OF COINTEGRATION EQUATION

Dependent Varialbe	Chi-Square	P-Value
Y	23.14593***	0.000317
D	56.99717***	0
I	69.72704***	0
R	55.38739***	0
E	16.03341***	0.006749
Embi+	23.499***	0.000271
S	26.70278***	0.000065

Source: Authors Elaboration.
Note: *** denotes significance at 1% level. "Y" represents the GDP growth rate; "D" represents the public debt; "I" represents the interest rate; "R" represents the inflation rate; "E" represents the exchange rate; "Embi+" represents the Embi+ and "S" represents the primary surplus.

The results of the Granger causality conclude that the GDP growth rate has a bidirectional relation with the debt ratio. Also, it is Granger caused by the exchange rate and by the primary surplus. Moreover, the debt ratio has a bidirectional relation with the interest rate, inflation rate and Embi+. Therefore, it is possible to say that the interest rate, inflation rate and Embi+ can influence the relation between debt and growth, since they affect the behaviour of the debt ratio. Moreover, these results are in accordance with the finds of Gadelha and Divino (2008), that also concluded that interest rate and Embi + Granger causes debt.

The VEC model satisfies the stability condition, since none of the roots of the model lies outside the unit circle. Results of the Roots of AR Characteristic Polynomial are presented in Figure 2. The autocorrelation LM test was applied to check for the presence of serial correlation in the error terms. Results conclude for no autocorrelation after lag seven, since we cannot reject the null hypothesis of no serial correlation at a significance level of 5%. Results of the LM test are presented in Table VIII.

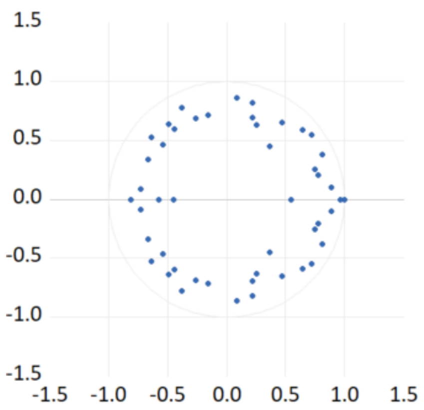

FIGURE 2 – ROOTS OF CHARACTERISTIC POLYNOMIAL.

In order to understand the whole scenario of the interactions between the variables in the system, we complemented the analyses of the causality by impulse response functions and variance decomposition. By the impulse response function, we can see the response of one variable, over a future period, to a shock of another variable in the VEC. Therefore, we analysed the response of GDP growth rate and debt ratio to innovations of the rest of the endogenous variables in the system over the period of 18 months. We used the method of generalized decomposition.

TABLE VIII

VEC RESIDUAL SERIAL CORRELATION LM TESTS

Lag	F-statistic	P-Value
1	1.882022	0.0003
2	2.000301	0.0001
3	1.738961	0.0015
4	1.377469	0.046
5	1.807478	0.0007
6	1.813465	0.0007
7	1.334598	0.0646
8	1.245115	0.1239
9	0.942524	0.5873
10	0.636017	0.9761

Source: Authors Elaboration.
Note: Null hypo paper : no serial correlation at lag h.

Figure 3, presents the response of the GDP growth rate to one standard innovation in the other endogenous variables of the VEC. Results suggest that a shock in debt ratio has negative effects on GDP growth rate, there are only a few positive effects during the third and fifth months. This behaviour is in accordance with the theory that government debt has a negative impact on growth. Moreover, the results of the first six months after a positive shock in primary surplus generates a positive impact on economic growth, validating the theory of expansionary fiscal consolidations[15]. Further, Matheson and Pereira (2016) concluded that fiscal multipliers related to spending and credit have dropped to near zero in Brazil between the global financial crisis in 2008 and 2014, thus non-Keynesian effects are more likely to prevail.

[15] See Afonso and Martins (2016) for a better understanding of expansionary fiscal consolidations.

The GDP growth rate shows a positive response to a shock in interest rates, and it is possible that an increase in short term interest rate could lead to an increase in savings, which may have a positive impact on the GDP growth rate. The response to a shock in the exchange rate is negative, meaning that a depreciation of the currency provokes a negative impact on the GDP, and this negative impact intensifies until the eighth month, then it starts to decrease, which may be related to capital imports. Inflation causes a negative impact on GDP growth rate during the first ten months, then it vanishes. Embi+ is negative for GDP growth rate during the first sixteen months. Both, inflation and Embi+ negative effects on growth may be related to bad expectations.

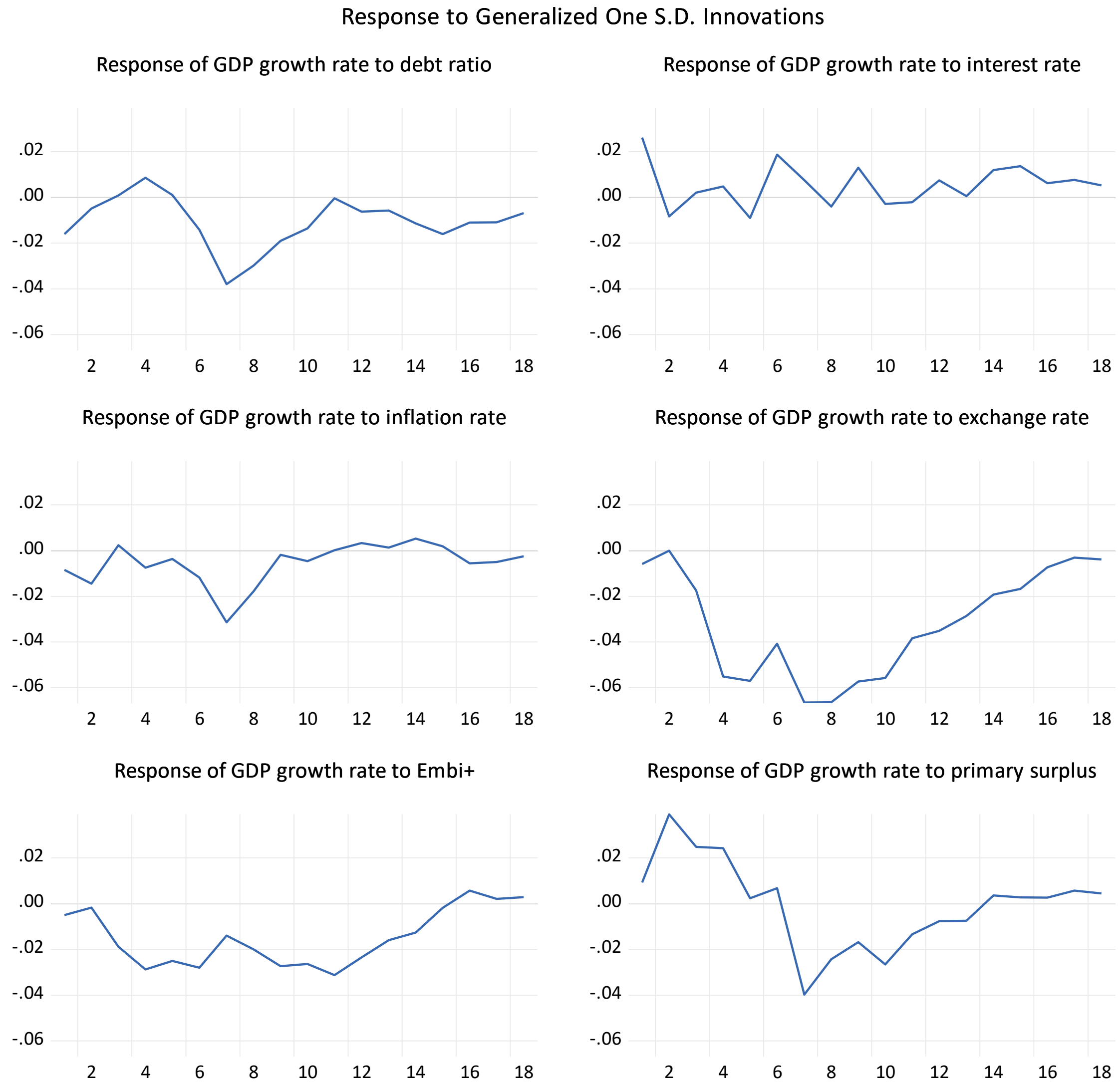

FIGURE 3 – Response of GDP growth rate to innovations on debt ratio, interest rate, exchange rate, Embi+ and primary surplus.

We also tested the response of debt to a shock on the system variables. Results are presented in Figure 4, they show that the debt ratio decreases when the GDP growth rate increases, that means that increases in GDP growth rate can reduces debt-to-GDP ratio. A shock on interest rate made debt increase over the first ten months. This behaviour is explained by part of the public debt being indexed to the interest rate. The exchange rate also makes debt increase over the first seven months. This behaviour is explained by the external debt, which may increase when there is a currency depreciation. Innovations in the inflation rate provokes a decrease in the debt ratio, in this case, although the monetary authority sets the interest rate independently of the fiscal authority, the debt ratio is somehow benefited by the seignioriage. Embi+ increases debt ratio over all the period, meaning that when Embi+ increases, expectations of the country get worse; therefore, investors demand more risk premium, which increases the debt.

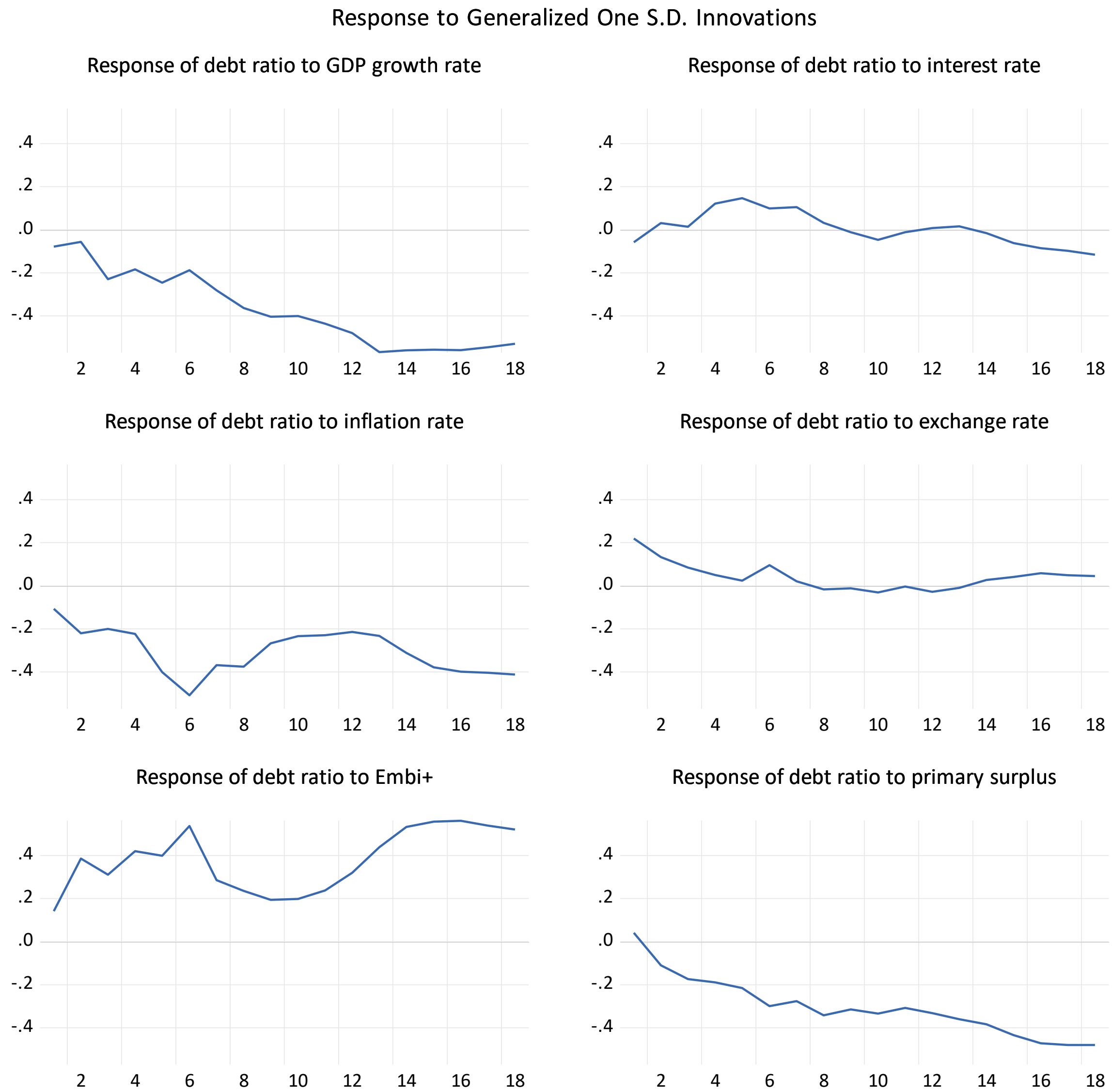

FIGURE 4 – Response of debt ratio to innovations on GDP growth rate, interest rate, inflation rate, exchange rate, Embi+ and primary surplus.

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The result validated the theory of expansionary fiscal consolidations for Brazil, since a decreasing debt ratio has positive impacts on economic growth. Moreover, as it is presented in Figure 5, debt shocks generate increases on inflation rate most of the time, corroborating the fiscal theory of the price level (FTPL)[16]. Debt shocks also generate interest rate increases, meaning that the monetary authority may try to control inflation. Additionally, it improves profitability of government bonds, which may be required by investors in response to the increase in debt ratio; Embi+ increases, since investors demand more risk premium and currency depreciates. Inflation rate, exchange rate and Embi+ will provoke a negative impact on GDP growth rate. Therefore, debt may directly provoke a negative impact on the GDP growth rate and indirectly have negative impact by changes in inflation rate, exchange rate and Embi+.

Furthermore, a shock on GDP growth rate provokes an increase on interest rate, mixed effects on inflation, appreciation of currency, decreases Embi+ and increases primary surplus. The effect of the last three will result in a further debt reduction. Results are presented in Figure 6. Thus, the increase in the GDP growth rate may also indirectly decrease the debt ratio by the effect of exchange rate, Embi+ and primary surplus.

[16] The FTPL posits that the increase in government debt increases demand and leads to price pressures.

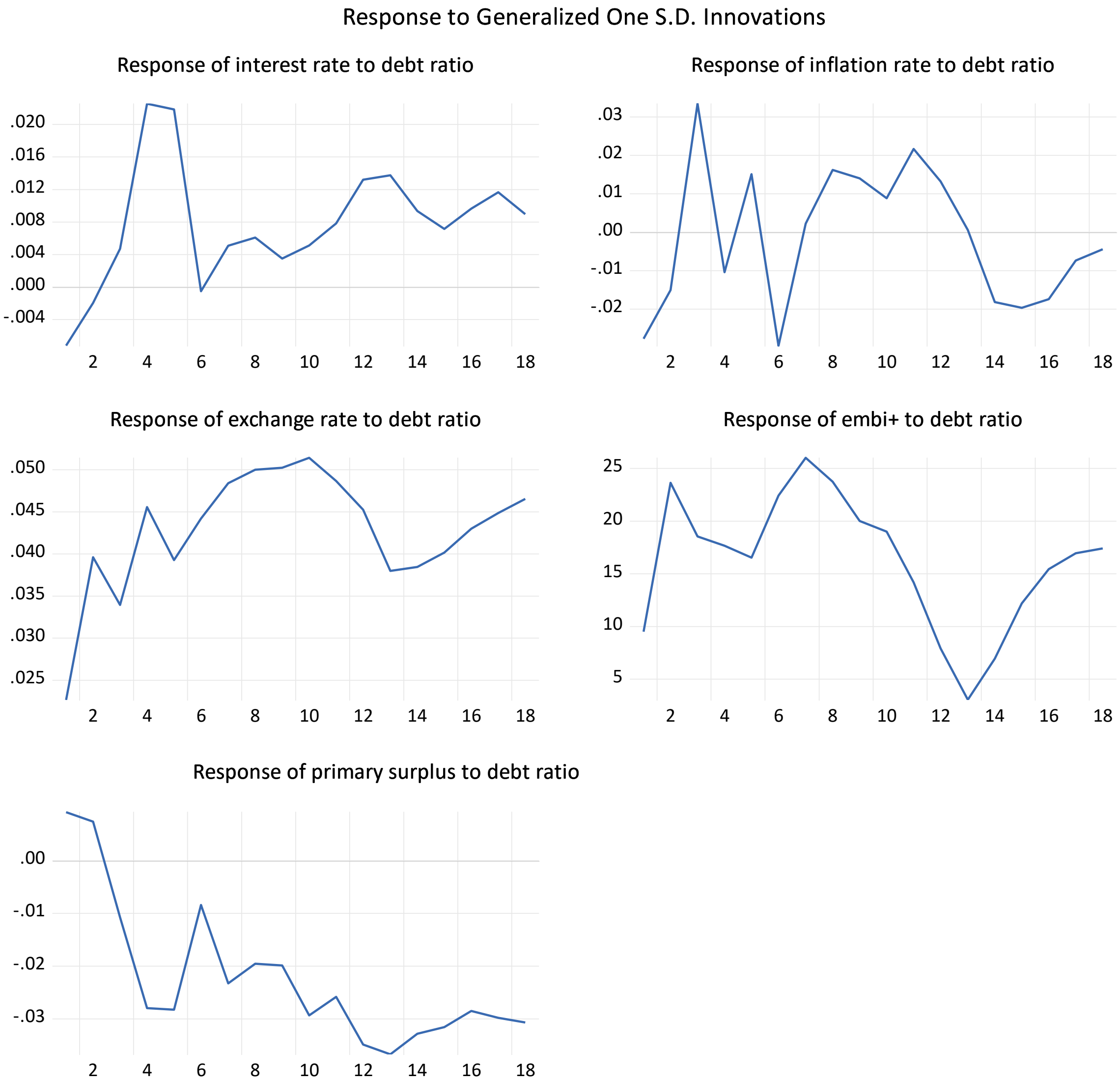

FIGURE 5 – Response of interest rate, inflation rate, exchange rate, Embi+ and primary surplus to innovations on Debt.

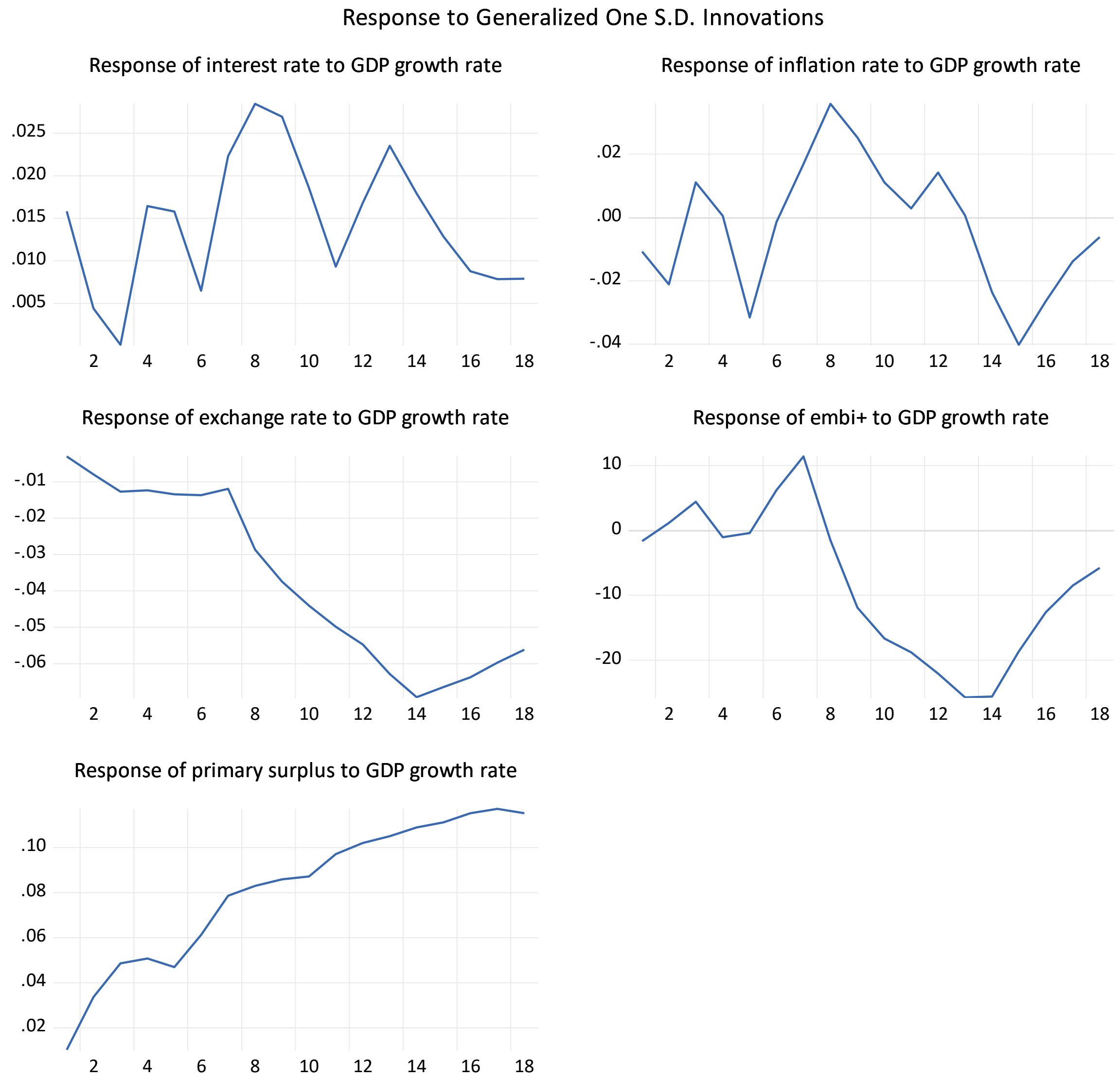

FIGURE 6 – Response of interest rate, inflation rate, exchange rate, Embi+ and primary surplus to innovations on GDP growth rate.

The variance decomposition quantifies the contribution of innovations in one variable to changes in another variable. Therefore, we aim to quantify the proportion of the variation in GDP growth rate and debt ratio that is related to each other and to the other endogenous variables in the VEC. Results are presented in Table IX and Table X, values are presented as percentage.

TABLE IX

VARIANCE DECOMPOSITION OF GDP GROWTH RATE

Variance Decomposition of GDP growth rate:								
Period	S.E.	GDP growth rate	Debt ratio	Interest Rate	Inflation Rate	Exchange Rate	Embi+	Primary Surplus
1	0.183151	100	0	0	0	0	0	0
2	0.209598	96.00433	0.023999	1.064935	0.098707	0.01221	0.001204	2.794612
3	0.222245	94.32781	0.111989	1.051877	0.202231	0.63681	0.272632	3.39665
4	0.239047	87.45872	0.426099	0.92354	0.188218	6.492936	0.317128	4.193356
5	0.247459	82.31036	0.410572	1.093467	0.17614	11.71551	0.29907	3.994875
6	0.253935	79.74094	0.591579	1.328337	0.480751	13.35328	0.530367	3.974745
7	0.269995	72.74237	2.157544	1.175015	2.05934	16.42001	0.78228	4.663444
8	0.279718	67.90714	3.08716	1.158759	2.448218	19.97127	0.826217	4.60124
9	0.286298	65.10216	3.333176	1.227076	2.368261	22.62983	0.802372	4.537124
10	0.292677	62.43653	3.376321	1.206154	2.295998	25.04519	0.77863	4.861186

Source: Authors Elaboration.
Note: Factorization by Cholesky Decomposition.

TABLE X

VARIANCE DECOMPOSITION OF DEBT RATIO

Variance Decomposition of debt ratio:								
Period	S.E.	GDP growth rate	Debt ratio	Interest Rate	Inflation Rate	Exchange Rate	Embi+	Primary Surplus
1	0.89585	0.767807	99.23219	0	0	0	0	0
2	1.153488	0.696747	88.87458	0.399082	1.941236	0.024476	7.448036	0.615844
3	1.395012	3.187934	83.56808	0.622883	2.398688	0.38676	8.258937	1.576718
4	1.602214	3.736209	76.55858	1.738067	3.248148	0.658492	12.06869	1.991818
5	1.771675	4.983667	67.26677	2.751705	7.880829	0.704996	14.23904	2.172995
6	1.975189	4.916541	58.76718	2.786985	12.69285	0.567856	17.41046	2.858131
7	2.103162	6.123217	55.67954	3.101031	14.20159	0.664024	16.73237	3.498234
8	2.248552	7.974579	53.11785	2.953769	14.97471	0.977567	15.47039	4.531138
9	2.401642	9.818936	52.86607	2.695913	14.03063	1.374447	13.99266	5.221348
10	2.550495	11.17809	52.80089	2.419296	12.97311	1.823604	12.87357	5.931436

Source: Authors Elaboration.
Note: Factorization by Cholesky Decomposition.

Table IX provides information about the variance decomposition of GDP growth rate. Most of the change is related to its past values, however one important part seems to be related to exchange rate, that increases it´s importance through time, followed by primary surplus and debt ratio. These results also reinforces the conclusion of the Granger causality, since these three variables appeared Granger causing GDP growth rate.

Table X provides information about the variance decomposition of debt ratio. GDP growth rate, Embi + and inflation contributes the most to changes in debt ratio. That validates results for Granger causality, since these variables appeared Granger causing debt ratio.

4.4.Bivariate Causality

The ARDL model in a bivariate causality allows us to have an embracing analysis of the relationship between both variables included. Since it does not demand the same number of lags for both variables, we do not run the risk of omitting important lags. However, in a bivariate causality we run the risk of omission of important variables. Therefore, both methodologies, VEC and ARDL, are going to be used as complementary to each other.

The first step in the analysis was to perform the Engle-Granger cointegration test, in which we used the AIC and the SIC criteria for the lag selection. Results are presented in Table XI. All the pairs presented cointegration for at least one side of the selection criteria used.

Therefore, we performed the ARDL model for all the pair of variables. The model was carried out using restricted constant in the trend specification, we also included the same dummies used in the VEC model as fixed regressor. After the results, we tested if the dummies and the constant were significant and excluded those that did not show to be significant in at least 10% of significance level.

TABLE XI

ENGLE-GRANGER COINTEGRATION TEST

Dependent Variable	Independent Variable	AIC	Lags	SIC	Lags
y	d	-2.236107	15	-4.322247***	1
d	y	-1.562334	4	-2.010215	0
y	i	-2.101870	15	-4.096740***	1
i	y	-2.796474	15	-2.194599	12
y	r	-2.099894	15	-4.143831***	1
r	y	-4.007762***	8	-7.566273***	0
y	e	-2.350242	15	-5.110468***	1
e	y	-0.637093	15	-3.402884*	0
y	Embi+	-2.046844	15	-4.146842***	1
Embi+	y	-1.947773	7	-2.695038	1
y	s	-3.158550*	15	-4.938623***	1
s	y	-1.466781	15	-1.466781	15
d	i	-0.307425	14	-1.290000	1
i	d	-3.498478**	14	-2.235684	12
d	r	-0.122924***	14	-1.270687	2
r	d	-4.017607	8	-7.621434***	0
d	e	-2.924036	2	-4.500774***	2
e	d	-3.256299*	4	-4.535044***	2
d	Embi+	-2.924036	2	-2.946314	2
Embi+	d	-3.256299*	4	-2.472982	0
d	s	-1.369190	14	-3.487583	0
s	d	-2.070137	13	-3.032596**	0

Source: Authors Elaboration.
Note: *, ** and *** denotes significance at 10%, 5% and 1% levels. “y” represents the GDP growth rate; “d” represents the public debt; “i” represents the interest rate; “r” represents the inflation rate; “e” represents the exchange rate; “Embi+” represents the Embi+ and “s” represents the primary surplus.

Hereafter, the Error Correction was included, when it presented to be statistically significant. Thus, we estimated the long run model of the error correction, otherwise we estimated the short run model, which is the ARDL with the differenced lagged variables without the error correction term. Results are presented in Table XII. The causality test was checked by the joint significance in the Wald test. Results show that in most of the cases, when GDP growth rate is the dependent variable, variables exhibited long run relationship, meaning that in the long run there is a univariate relationship from the variables selected to GDP growth rate, except for debt, which exhibited only short run relation with GDP growth rate. On the other hand, GDP growth rate also Granger causes the pattern of most of the variables, although in the short run, except for the exchange rate. The opposite relation occurred for the debt, which appeared to have a long run relationship with most of the variables, only when it was the independent variable, however, in the short run debt is Granger caused by the pattern of most of the variables.

Primary surplus is not Granger caused neither by GDP growth rate nor by debt. The same was found in the VEC Granger causality.

TABLE XII

ENGLE-GRANGER CAUSALITY TEST

Null Hypotesis	Model	OBS	F-statistic	P-value	Causality	ADL
Δ*D* does not Granger causes Δ*Y*	Short run	258	3.780155	0.0241	Yes	(4,2)
Δ*Y* does not Granger causes Δ*D*	Short run	260	6.927241	0.009	Yes	(2,2)
Δ*I* does not Granger causes Δ*Y*	Long run	250	15.86195	0.0000	Yes	(12,12)
Δ*Y* does not Granger causes Δ*I*	Short run	250	12.58429	0	Yes	(12,12)
Δ*R* does not Granger causes Δ*Y*	Long run	250	5.883427	0.0032	Yes	(12,6)
Δ*Y* does not Granger causes Δ*R*	Long run	254	14.71783	0	Yes	(4,2)
Δ*E* does not Granger causes Δ*Y*	Long run	250	15.90652	0.0000	Yes	(12,7)
Δ*Y* does not Granger causes Δ*E*	No relationship	-	-	-	No	-
Δ*Embi+* does not Granger causes Δ*Y*	Long run	250	5.195388	0.0005	Yes	(12,11)
Δ*Y* does not Granger causes Δ*Embi+*	Short run	256	2.857744	0.0593	Yes	(6,6)
Δ*S* does not Granger causes Δ*Y*	Long run	250	17.01869	0.0000	Yes	(12,6)
Δ*Y* does not Granger causes Δ*S*	No relationship	-	-	-	No	-
Δ*I* does not Granger causes Δ*D*	No relationship	-	-	-	No	-
Δ*D* does not Granger causes Δ*I*	Long run	250	7.955811	0	Yes	(12,10)
Δ*R* does not Granger causes Δ*D*	Short run	254	4.443769	0.0000	Yes	(12,7)
Δ*D* does not Granger causes Δ*R*	Long run	254	22.15795	0	Yes	(8,7)
Δ*E* does not Granger causes Δ*D*	Short run	260	12.82875	0.0004	Yes	(2,0)
Δ*D* does not Granger causes Δ*E*	Long run	261	15.00638	0	Yes	(1,1)
Δ*Embi+* does not Granger causes Δ*D*	Short run	253	12.14643	0.0000	Yes	(9,6)
Δ*D* does not Granger causes Δ*Embi+*	Long run	254	3.863187	0.0022	Yes	(1,8)
Δ*S* does not Granger causes Δ*D*	Short run	261	3.441240	0.0647	Yes	(0,1)
Δ*D* does not Granger causes Δ*S*	No relationship	-	-	-	No	-

Source: Authors Elaboration.
Note: F-statistic is the result of the application of the Wald test for the joint coefficients of the dependent variable. "Δ" represents the variation, "y" represents the GDP growth rate; "d" represents the public debt;

“i” represents the interest rate; “r” represents the inflation rate; “e” represents the exchange rate; “Embi+” represents the Embi+ and “s” represents the primary surplus.

4.5.Results

In the first exercise we estimated the static relationship between the group of variables. In this part we did not allow for lagged variables impacting in the dependent variable, since the primordial objective was to understand if the relation would change after the Constitutional ceiling amendment and after changing in debt ratios. Results showed the same signals and almost small changes in the coefficients. In all the models employed we find debt impacting positively in GDP growth rate.

Hereafter, we studied the causality between the variables, using VEC in a multivariate analysis, and ARDL the bivariate one. The bivariate analysis allowed the impact of lagged variables and showed a similar result as the static model. In this methodology debt Granger causes GDP growth rate in a positive way in the short run. Moreover, GDP growth rate is Granger caused by the rest of the system of variables, but in a negative way. On the other hand, the multivariate analysis presented a different result, where debt also Granger causes GDP growth rate, but in a negative way and in the long run.

Table XIII presents a comparison of both methodologies for the cases that we find Granger causality. In the VEC, the signal is a result of the cumulative values of the impulse response function. In the ARDL are the values of the summation of the coefficients of the lagged values of the dependent variables. Overwritten letters “S” and “L” represent that we find Granger causality respectively in short and long run models.

This result brings us the question: Why could debt impact positively on the GDP growth rate in the short run and negatively in the long run? The answer to this question takes in consideration the difference between ARDL and VEC methodologies. Since ARDL runs the risk of omission of important variables and VEC runs the risk of the omission of important lags, both analyses should be used in a complementary way. Results of the VEC consider the impact that debt may generate in other variables, that will also impact on GDP growth rate, such as inflation rate, exchange rate and Embi+, that are positively Granger caused by debt and negatively Granger causes GDP growth rate. That is, when debt increases, inflation rate, exchange rate and Embi+ also increases, however all of them will work to decrease growth. Moreover, the impulse response of the VEC showed positive impacts of debt on growth during the third and fifth periods, which is in accordance with ARDL results. Therefore, it is possible to say that the short run causality from debt to growth is positive, however this relationship changes in the long run, when the causality becomes negative, and part of it is related to the impact of inflation rate, exchange rate and Embi+.

TABLE XIII

COMPARISON OF RESULTS OF GRANGER CAUSALITY: VEC AND ARDL

Causality	VEC	ARDL	Causality	VEC	ARDL
D ⟶ Y	Negative^{L}	Positive^{S}	Y ⟶ D	Negative^{L}	Negative^{S}
I ⟶ Y		Negative^{L}	Y ⟶ I	Positive^{L}	Positive^{S}
R ⟶ Y		Negative^{L}	Y ⟶ R	Negative^{L}	Positive^{L}
E ⟶ Y	Negative^{L}	Negative^{L}	Y ⟶ E		
Embi+ ⟶ Y		Negative^{L}	Y ⟶ Embi+	Negative^{L}	Positive^{S}
S ⟶ Y	Positive^{L}	Negative^{L}	Y ⟶ S		
I ⟶ D	Positive^{L}		D ⟶ I	Positive^{L}	Negative^{L}
R ⟶ D	Negative^{L}	Negative^{S}	D ⟶ R	Positive^{L}	Positive^{L}
E ⟶ D		Positive^{S}	D ⟶ E	Positive^{L}	Positive^{L}
Embi+ ⟶ D	Positive^{L}	Positive^{S}	D ⟶ Embi+	Positive^{L}	Positive^{L}
S ⟶ D		Negative^{S}	D ⟶ S		

Source: Authors Elaboration.
Note: Overwritten S represents Granger causality only in the short run and overwritten L in the long run. "Y" represents the GDP growth rate; "D" represents the public debt; "I" represents the interest rate; "R" represents the inflation rate; "E" represents the exchange rate; "Embi+" represents the Embi+ and "S" represents the primary surplus.

Another approach that one could takes to explain the difference in the relationship from debt to growth in short and in long run is that fiscal multipliers are not long lasting, therefore they are more likely to prevail in short run than in long run, while in long run crowding-out effects are more likely to prevail. Results of Matheson and Pereira (2016) shows that fiscal multipliers in Brazil are short lived. Moreover, they came to the same conclusion of Gadelha (2011) about the fiscal synchronization, explaining that a surprising increases in government spending in a given quarterly is likely to generate a consolidation in the later on. Therefore, in long run, crowding-out effects are more expected than Keynesian multipliers.

On the other hand, the causality from growth to debt is also negative, meaning that increasing in GDP growth rate causes reduction in debt-to-GDP ratio. Moreover, part of it is related to the impact of growth on Embi+, since growth negatively Granger causes Embi+ in the long run, that in turns, positively Granger causes debt; that is, when the GDP growth rate increases, it decreases Embi+ in the long run, therefore the following reduction on debt is related to growth directly and indirectly.

Primary surplus showed to be completely exogenous in the VEC, in the sense that it was not Granger caused by none of the variables. In ARDL it appeared to be statistically significant when it was dependent only by its lagged value. This may be explained by the findings of Gadelha (2011) who concluded by a synchronization between government revenues and expenditures, that makes primary surplus much more dependent on both variables than in those used in this study. When primary surplus is the independent variable, it decreases debt in the short run; however, it presents opposite results for its relation with the GDP growth rate, which does not allow us to make a reliable statement about this relationship.

Interest rate presented negative long run Granger causality with GDP growth rate only in the ARDL and positive long run Granger causality with debt only in the VEC, the last proposition is in accordance with the findings of Gadelha and Divino (2008). When interest rate is the dependent variable, it showed to be positively Granger caused by GDP growth rate both in the short and long run. However, it is not possible to make any statement about its dependence of debt, since it has presented opposite signals in the VEC and in the ARDL.

5. Conclusion and Policy Implications

The objective of this paper was to understand the relationship between public debt and economic growth in Brazil. Moreover, we aimed to understand the interaction between debt and growth with other variables such as: interest rate, inflation rate, exchange rate and Embi+. To achieve that objective we have estimated growth equations, and performed multivariate and bivariate Granger causality tests, by applying VEC and ARDL methodologies.

We concluded that government debt and growth have a bi-directional relationship, meaning that one variable causes and is caused by each other. Although the presence of causality in both directions, the behaviour is not the same in the short and in the long run. Debt may improve growth in the short run, however, it can be harmful to growth in the long run, not only by its direct relation to the GDP growth rate, but by its indirect impact over inflation rate, exchange rate and Embi+. On the other hand, economic growth reduces debt, both in the short and long run, and there is also an indirect impact of growth in debt by the reduction that growth causes in Embi+.

The important policy implication of this result is that if we can better understand the relation between debt and inflation rate, exchange rate and Embi+, we may also be able to soften the negative impact of debt on growth, by the use of other policies that can impact on this variable.

Another issue that came up is why studies applied to other countries could find a positive long run relationship between debt and growth, as the findings of Gómez-Puig and Sosvilla-Rivero (2015) for Austria, Finland and France. What is the difference between Brazil and other countries that found this positive relation? Maybe the answer is related to what this debt is used for, or else, the negative impact of other variables such as Embi+ is not that strong as it is in Brazil. These questions can be evaluated more deeply by other studies and should be taken into consideration by the policy makers.

In addition, the fact that debt may improve growth in the short run and harm it in the long run also emphasizes the trade-off faced by governments to correctly evaluate if it is time to promote aggregate demand or to implement austerity measures. The actual situation of high debts and economic recession also made the decision harder, since some austerity measures can deteriorate growth in the short run. Therefore, evaluating the quality of public expenditures, namely its efficiency and effectiveness could be the better way to help the decision about in what policy or program to adopt austerity or expansionary measures.

Further, bills such as the PEC 187/2019 that propose the use of resources of public funds to pay down debts seems to be a more efficient way to use this feature, since part of it has not been used and at the same time government pays interest on its debt. Similarly, the use of part of Dollar reserves of the Central Bank to pay down public debt may be a good solution, since the current exchange rate depreciation increased a lot the Central Bank´s profit on its reserves for much more than what is necessary to face possible future losses. Furthermore, measures such as the use of part of the amount saved by austerity measures in infrastructure projects, presented in the PEC 188/2019 may also be growth inducing and reduce public debt.

Finally, we have not included external public debt in our analysis because since October 2006 the Brazilian net external public debt is negative. This occurs because of the effort made by the government to increase dollars reserves, which decreases the vulnerability related to exchange rate depreciations. However, we think that further work could evaluate the interaction between external debt, dollar reserves, exchange rate and Embi+. Moreover, we think that the impact from Embi+ in external debt may be much stronger than in total debt.

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